Title: Factorization Algebras in Quantum Field Theory

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Abstract:

I will introduce factorization algebras as an approach to understanding quantum field theory. I will outline some of the benefits of this approach, as well as how it connects with other topics in mathematical physics.

Classical Picture

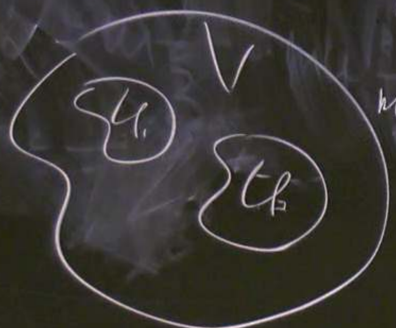
Classical field thy
= solve PDE on M

Observable = $\mathcal{O}$(sol'ns to my PDE)

$U \subseteq M$ $\quad \mathcal{O}(\text{Sol'ns}(U)) =: \mathcal{F}(U)$

$U \subseteq V$ $\quad \mathcal{F}(U) \longrightarrow \mathcal{F}(V)$

$$A \longmapsto A(f|_U)$$



$$m: \mathcal{F}(U_1) \otimes \mathcal{F}(U_2) \longrightarrow \mathcal{F}(V)$$

Truly local ops

$$\lim_{r \to 0} \mathcal{F}(D(x,r))$$

States: $\langle - \rangle : \mathcal{F}(M) \to \mathbb{C}$

Correlation fns $\quad \langle m(\mathcal{O}_1, \dots, \mathcal{O}_n) \rangle$

Mechanics

Quantization: deformation quant.

$M \rightsquigarrow T^*M \rightsquigarrow C^\infty(T^*M) = A$ Poisson alg

comm. product

$\{-,-\}$

$\left(A[[\hbar]], [-,-]\right)$ $\quad [a,b] = \hbar\{a,b\} + O(\hbar^2)$

Ex: $\phi \in C^\infty(M)$

$$S(\phi) = \int_M \phi(\Delta + m^2)\phi$$

$$\mathrm{Sol'n}(U) = \{(\Delta + m^2)\phi = 0\}$$

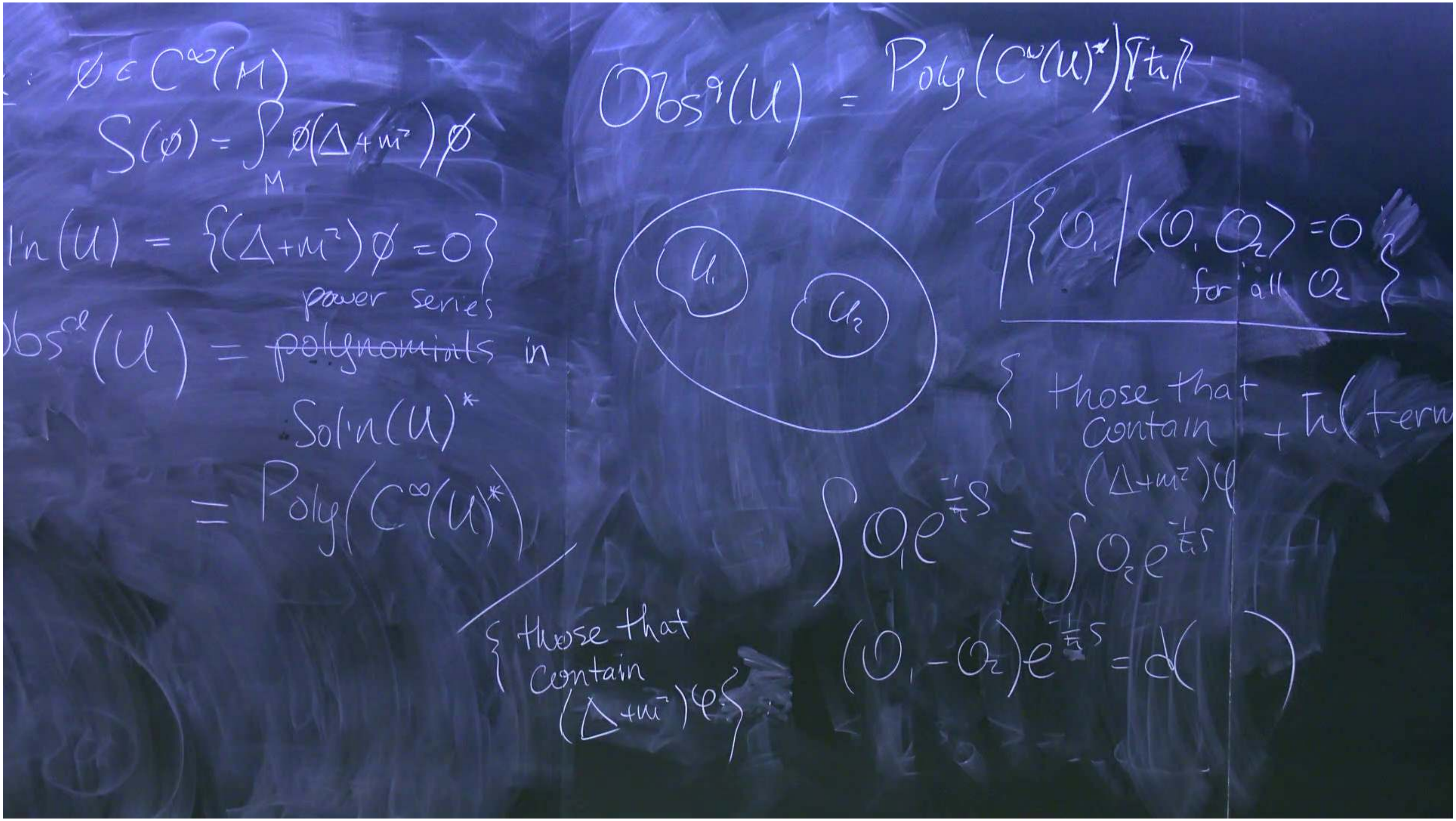
$$\mathrm{Obs}^{cl}(U) = \text{~~polynomials~~ power series in } \mathrm{Sol'n}(U)^*$$

$$= \mathrm{Poly}(C^\infty(U)^*) \Big/ \{\text{those that contain } (\Delta + m^2)\varphi\}$$

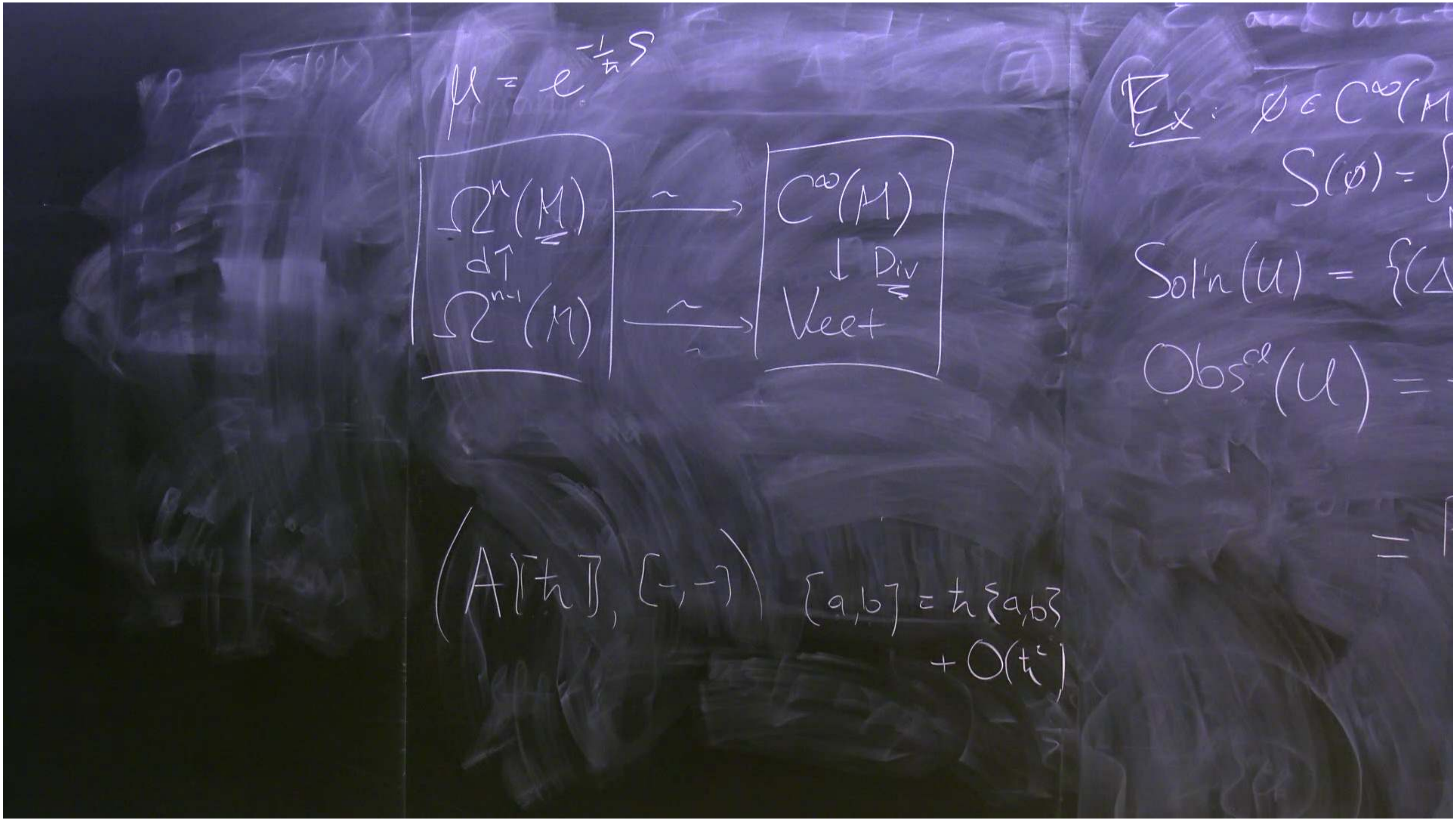
Truly

States

Correlat fns



$\phi \in C^\infty(M)$
$S(\phi) = \int_M \phi(\Delta + m^2)\phi$
$\mathrm{Sol'n}(U) = \{(\Delta + m^2)\phi = 0\}$
$\mathrm{Obs}^{cl}(U) =$ ~~polynomials~~ power series in $\mathrm{Sol'n}(U)^*$
$= \mathrm{Poly}(C^\infty(U)^*)$
{ those that contain $(\Delta + m^2)\varphi$ }
$\mathrm{Obs}^q(U) = \mathrm{Poly}(C^\infty(U)^*)[[\hbar]]$
U_1
U_2
$\{ O_1 \mid \langle O_1 O_2 \rangle = 0$ for all $O_2 \}$
{ those that contain $(\Delta + m^2)\varphi$ + $\hbar$(term
$\int O_1 e^{-\frac{1}{\hbar}S} = \int O_2 e^{-\frac{1}{\hbar}S}$
$(O_1 - O_2) e^{-\frac{1}{\hbar}S} = d(\quad)$


$\mu = e^{-\frac{1}{\hbar}S}$
$\Omega^n(M) \xrightarrow{\sim} C^\infty(M)$
$d\uparrow \quad \downarrow \mathrm{Div}$
$\Omega^{n-1}(M) \xrightarrow{\sim} \mathrm{Vect}$
$(A[[\hbar]], [-,-])$
$[a,b] = \hbar\{a,b\} + O(\hbar^2)$
Ex: $\phi \in C^\infty(M$
$S(\phi) =$
Sol'n$(U) = \{($
$\mathrm{Obs}^{cl}(U) =$