Title: 1/2-BPS line defects in 4d N = 2 SQFTs via Cohomological Hall algebras

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Abstract:

After a brief reminder/introduction of relevant facts about 4d N=2 SQFTs (BPS things, twisting, wall-crossing, line defects) I will try to sketch our proposal for the category of 1/2-BPS line defects in terms of certain bimodules over Cohomological Hall algebra of the BPS quiver; maybe even say a few words about why we believe it is a correct thing to do.

1/2-BPS line defects in 4d N = 2 SQFTs via Cohomological Hall algebras

Nikita Grygoryev

Based on the joint work with
Davide Gaiotto and Wei Li
2406.07134.

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Main Claim: The category of $\frac{1}{2}$-BPS line defects $\mathbb{L}$ in a 4d $\mathcal{N}=2$ SQFT $\mathcal{T}$ is described by a full monoidal subcategory of

$\mathcal{H}_Q$-bimod$(\mathcal{C}^{IR})$ generated by $\{\mathcal{B}_f \mid$ for all framings f of $Q\}$

where:

Q – BPS quiver of $\mathcal{T}$ $\quad \mathcal{C}^{IR}$ – line defects in IR

$\mathcal{H}_Q$ – Cohomological Hall algebra

some explicit monoidal category

some associative algebra in $\mathcal{C}^{IR}$

$\mathcal{B}_f$ – explicit $\mathcal{H}_Q$-bimodule assigned to the framing Q^f

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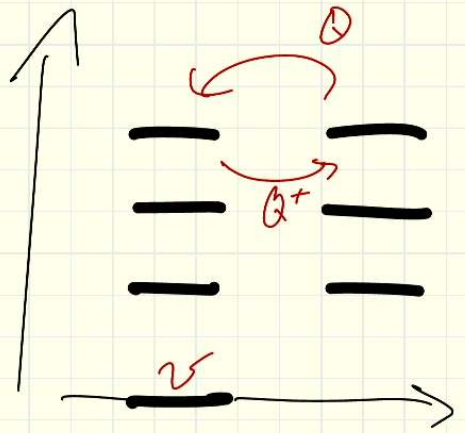
0. BPS things: $\mathcal{N}=2$ SQM example

$$H = \{Q, Q^\dagger\}$$

$$I := \mathrm{tr}_{\mathcal{H}} (-)^F e^{-\beta H}$$

$$= \mathrm{tr}_{\mathcal{H}^{BPS}} (-)^F e^{-\beta H} = 1$$

$$\mathcal{H}^{BPS} := \{\text{Cohomology of } \mathcal{H} \text{ w/ respect to } Q\} = \frac{\ker Q}{\mathrm{Im}\, Q} = \mathbb{C}[v]$$



$$Qv = Q^\dagger v = 0$$

BPS condition

Note: $H\big|_{\mathcal{H}^{BPS}} = 0$ ← topological time direction

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1. BPS things in SQFTs

T — a 4d $\mathcal{N}=2$ SQFT

e.g.:
- SYM w/ group G & matter rep V
- Argyres-Douglas (A_n, A_m)

Claim: BPS *things* are easy to study

Why? Twists! • Choose a supercharge $Q^2 = 0$

• Consider a new theory $T \rightsquigarrow T_Q$ s.t.

1. things (T_Q) "=" Q-BPS things (T)
2. T_Q has a lot of topological / holomorphic directions

For me: • Q s.t. T_Q is $\mathbb{R}^2 \times \mathbb{C}$ (topological × holomorphic)

• things = $\frac{1}{2}$-BPS line defects @ $\mathbb{R}\times\{0\}\times\{z\} \subset \mathbb{R}^2\times\mathbb{C}$

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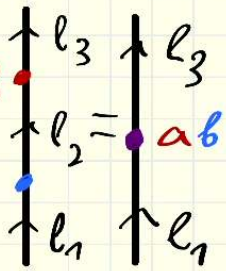
2. Lines in 4d HT theories (e.g. $\mathcal{T} = \mathcal{T}_{\mathbb{Q}}$)

1. Line($\mathcal{T}$) is a category

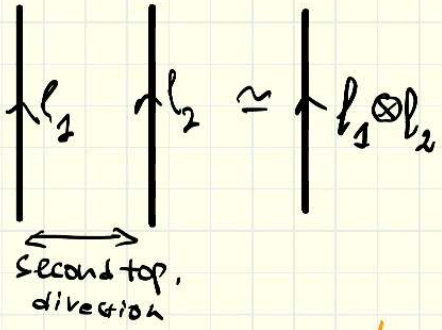
$b \in \mathrm{Hom}(\ell_1, \ell_3)$, $a \in \mathrm{Hom}(\ell_2, \ell_3)$



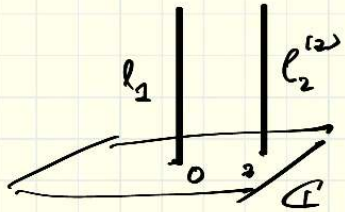
$\mathrm{Obj} = \{$ lines along fixed line in top. direction $\}$

$\mathrm{Hom}(\ell_1, \ell_2) = \{$ top. local operator at a junction of $\ell_1 \& \ell_2 \}$

2. Line($\mathcal{T}$) is a monoidal category



3. Line($\mathcal{T}$) has a system of renormalized r-matrices



$R(z): \ell_1 \otimes \ell_2^{(z)} \xrightarrow{\sim} \ell_2^{(z)} \otimes \ell_1$

$R(z) = z^{\Lambda(\ell_1, \ell_2)} \left(r_{1,2} + O(z) \right)$

Not iso in general!

$r: \ell_1 \otimes \ell_2 \to \ell_2 \otimes \ell_1$

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3. BPS Quiver

Claim: For many 4d $\mathcal{N}=2$ theories the spectrum of BPS particles is recovered from a quiver Q

- IR charge lattice $\Gamma = \mathbb{Z}^{Q_0}$
- Dirac pairing $\langle \gamma, \gamma' \rangle = \sum_{i,j} \left(|i \to j| - |j \to i| \right) \gamma_i \gamma'_j$

Def: Quantum torus: $X_\gamma X_{\gamma'} = q^{\langle \gamma, \gamma' \rangle} X_{\gamma+\gamma'}$

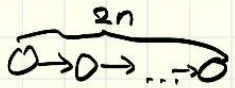
Def: DT-invariant of Q

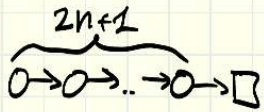
$$DT(Q) = \sum_{\gamma \in \Gamma^+} \frac{q^{\chi_E(\gamma,\gamma)}}{\prod_{i \in Q_0} (q^2;q^2)_{\gamma_i}} X_\gamma$$

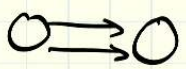
Def: Quantum dilogarithm

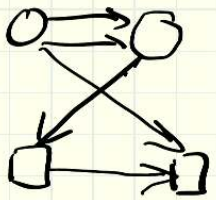
$$\Phi(X_{\gamma_i}) \equiv \prod_{m=0}^{\infty} (1 + X_{\gamma_i} q^{2m+1}) = DT(A_1)$$

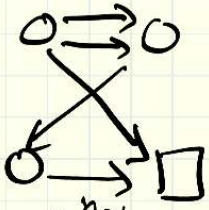
e.g.

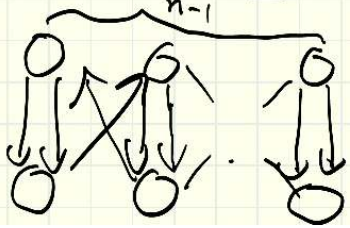
$AD(A_1, A_{2n})$ 

$AD(A_1, A_{2n+1})$ 

$SU(2)$ YM 

$U(2)$ YM 

$SU(2)$ $N_f = 4$ 

$SU(n)$ YM 

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E.g. $Q = A_2$ $\quad 0 \to 0$ $\quad AP(A_1, A_2)$

$$DT(A_2) = \underbrace{\Phi(X_{\gamma_2})\,\Phi(X_{\gamma_1})}_{\text{2 particles}} = \underbrace{\Phi(X_{\gamma_1})\,\Phi(X_{\gamma_1+\gamma_2})\,\Phi(X_{\gamma_2})}_{\text{3 particles}}$$

Claim: For a given theory, DT is recovered from how it conjugates cluster functions = holomorphic functions on CB on $S^1 \times \mathbb{R}^3$ =

vevs of lines on S^1 = $K_0(\text{Line}_J)$

Quest: Find Line_J that categorifies cluster algebra A_Q

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6. Example: $AD(A_1, A_2)$ $Q = A_2$

- Characters are cluster variables

$X_{-\gamma_1}$,	$B_0 \simeq b_{-\gamma_1}\mathcal{H}$
X_{γ_2},	$B_1 \simeq b_{\gamma_2}\mathcal{H}$
$X_{\gamma_1} + X_{\gamma_1+\gamma_2}$,	$B_2 \simeq b_{\gamma_1}\mathcal{H} \oplus e_0^2 b_{\gamma_1}\mathcal{H}$
$X_{-\gamma_2} + X_{\gamma_1-\gamma_2} + X_{\gamma_1}$,	$B_3 \simeq b_{-\gamma_2}\mathcal{H} \oplus e_0^1 b_{-\gamma_2}\mathcal{H} \oplus e_0^2 e_0^1 b_{-\gamma_2}\mathcal{H}$
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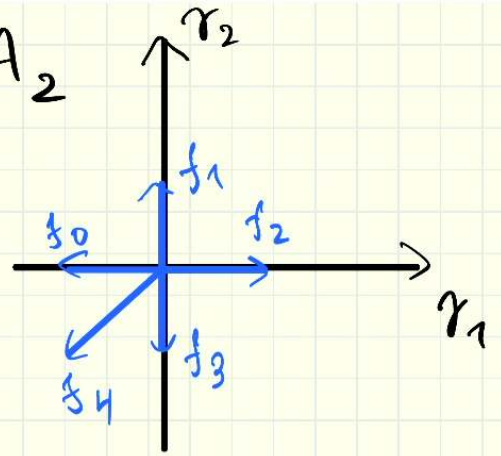

- Isos & Seses that become mutations in K_0

$$B_a \otimes B_{a+1} \simeq \mathbf{q}^{-2} B_{a+1} \otimes B_a$$

$$0 \to \mathbf{q} B_a \to B_{a+1} \otimes B_{a-1} \to \mathcal{H} \to 0$$

$$0 \to \mathcal{H} \to B_{a-1} \otimes B_{a+1} \to \mathbf{q}^{-1} B_a \to 0$$

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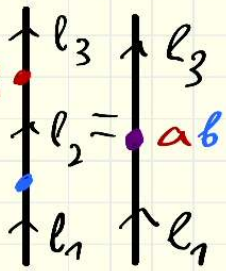

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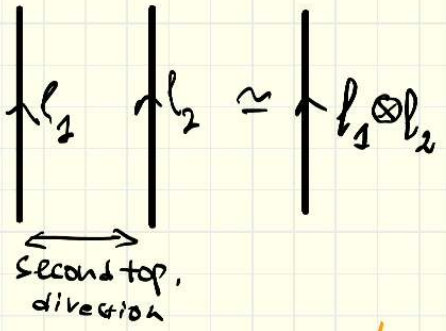

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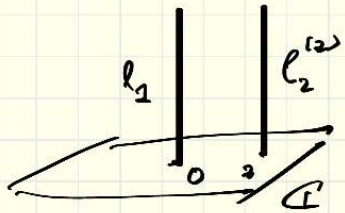

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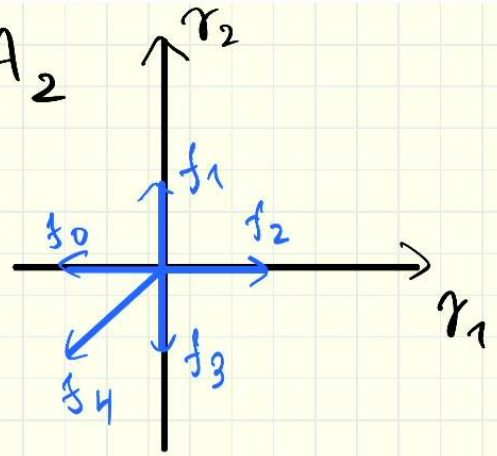

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