Title: Classification of non-Fermi liquids and universal superconducting fluctuations

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Abstract:

Critical fluctuations coupled to Fermi surfaces can destroy conventional Fermi liquids, giving rise to diverse non-Fermi liquids that may remain metallic or become superconducting at zero temperature. I will present a classification of non-Fermi liquids with globally convex hot Fermi surfaces based on the concept of projective fixed points, which account for the flow of the Fermi momentum under renormalization. This framework organizes non-Fermi liquids into seven super-universality classes, each characterized by distinct superconducting fluctuations. Stable non-Fermi liquids exhibit universal but non-scale-invariant pairing interactions, while unstable ones display universal constraints on pairing symmetry, superconducting transition temperature, and its scaling with the Fermi momentum. I will discuss physical examples and a toy model that captures the essential universal properties of these classes.

Classification of non-Fermi liquids and universal superconducting fluctuations

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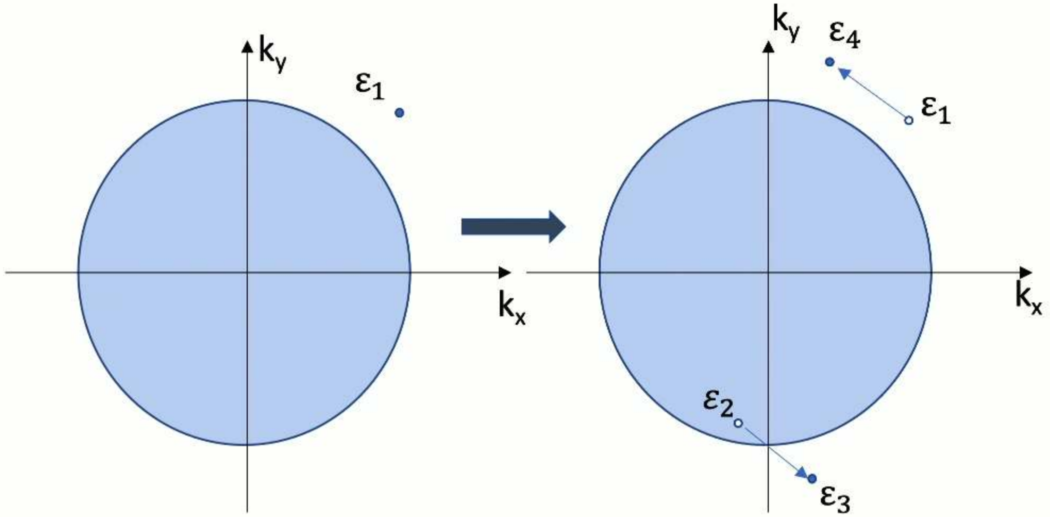
Introduction

Ordinary Metals and Fermi Liquids

- Ground state of non-interacting metals- Fermi sea
- Many body eigen state – single electron levels
- [1-3] showed FS –well defined in presence of strong interactions
- Common transport properties- $C_v \propto T, \rho \propto T^2$

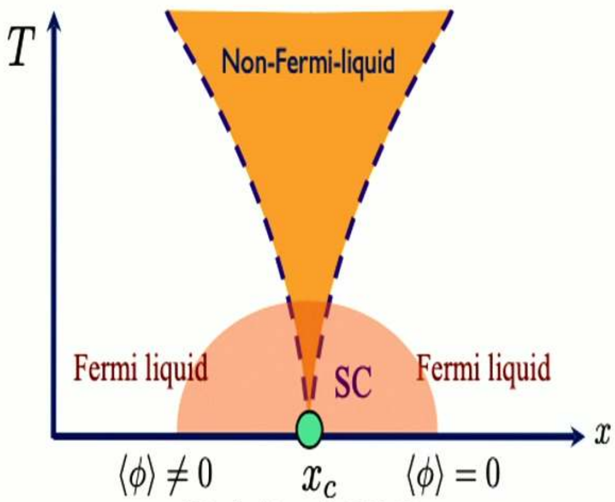


Lifetime: $\frac{1}{\tau} = \alpha V \delta^2$

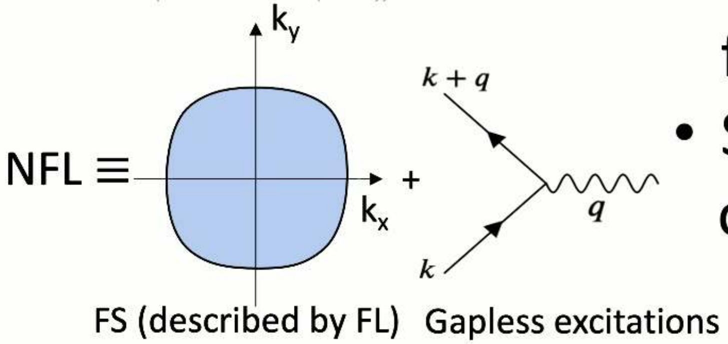
δ - $\varepsilon_1 - \varepsilon_f$
α - kinematic constants
V – microscopic interaction

[1] Landau, L. D. (1957), [2] Shankar, R. (1994), [3] Polchinski, J. (1992)

Non-Fermi liquids



(Metlitski et al. (2014))



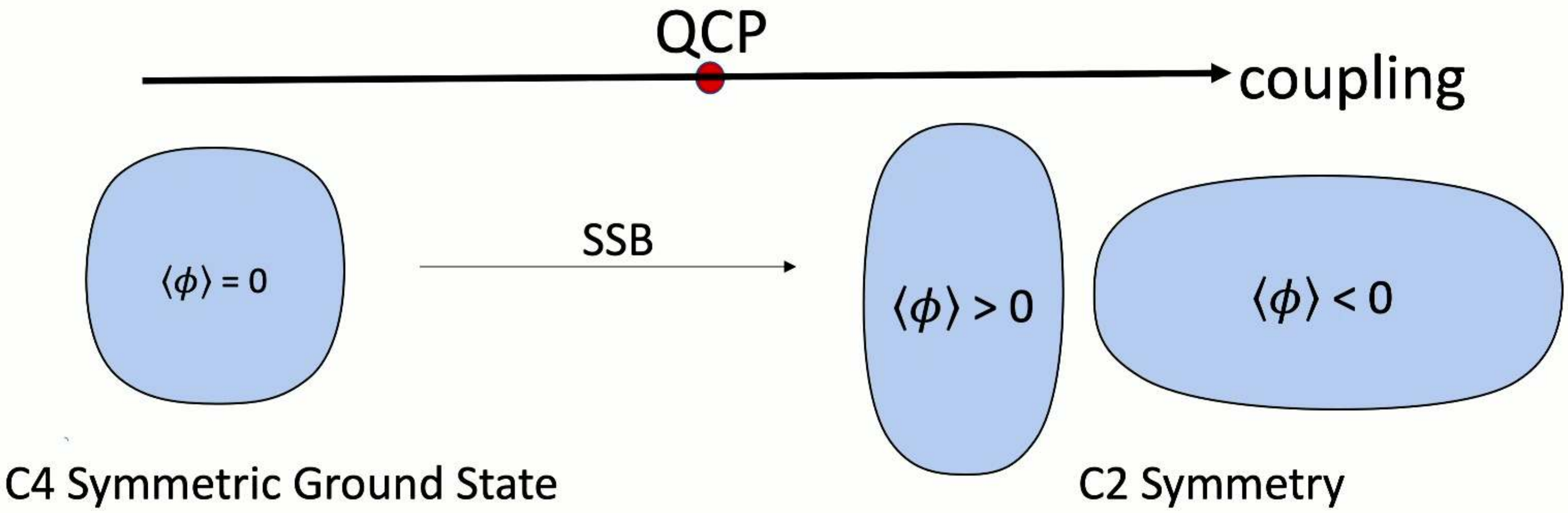
- Near QCP NFL behaviour characterized by $C_v \propto T\log\left(\frac{1}{T}\right)$, $T^n\ (n \neq 1)$, $\rho \propto T^n\ (n < 2)$
- Examples- (heavy fermion compounds, cuprates and pnictides)

- At QCP, $\xi \to \infty$, long- range (gapless) interactions mediate the scattering of fermions
- Singular interactions in energy space offset the lifetime of single particle excitation

$$\frac{1}{\tau} = \alpha[V(\delta)]^2\delta^{\,2} > \delta$$

NFL – Hot Fermi Surfaces

1. Quantum critical non-Fermi liquids – Ising-nematic



2. Gauge field non-Fermi liquids – Composite Fermi liquids (HLR), Spinon Fermi surface

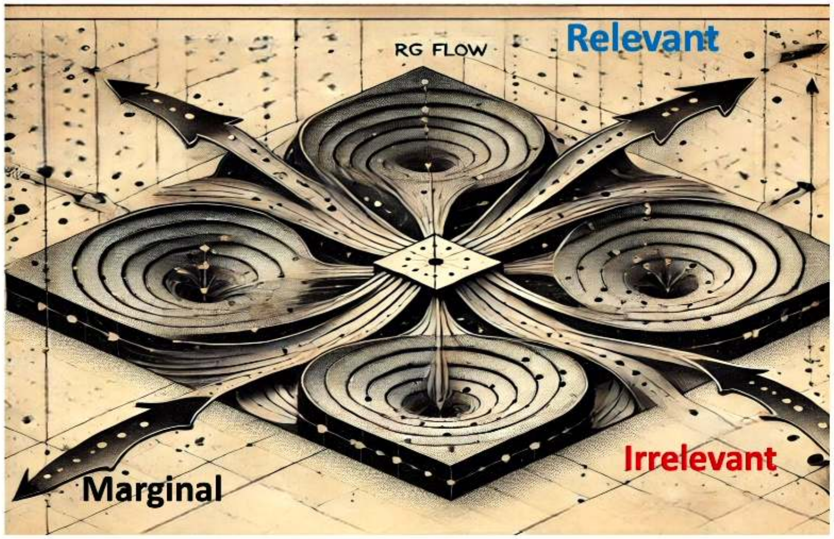
Goal

Low-energy description

- Systematic framework -> RG – to identify IR fixed points of a theory -> characterizes different phases of matter and universality classes

- Express low-energy observables – {marginal, relevant} parameters

[Wilson, Kadanoff]



Picture describing a critical manifold

Complexities in Metals

- Patch theory – Expansion around a point in the momentum space - small part of low energy electrons and gapless modes *[Metlitski, Sachdev ; Dalidovich, SSL]*

- Presence of Fermi surface facilitates -> extensive low-energy fermions and large number of gapless degrees of freedom (for example in NFL)

- For example, SC fluctuations – captured by four fermion interaction, scattering of zero momentum pair of electrons around Fermi surface

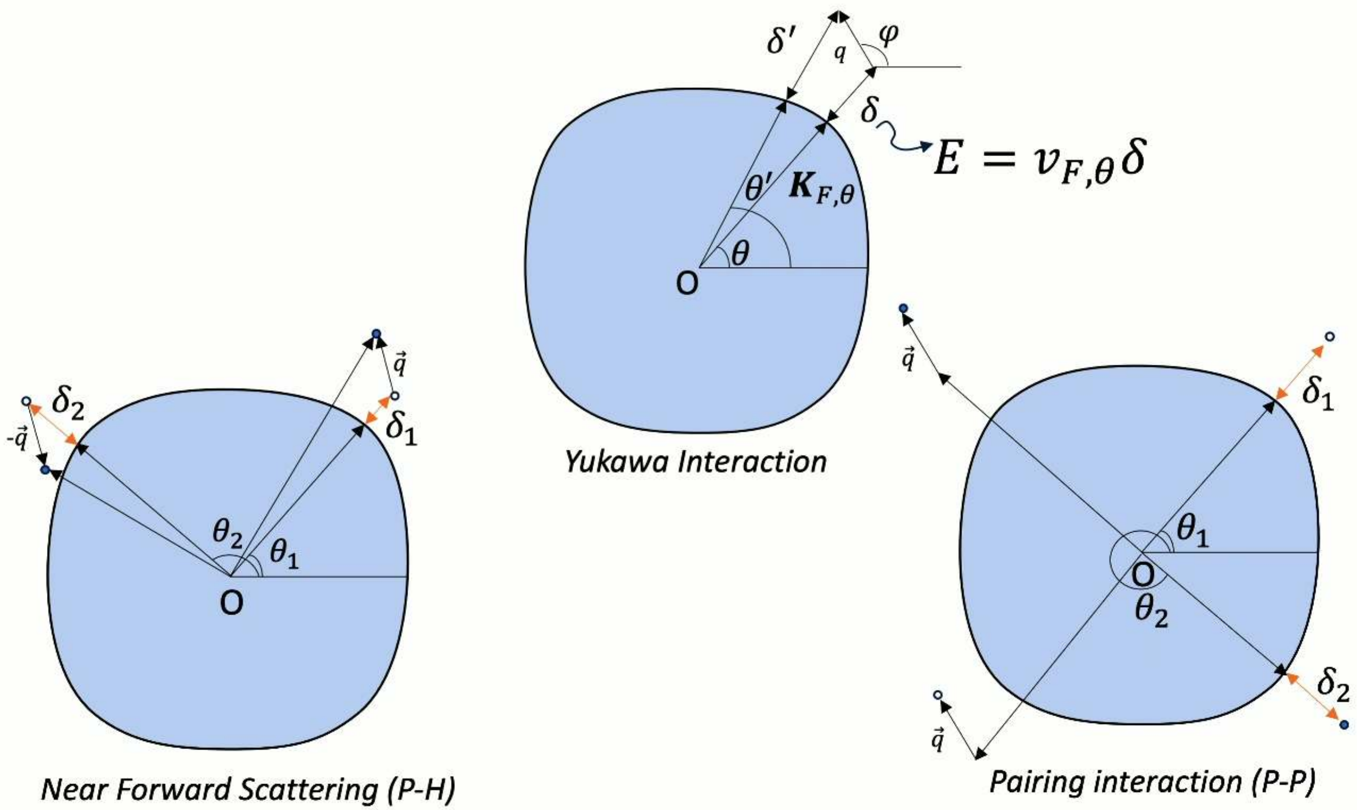
- parameters of theory – no longer finite dimensional -> couplings enlarged to coupling functions [*Borges et al.*]

- **Minimal** framework needed -> Field theoretic functional RG that keeps **all low energy data in our theory**

Setup

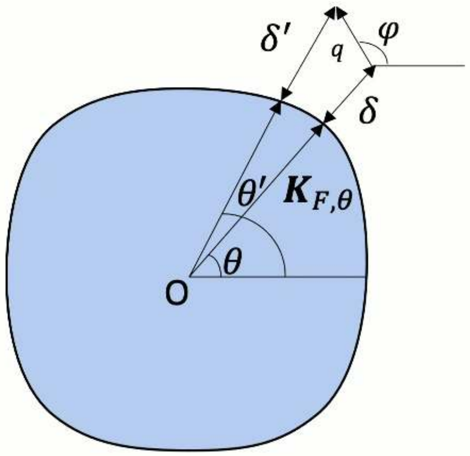
Parameters of the theory

Full low-energy field theory specified by $\{\boldsymbol{K}_{F,\theta}, v_{F,\theta}, e_{\theta',\theta}, \lambda_{\theta_1,\theta_2}(\vec{q})\}$



Projective fixed points (PFP)

Projective nature of fixed points



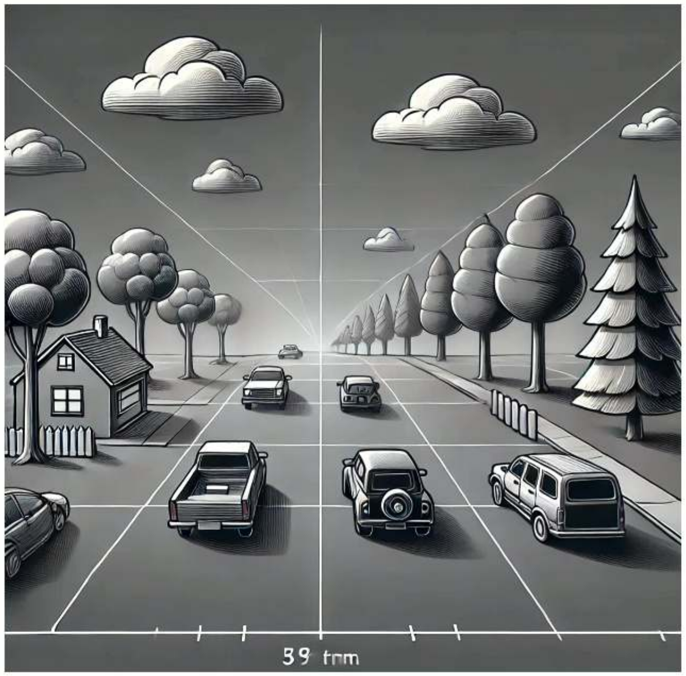
$$\Delta\theta \sim q/\boldsymbol{K}_{F,\theta}$$

Under the scale transformation (b>1)

$$q \to q/b$$

$$\boldsymbol{K}_{F,\theta} \to b\boldsymbol{K}_{F,\theta}$$

Projective nature of fixed points

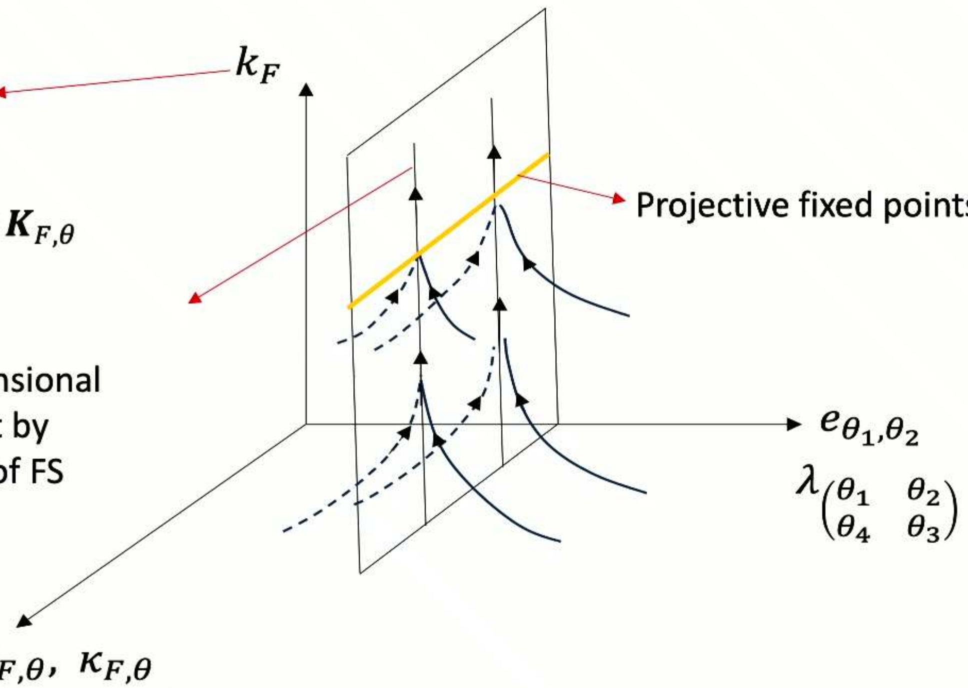


Perceived size of the object – dimensionless angular width measured in units of proximity to object

$$k_F = \boldsymbol{k}_F/\mu$$

$$\boldsymbol{k}_F = \frac{1}{2\pi}\int d\theta\, \boldsymbol{K}_{F,\theta}$$

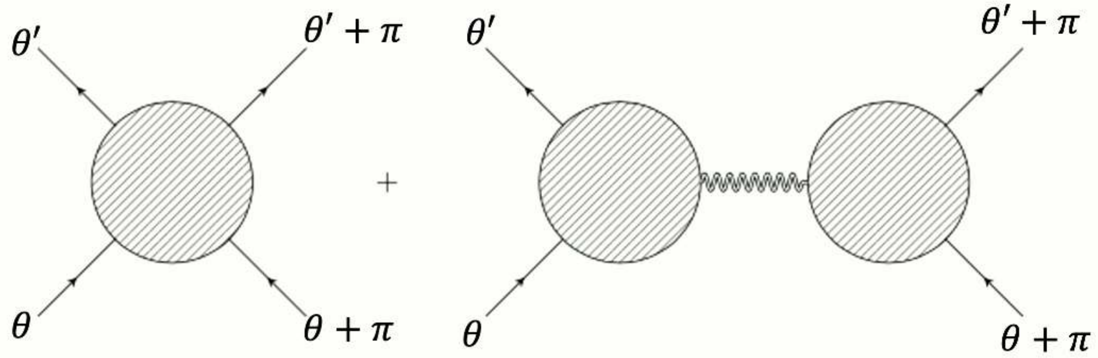
A one-dimensional manifold set by overall size of FS



$\kappa_{F,\theta} = \boldsymbol{K}_{F,\theta}/\boldsymbol{k}_F$ Shape function (marginal)

Effective two body interaction

$V'_{\theta,\theta'}$ =

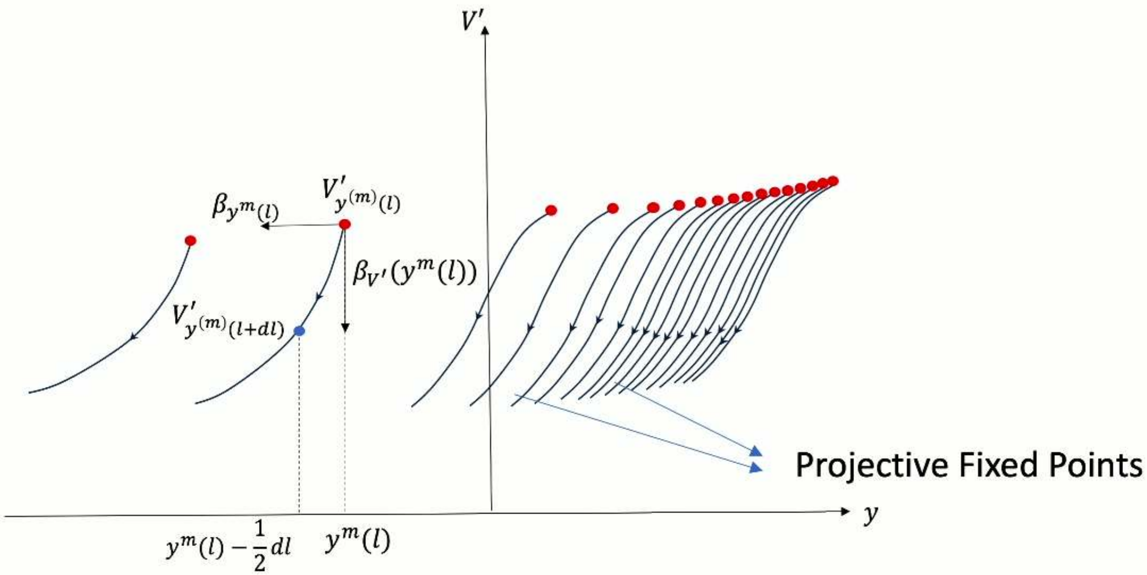


1PI 4-point function

Two 1PI 3-point function connected with boson propagator

RG Equation

Vector field equation → $\frac{\mathrm{d}}{dl}(y, V') = \left(-\frac{1}{2}, \beta_{V'}(y)\right)$ $\qquad l = \log\frac{\Lambda}{\mu}$



RG Equation

Vector field equation → $\frac{\mathrm{d}}{dl}(y, V') = \left(-\frac{1}{2}, \beta_{V'}(y)\right)$ $\quad l = \log\frac{\Lambda}{\mu}$

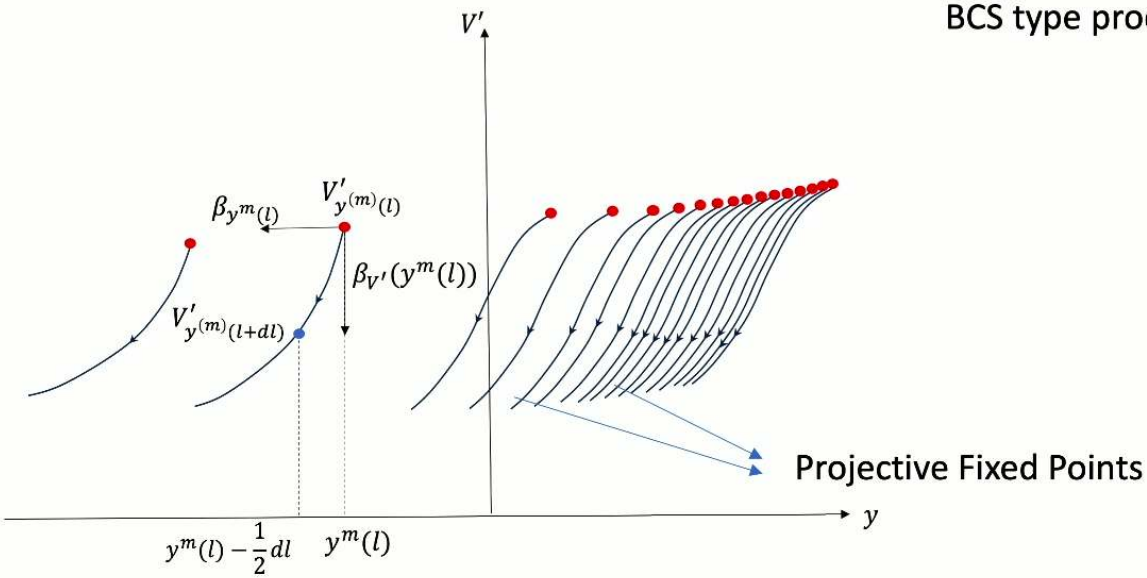
horizontal velocity

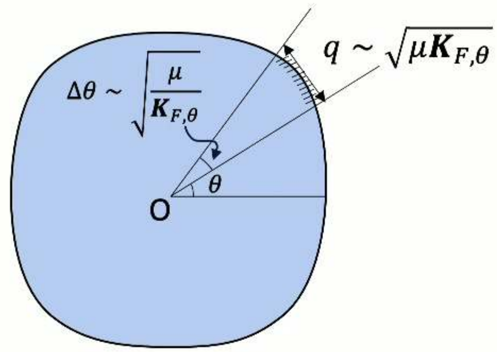
vertical velocity: $\beta_{V'}(y) = -R_d {V'_y}^2 + \left(\frac{1}{2} - \mathbf{H}_d\right) V'_y + S'_y$

$\mathbf{H}_d(\Delta, z, \eta_\psi)$

BCS type processes

Interaction generated from gapless boson





Classification

PFP Equation

$$\frac{\mathrm{d}}{dl}(y, V') = \left(-\frac{1}{2}, \beta_{V'}(y)\right) \quad \rightarrow \quad \frac{1}{2}\partial_y {V'_y}^{*} + \beta_{V'^*}(y) = 0$$

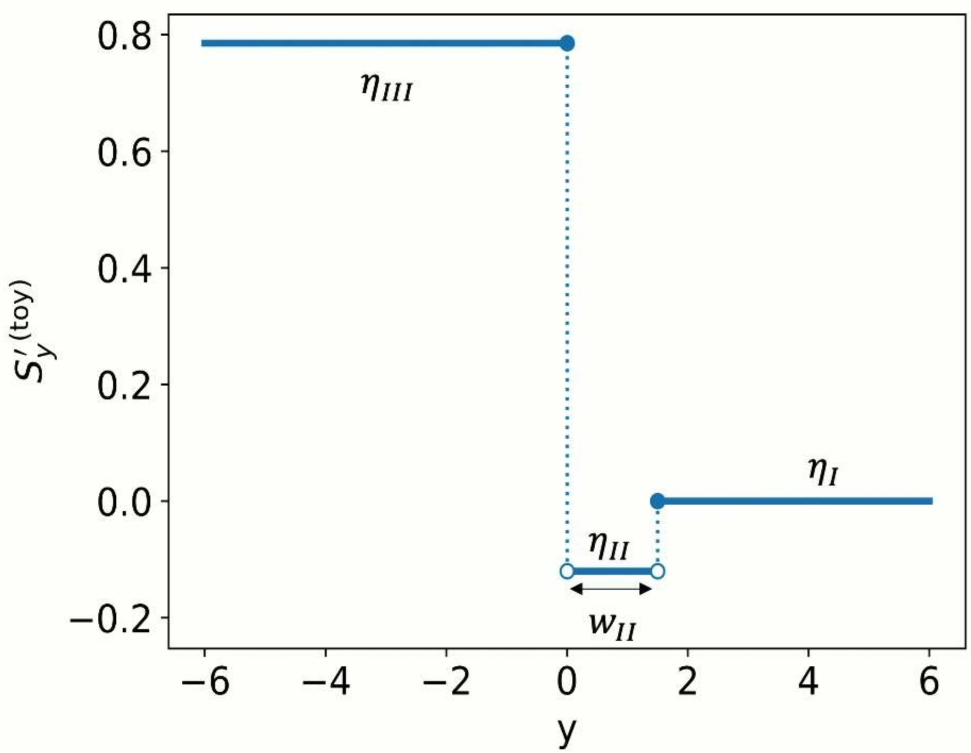
Flow equation

Fixed Point Equation

Solutions with different topologies – distinct super-universality classes

Each super-universality class consists of different types of nFL
that can be accessed through the choice of bare four-fermion coupling

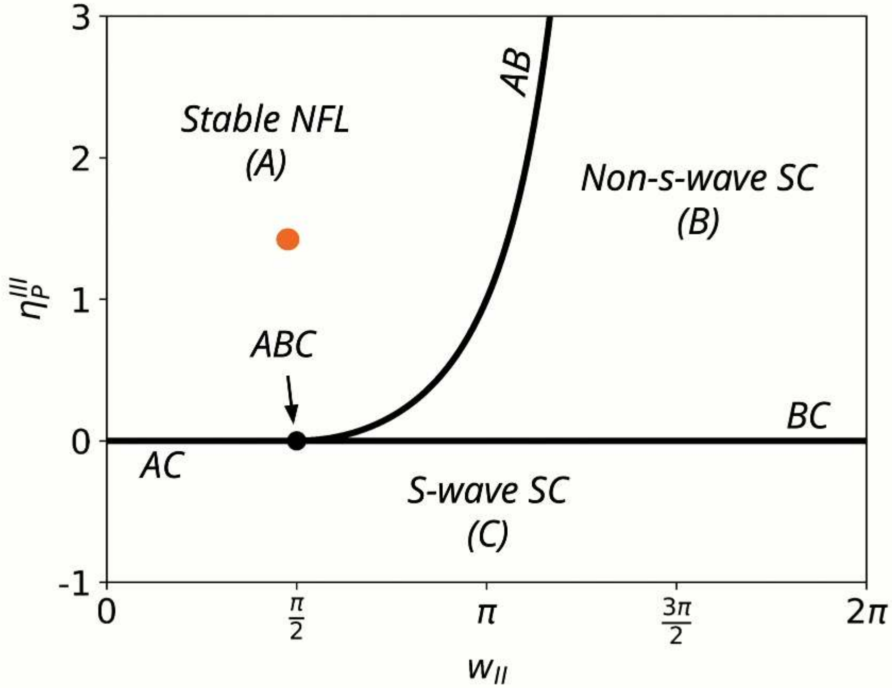
Toy model



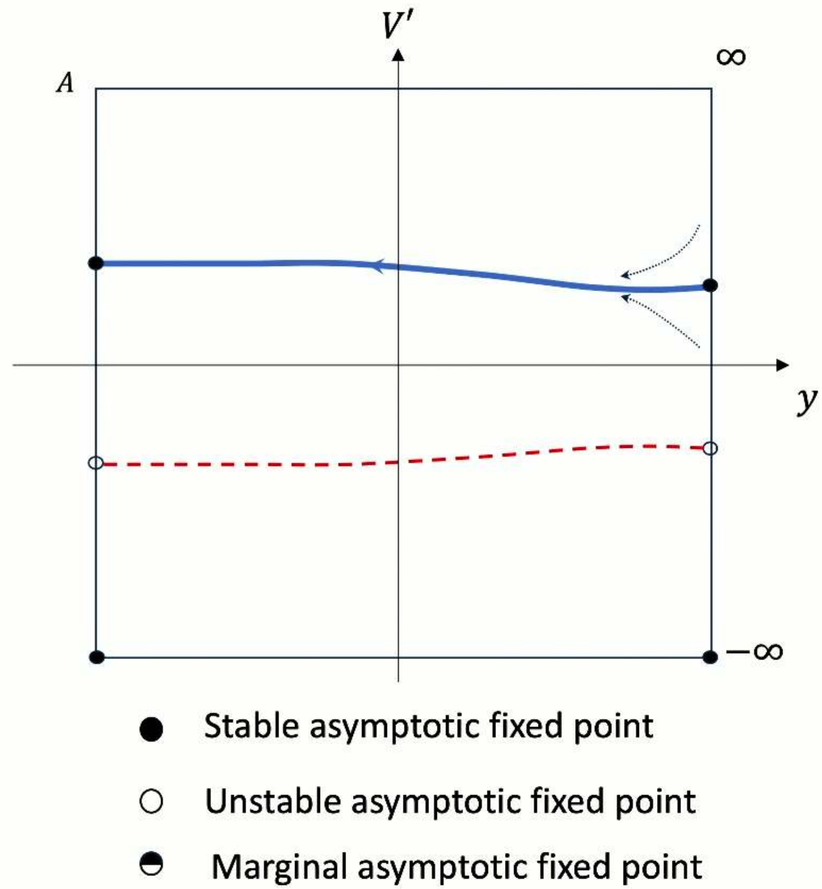
$$\eta_{P,y} = (2\mathbf{H}_d - 1)^2 + 16 R_d S_y'$$

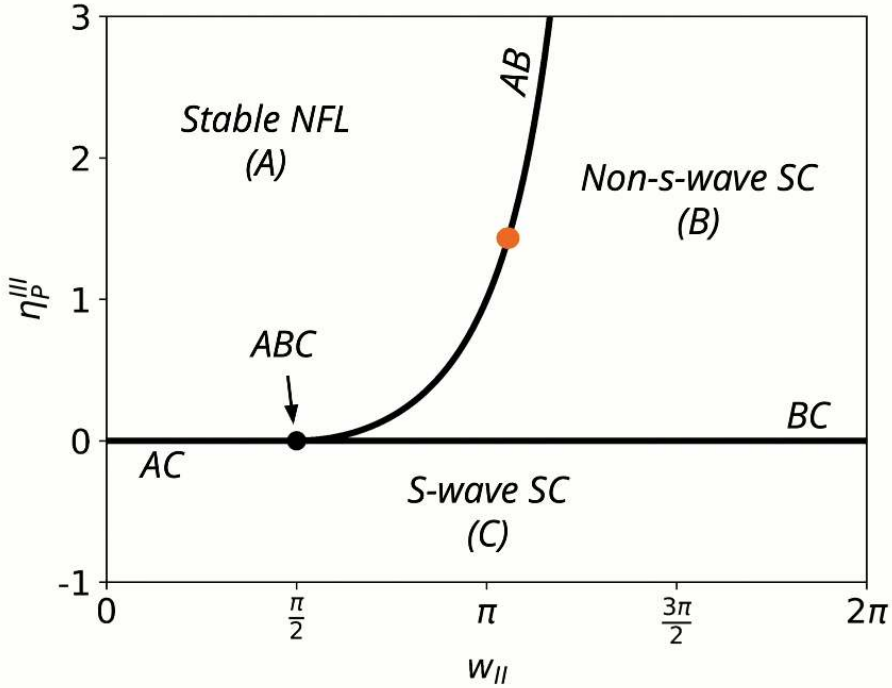
- η_I fixed by $\mathbf{H}_d$
- Tuning parameters

$$\eta_{II}, \eta_{III}, w_{II}$$



Phase diagram of distinct universality classes with
$\boldsymbol{H}_d = 1, \eta_{II} = -1$





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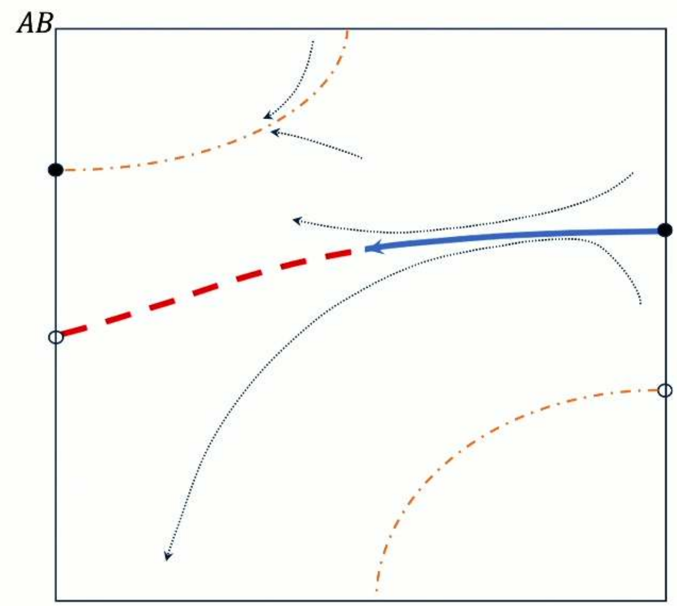
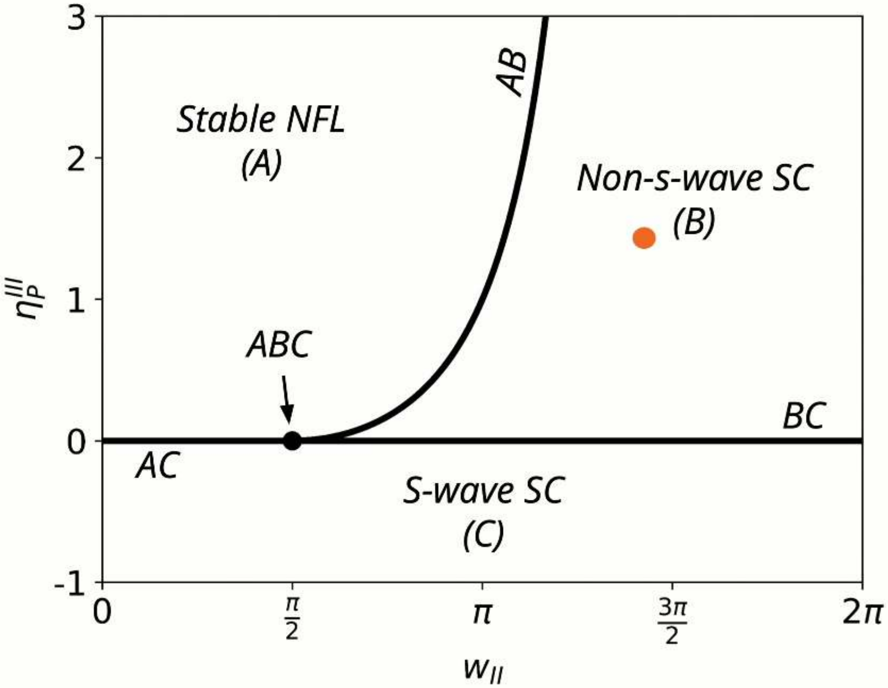
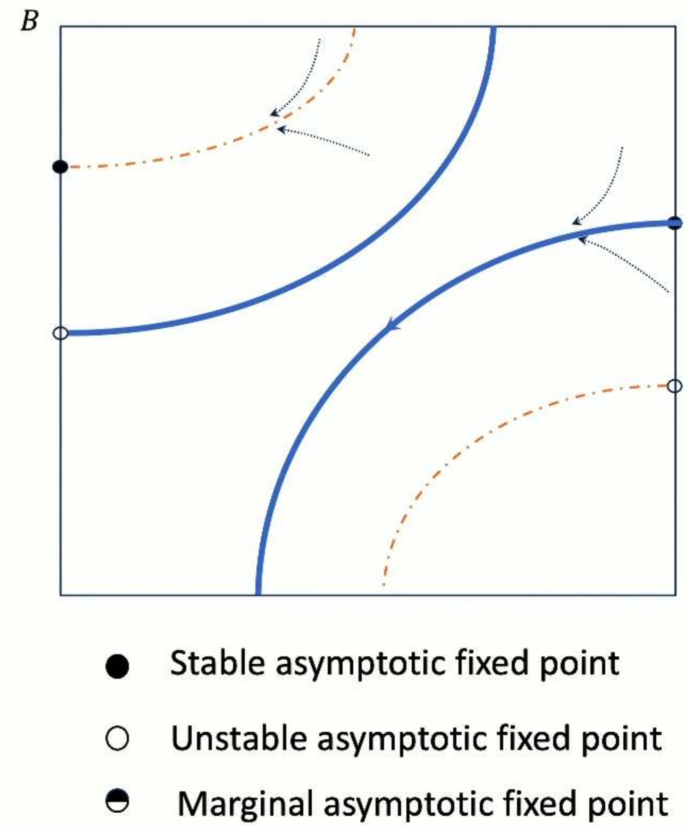
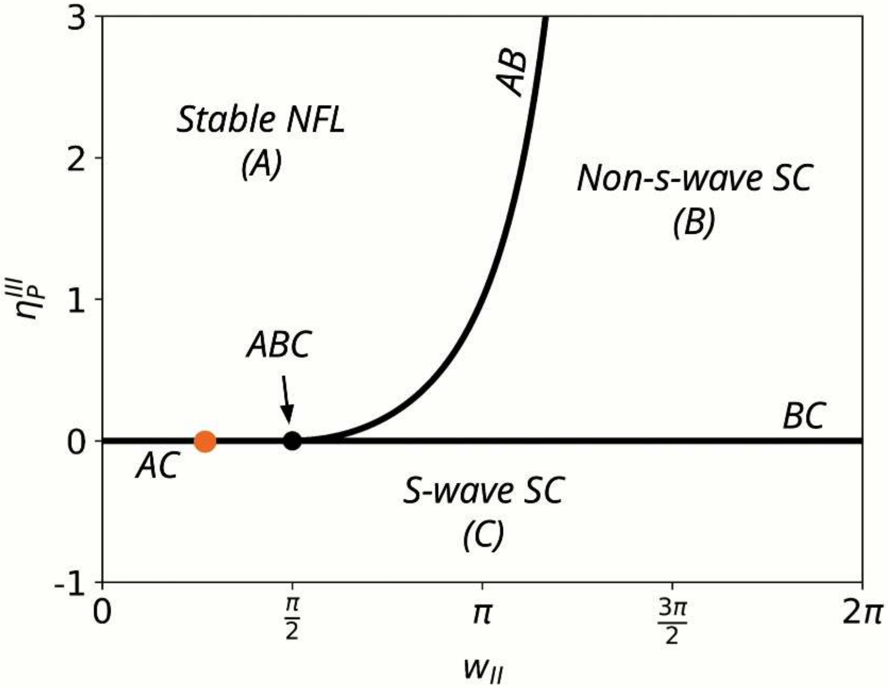


- Stable asymptotic fixed point
- Unstable asymptotic fixed point
- Marginal asymptotic fixed point



Phase diagram of distinct universality classes with $\boldsymbol{H}_d = 1, \eta_{II} = -1$





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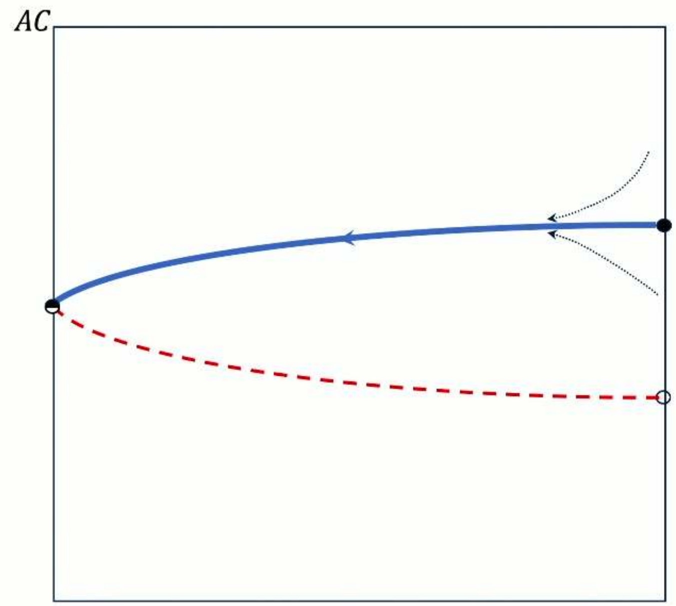

- Stable asymptotic fixed point
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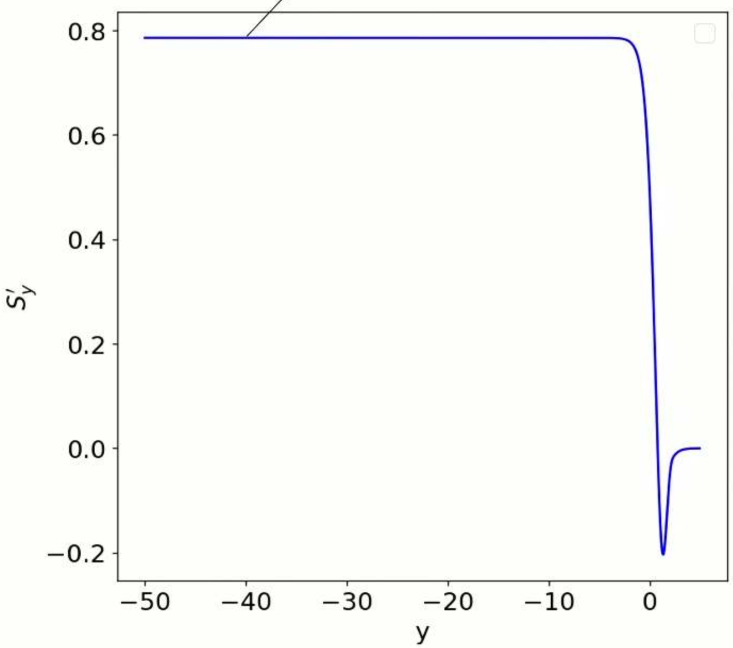
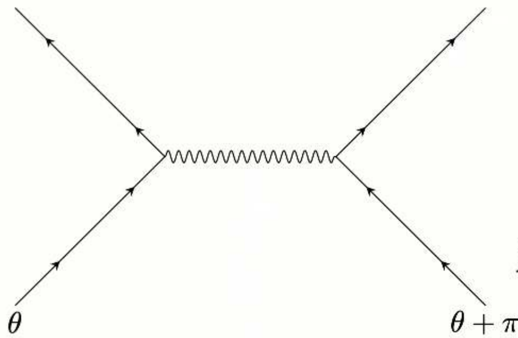
- Marginal asymptotic fixed point

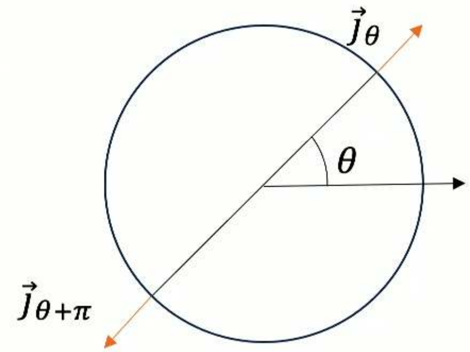
U(1) gauge field

Repulsive in s-wave channel

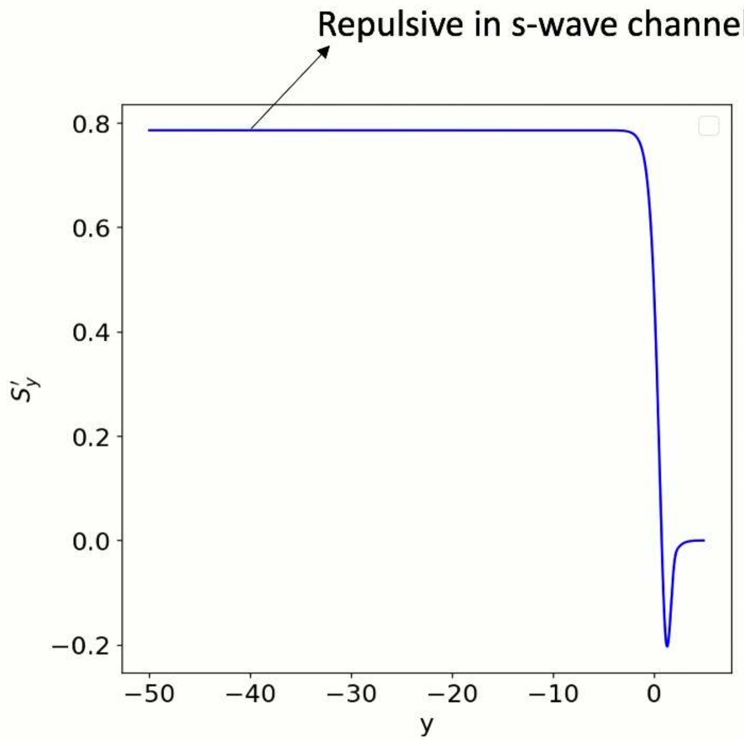


$$S_{int} \sim -\vec{J}_\theta \cdot \vec{J}_{\theta+\pi} \psi^\dagger_\theta \psi_\theta \psi^\dagger_{\theta+\pi} \psi_{\theta+\pi}$$





U(1) gauge field



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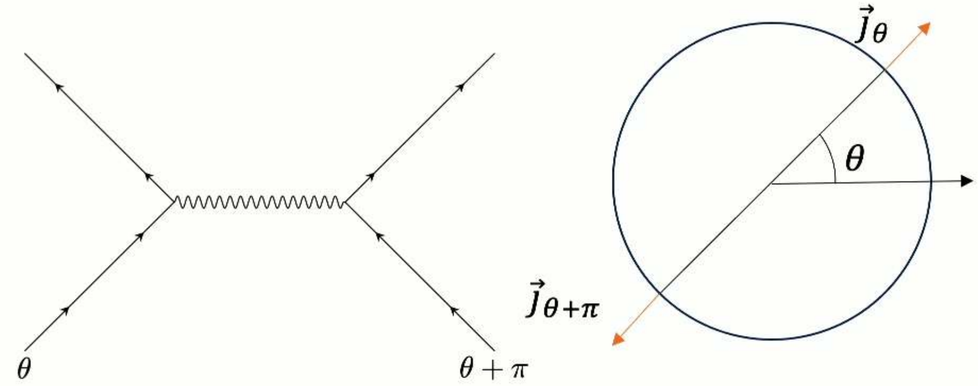
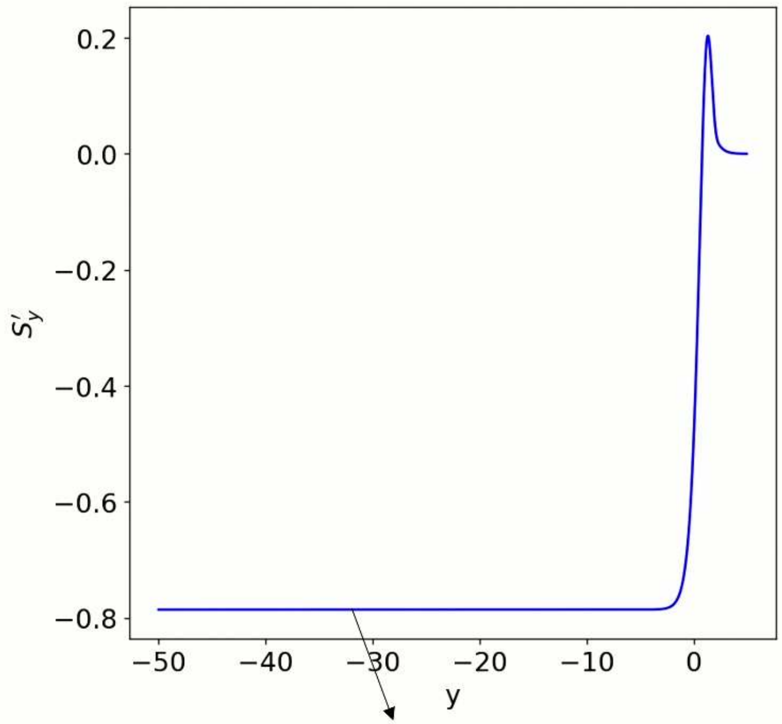
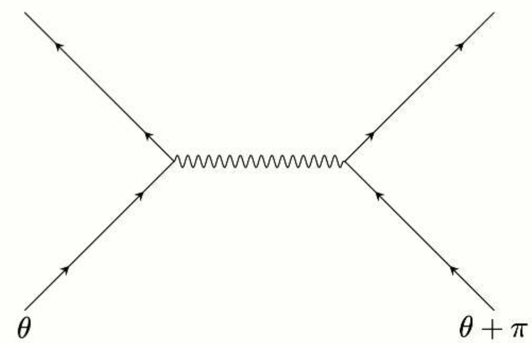


Ising-nematic

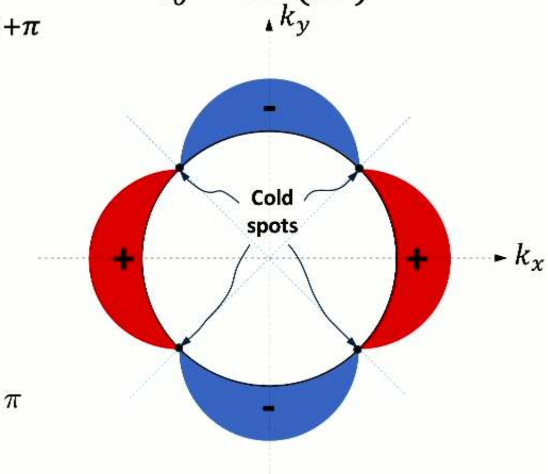


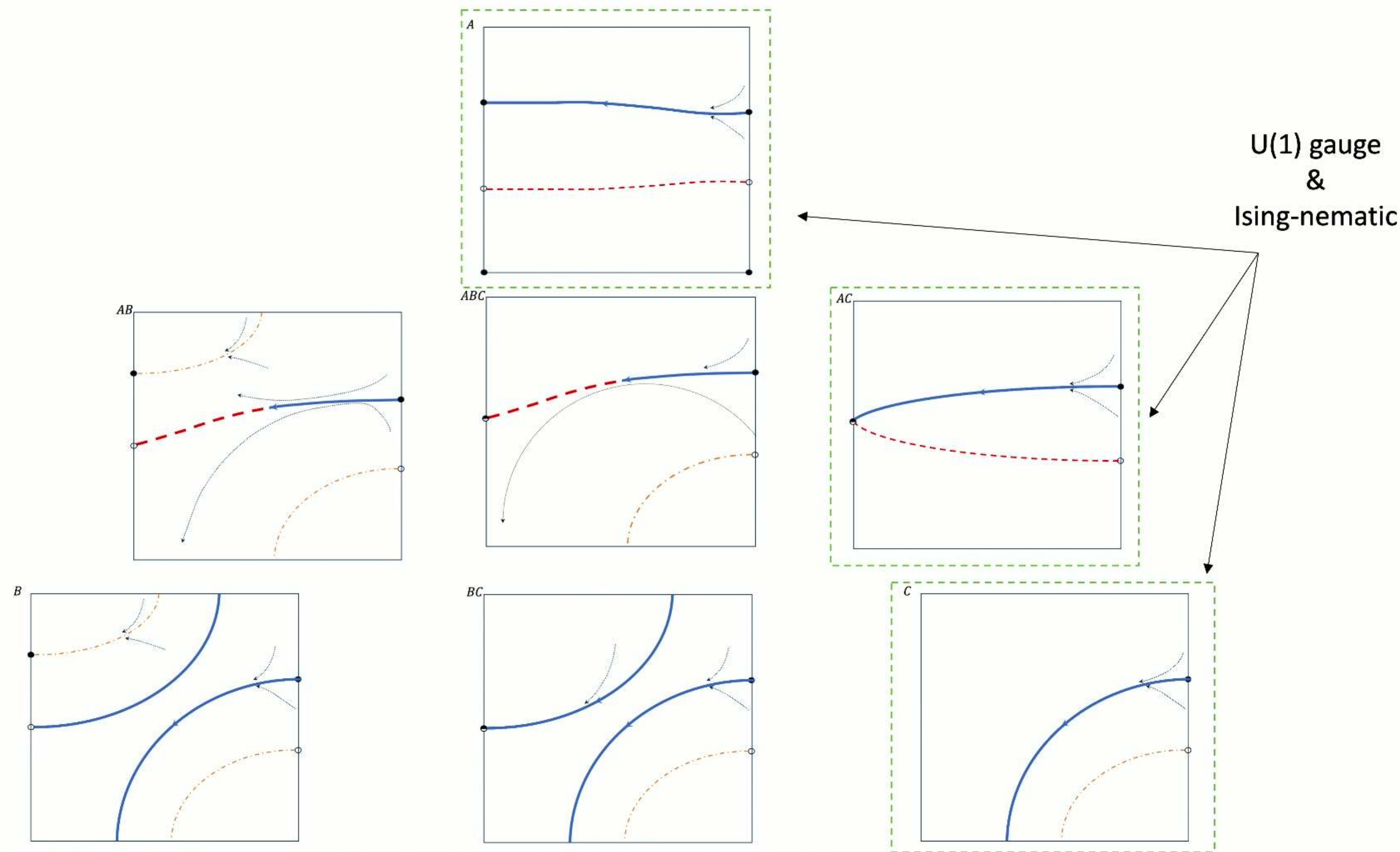
Attractive in s-wave channel

$$S_{int} \sim -e_\theta e_{\theta+\pi}\, \psi^\dagger_\theta\, \psi_\theta\, \psi^\dagger_{\theta+\pi}\, \psi_{\theta+\pi}$$



$$e_\theta \sim \cos(2\theta)$$




A
AB
ABC
AC
B
BC
C
U(1) gauge
&
Ising-nematic

Conclusion

- Developed the theory of full Fermi surface
- Notion of projective fixed points
- Using toy model, seven super-universality classes are identified
- Withing each super-universality class, stable or unstable NFL realized
- Classes A, AC and C realized in the known physical examples