**Title:** D-branes and the planar limit of Chern-Simons theory I: Link invariants

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**Abstract:**

We revisit the Holographic duality between SU(N) Chern-Simons theory and the A-model Topological String Theory. We develop a strategy to systematically compute the large N saddles for correlation functions of Wilson lines in antisymmetric powers of the fundamental representation. The mathematical structures which appear in the calculation match in detail the data of dual A-model D-branes. Suriyah Rajalingam's talk is giving another perspective about the same project.

# D-branes and the Planar Limit of Chern–Simons Theory I: Link Invariants

Sergio Sanjurjo

Perimeter Institute for Theoretical Physics

October 2025

# The Paper

**D-branes and the planar limit of Chern–Simons theory I: Link invariants**, *arXiv:2506.03246, June 2025*

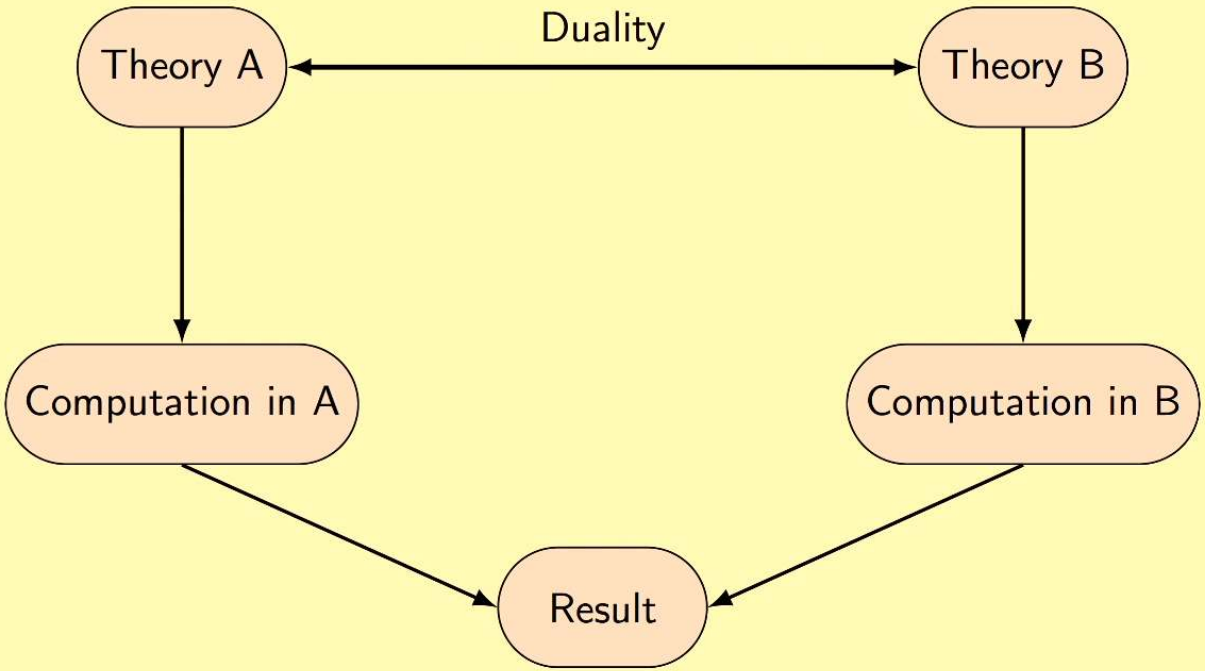
**Davide Gaiotto**

**Suriyah Rajalingam**

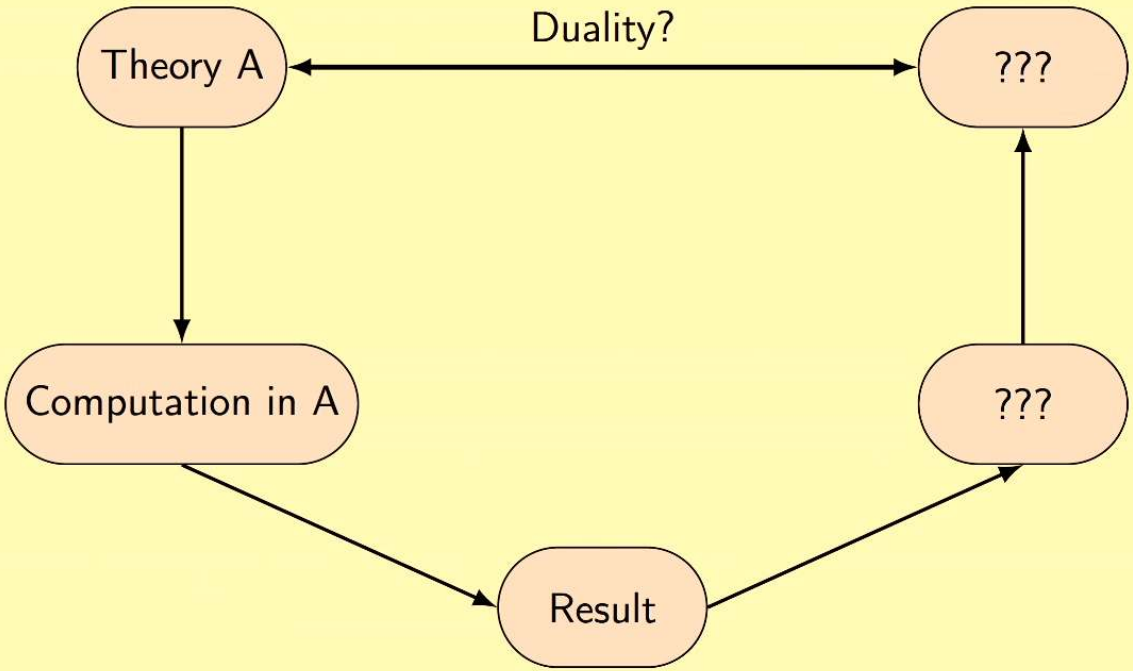
## Objectives for Today

1. Present my perspective on the importance of dualities
2. Outline the central ideas and main results of the paper
3. Discuss the broader goals and open directions of the project

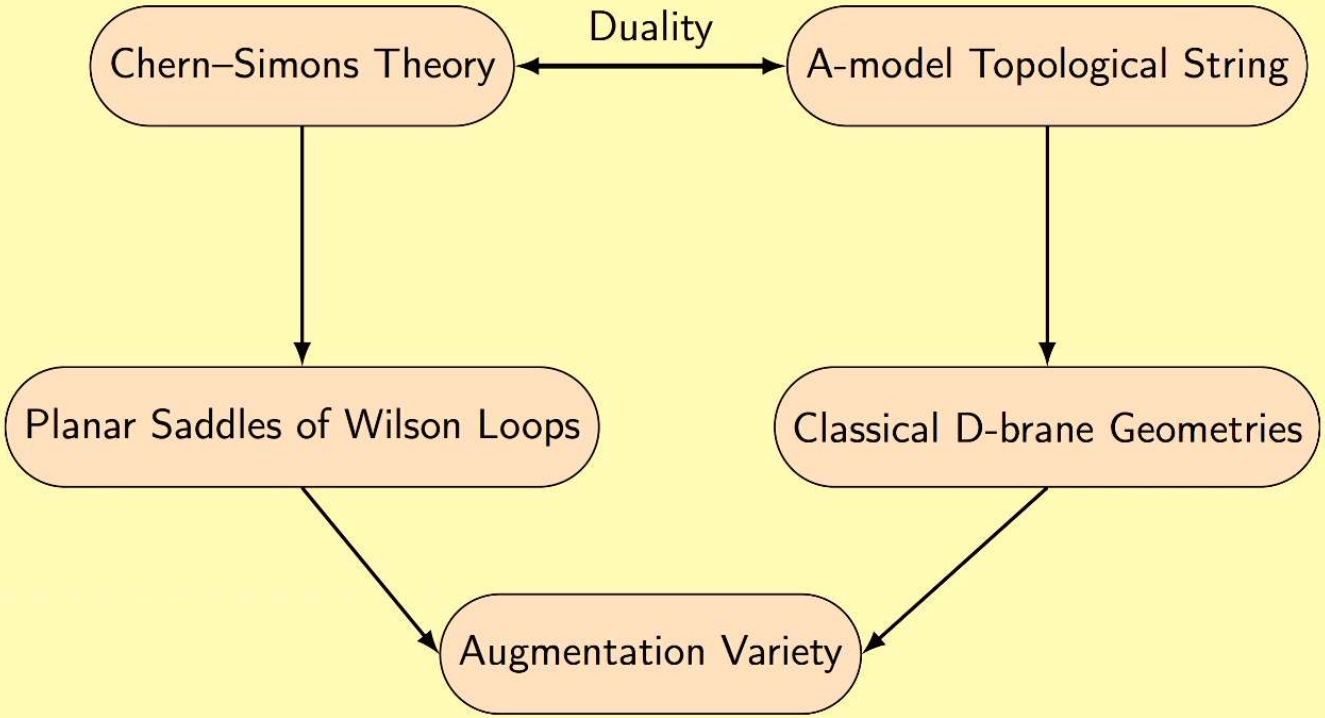
# A Sketch of Dualities



# The Power of Dualities II



# Learning More about Dualities

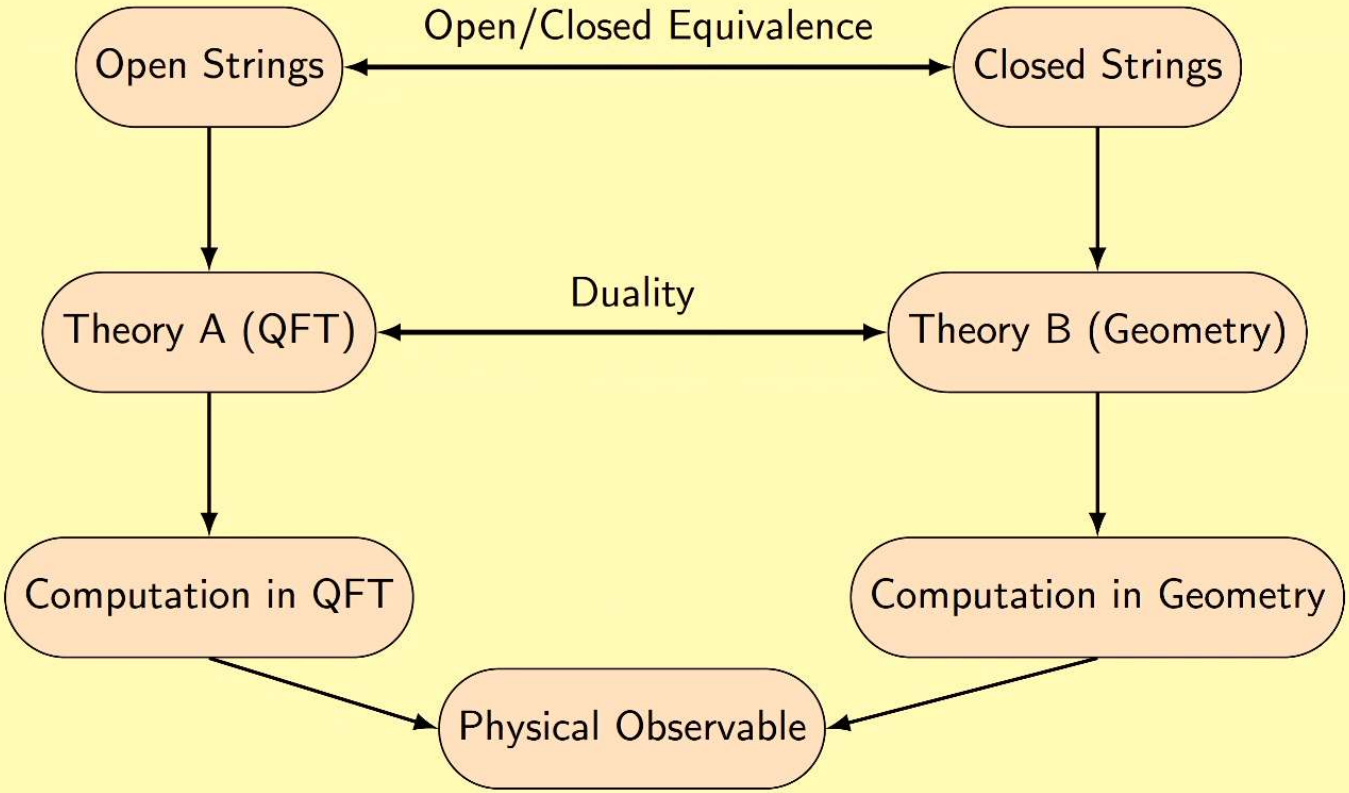


# A Recipe for QFT/Geometry Dualities

**QFT on $M \iff$ Geometry on $Y$**

1. Start with a string theory on a target space $X$
2. Introduce D-branes wrapping a submanifold $M \subseteq X$
3. Take a decoupling limit: the IR dynamics give a QFT on $M$
4. Replace D-branes by their backreaction: $X$ deforms to $Y$
5. Take a classical limit, suppressing string/quantum effects on $Y$

# A Sketch of the Recipe



## Super Yang-Mills and Supergravity

**$U(N)$ $\mathcal{N} = 4$ SYM $\Longleftrightarrow$ Type IIB SUGRA on $\mathrm{AdS}_5 \times S^5$**

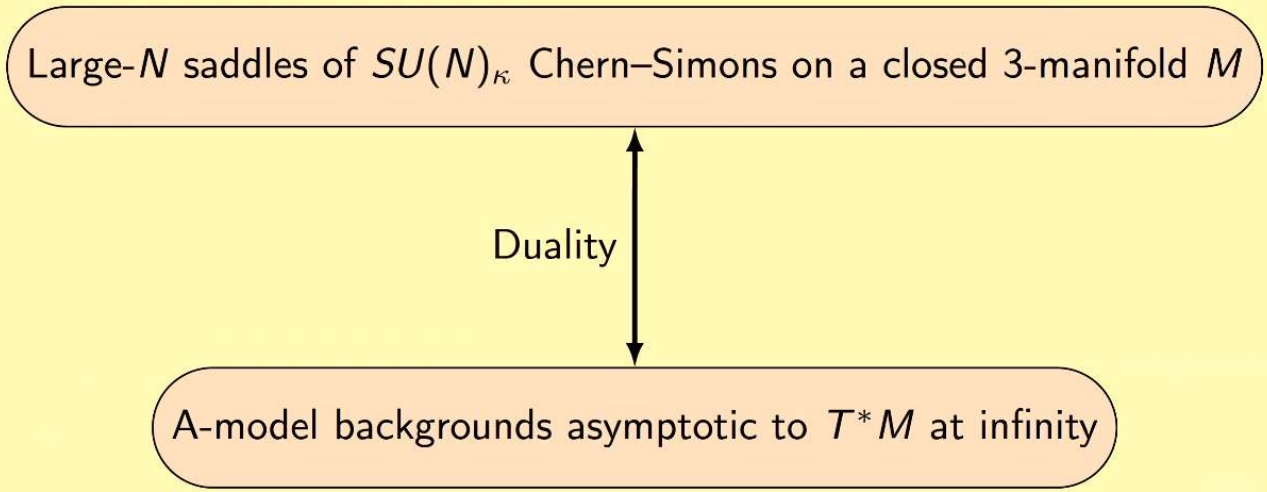
1. Start with type IIB string theory on $\mathbb{R}^{1,9}$
2. Place $N$ D3-branes along $\mathbb{R}^{1,3} \subset \mathbb{R}^{1,9}$
3. Take $\ell_s \to 0$: the IR is $U(N)$ $\mathcal{N} = 4$ SYM on $\mathbb{R}^{1,3}$
4. Backreaction: $\mathbb{R}^{1,9}$ deforms to $\mathrm{AdS}_5 \times S^5$
5. Take $g_s \to 0$, $N \to \infty$, $\lambda = g_s N \gg 1$: SUGRA on $\mathrm{AdS}_5 \times S^5$

# Chern–Simons theory and A-model Geometry

## $U(N)$ Chern–Simons $\Longleftrightarrow$ Genus-0 A-model

1. Start with the A-model topological string on $T^*S^3$
2. Place $N$ A-branes along $S^3 \subseteq T^*S^3$
3. The dynamics give $U(N)_\kappa$ CS on $S^3$, with $\kappa + N = \frac{2\pi i}{g_s}$
4. Backreaction: $T^*S^3$ deforms to the resolved conifold
5. Take $g_s \to 0$, $N \to \infty$, $t = g_s N$ fixed: genus-0 A-model

# Duality Expectations



# Large-$N$ Limit and Holography

## Planar Limit

Gauge Group $SU(N)$ at level $\kappa$ with $N, \kappa \to \infty$

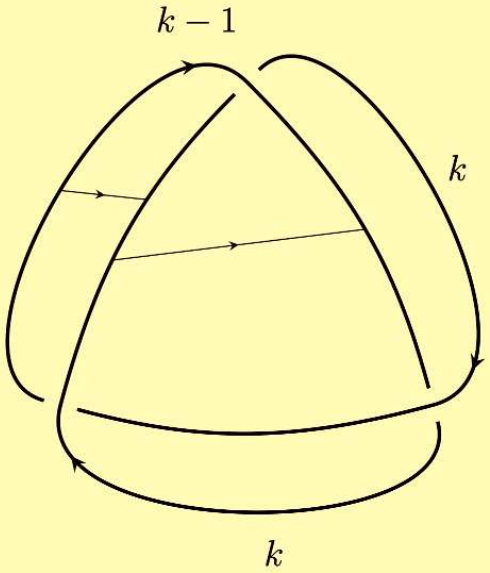
## The Dictionary of Representations

Fundamental $\longrightarrow$ String $\qquad$ $O(N)$ indices $\longrightarrow$ D-brane

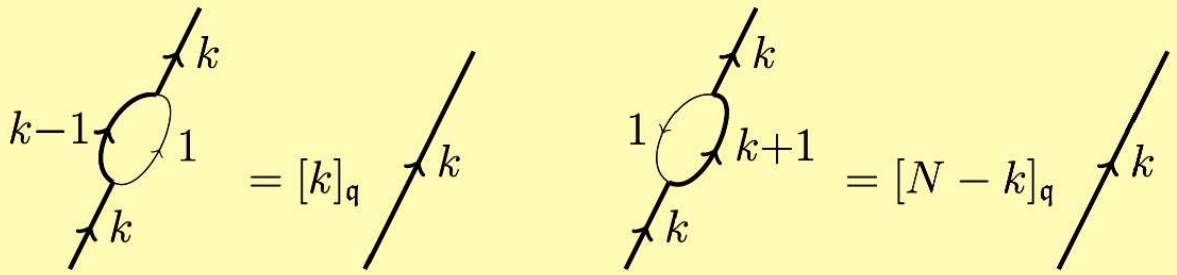
# Fundamental Modifications

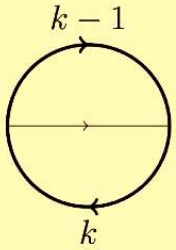
## Fermionic Modification

$N$ 1d fermions $\longrightarrow$ Antisymmetric reps $\Lambda^k \mathbb{C}^N$ $\longrightarrow$ D-brane



# Diagrammatics for Antisymmetric Powers




$$\longrightarrow \quad [k]_{\mathfrak{q}}\langle W_{\Lambda^k\mathbb{C}^N}\rangle = [N-k+1]_{\mathfrak{q}}\langle W_{\Lambda^{k-1}\mathbb{C}^N}\rangle$$

# The Planar Limit

## Planar Variables

$$\mathfrak{q} \to 1 \qquad \mathfrak{q}^k \to \mu \qquad \mathfrak{q}^N \to g \qquad \frac{\langle W_{\Lambda^{k-1}\mathbb{C}^N} \rangle}{\langle W_{\Lambda^k \mathbb{C}^N} \rangle} \to \lambda$$

## Unknot in the fundamental

$$(\mathfrak{q} - \mathfrak{q}^{-1}) \langle W_{\mathbb{C}^N} \rangle \to g - g^{-1}$$

## Unknot's Augmentation Variety

$$\frac{[k]_{\mathfrak{q}}}{[N-k+1]_{\mathfrak{q}}} = \frac{\langle W_{\Lambda^{k-1}\mathbb{C}^N} \rangle}{\langle W_{\Lambda^k \mathbb{C}^N} \rangle} \quad \longrightarrow \quad g^2\lambda - \mu^2\lambda + \mu^4 - \mu^2 = 0$$

# The Holographic Dual Story

## Augmentation Variety Recipe

1. Consider an appropriate cotangent bundle
2. Add $N$ D-branes in the base to get Chern–Simons theory
3. Replace them by their backreaction and take the 't Hooft limit
4. Identify appropriate lagrangian D-branes
5. Discover the augmentation variety as their moduli

# The Backreaction

## The Recipe

1. Start with $T^*M$ and a flux $d\omega$ generated by the D-branes.
2. Compute a symplectic form $\omega$ that accounts for the flux.
3. Find new coordinates in which $\omega$ is canonical.

## The deformed $A_1$ singularity

$$T^*S^2 \longrightarrow \mathcal{M}_t : \; xy + z(z-t) = 0$$

# Understanding D-branes

## Building D-branes through recipes

- There may be recipes that give the same D-brane
- There may be D-branes not created by any recipe

## Identifying D-branes through signatures

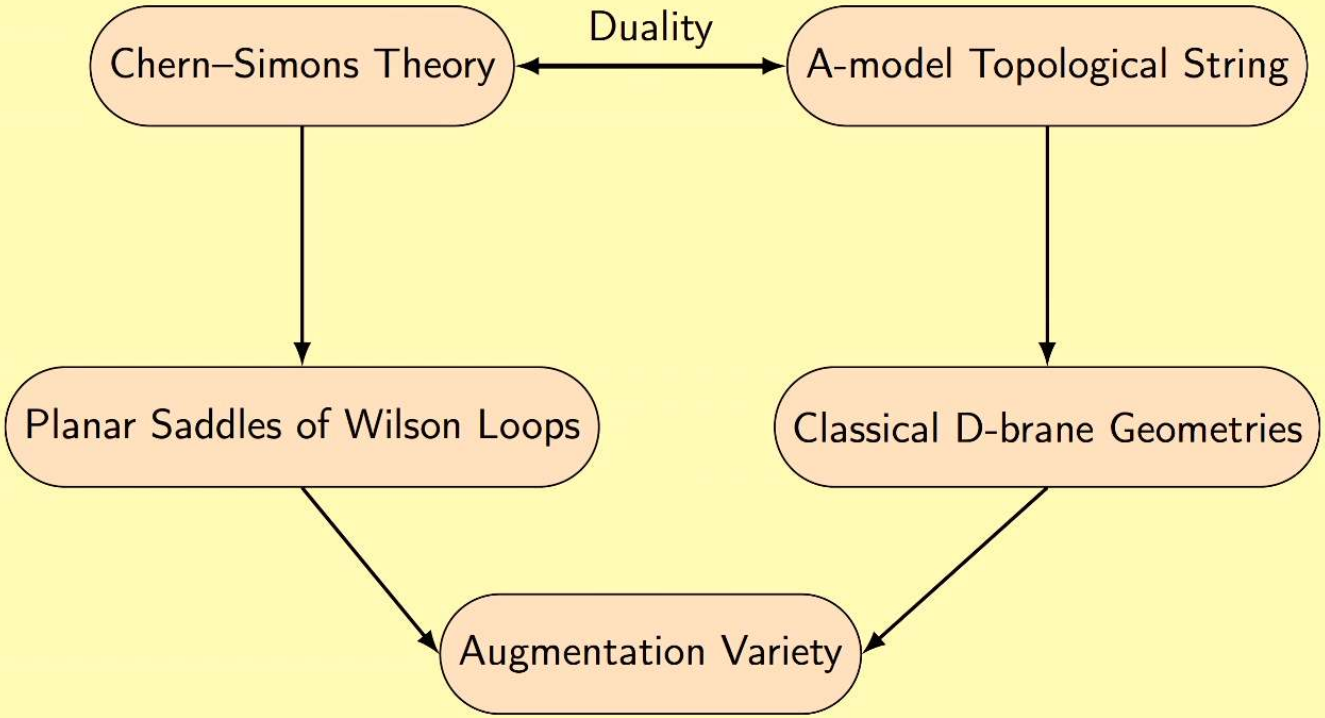
- There may be D-branes with the same signature
- There may be signatures not associated with any D-brane

# The Appropriate D-branes

## A Sketch of the Construction

1. There are simple Lagrangian D-branes on $\mathcal{M}_t$, called $L_+$ and $L_-$
2. They form families identified with moduli spaces of Higgs bundles through a spectral curve construction
3. A hyperkähler rotation turns them into moduli spaces of flat connections
4. This produces augmentation varieties as the moduli spaces of the deformation of combinations of $L_+$ and $L_-$

# The Sketch Again



# Results

## Holographic Match

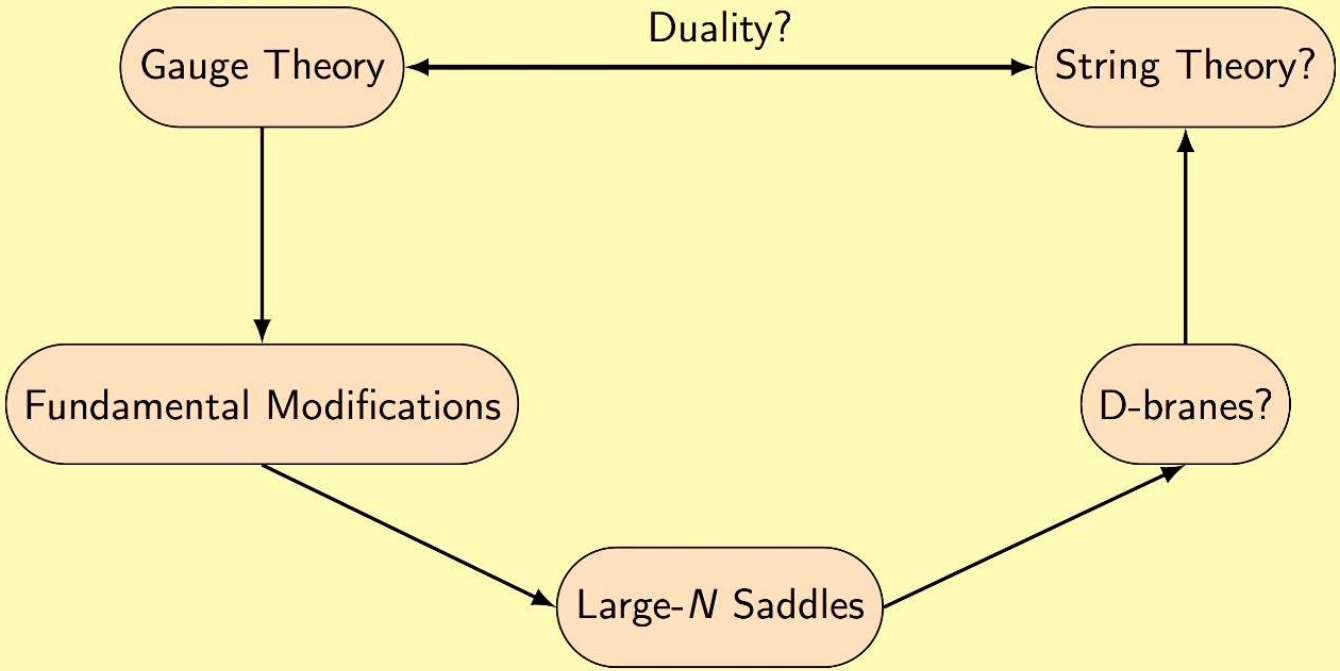
Chern–Simons theory large-$N$ saddles and A-branes for arbitrary knot/link and antisymmetric power of the fundamental on:

- an arbitrary 3d manifold with $g \to 1$
- $S^2 \times S^1$ with arbitrary $g$
- $S^2 \times \mathbb{R}$ and $\mathbb{R}^3$ with arbitrary $g$, except for the caps constraints

## Augmentation Varieties

A new way of computing augmentation varieties, which should match the DGA constructions coming from symplectic geometry

# Discovering String Theory Duals



## The Program

1. Start with a gauge theory
2. Add a fundamental modification: fundamental matter with a choice of large-$N$ saddle for the 't Hooft expansion
3. Find a D-brane that reproduces the saddle in the backreacted geometry
4. Repeat for all possible fundamental modifications
5. Define the string theory dual via the boundary conditions specified by the D-branes

# Outlook and Future Goals

- Understanding the Planar Limit
- Understanding String Theory
- Achieving Mathematical Holography
- Achieving Physical Holography

¡Thank you!