Title: Lorentzian Quasicrystals and the Irrationality of Spacetime

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Abstract:

Ordered structures that tile the plane in an aperiodic fashion - thus lacking translational symmetry - have long been considered in the mathematical literature. A general method for the construction of quasicrystals is known as *cut-and-project* (CNP for short), where an irrational slice ''cuts'' a higher-dimensional space endowed with a lattice and suitably chosen lattice points are further ''projected'' down onto the slice to form the vertices of the quasicrystal. However, all of the known examples of CNP quasicrystals are Euclidean. In this talk, after presenting the main ingredients of the Euclidean prescription, we will extend it to Lorentzian spacetimes and develop Spacetime CNP. This will allow us to discuss the first-ever examples of spacetime quasicrystals, one in (1+1)- and another in (1+3)-dimensional spacetime. Finally, we will argue why the latter construction might be relevant for *our Lorentzian spacetime*. In particular, we shall appreciate how the picture of a quasi-crystalline spacetime could provide a potentially new string-compactification scheme that can naturally accommodate for the hierarchy problem and the smallness of our cosmological constant. Lastly, we will briefly comment on its relevance to quantum gravity; first, as a conformal Lorentzian structure of no intrinsic scale, and second through the connection of quasicrystals to quantum error-correcting codes.

Lorentzian Quasicrystals and the *irrationality* of Spacetime

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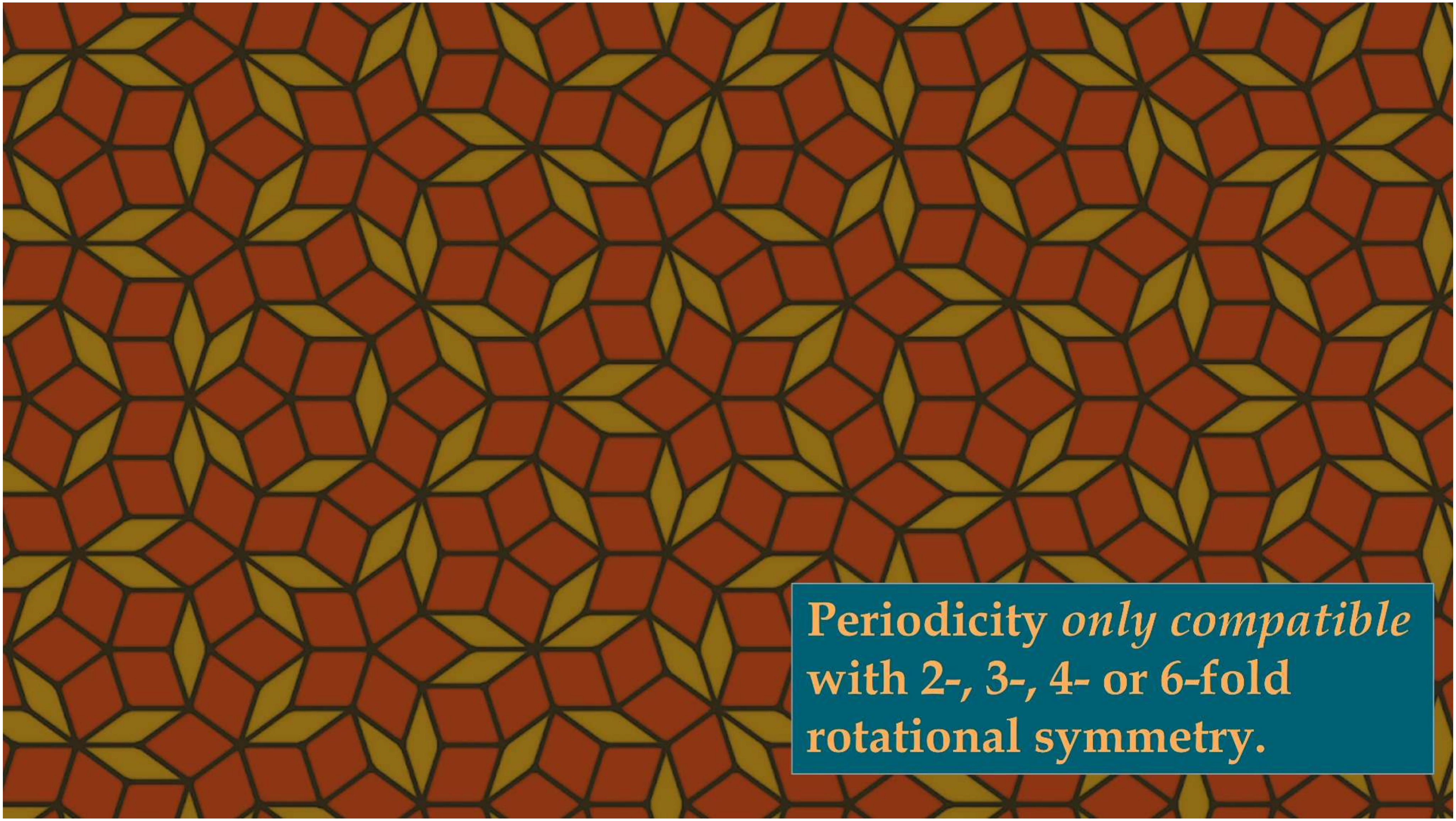
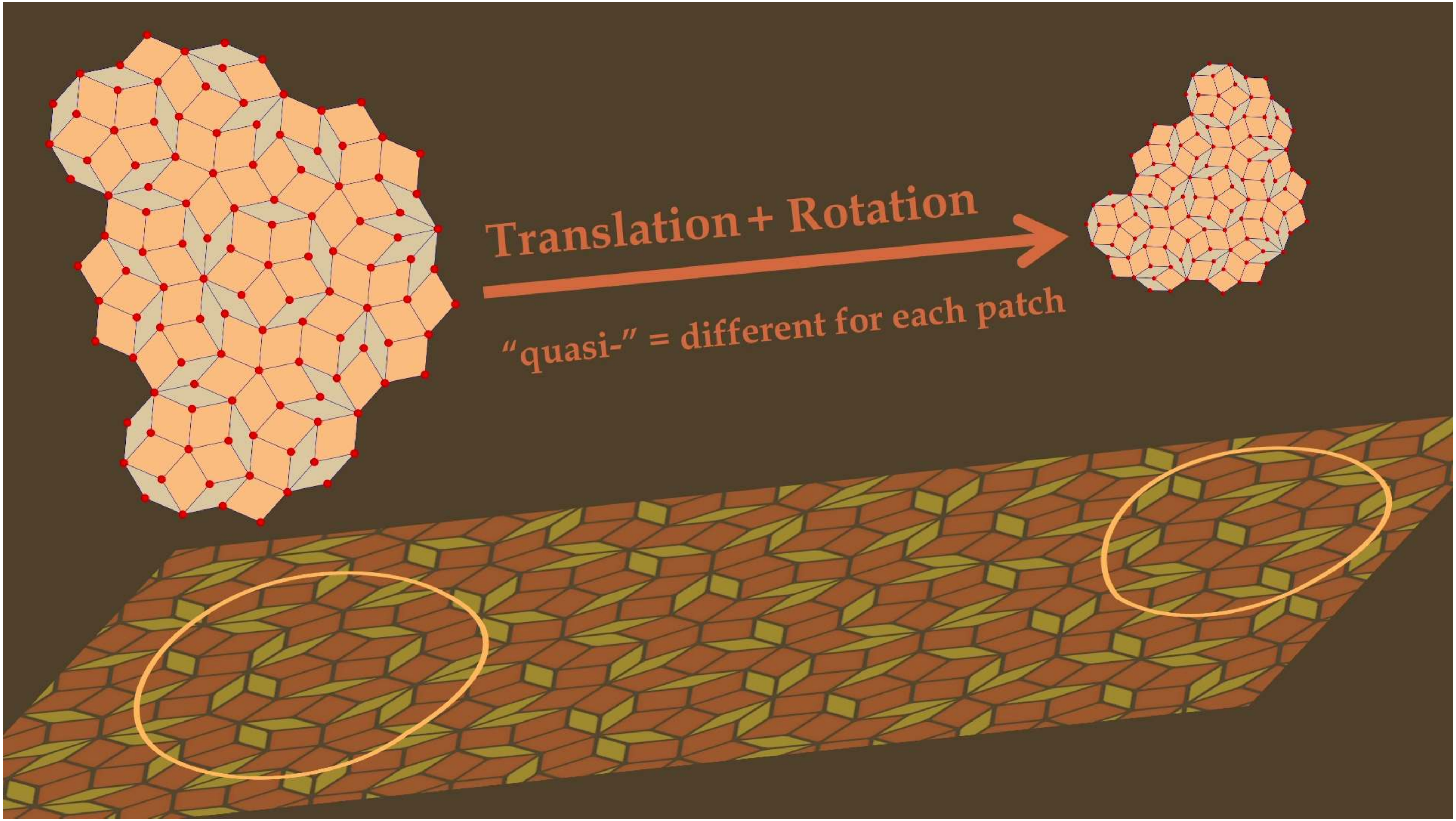
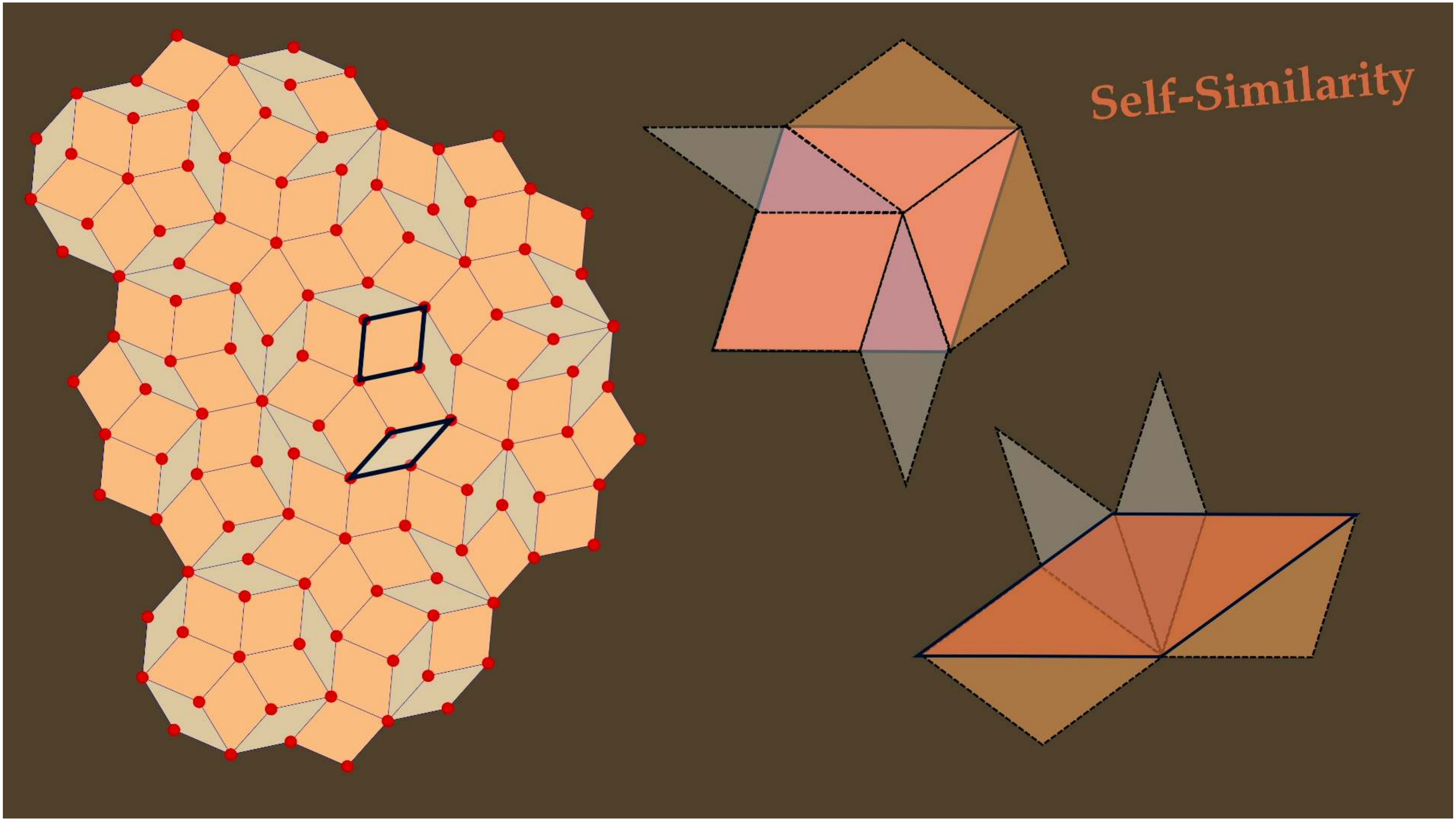
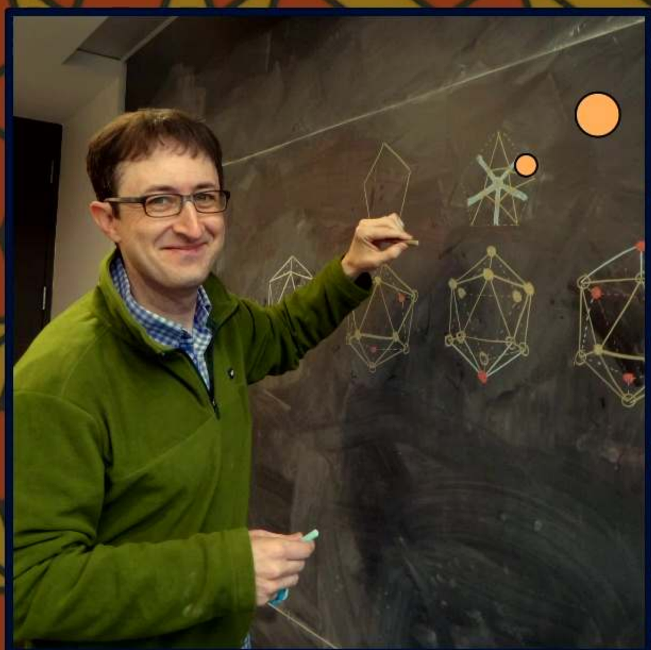
Grad Conference @ Perimeter Institute, October 16th, 2025

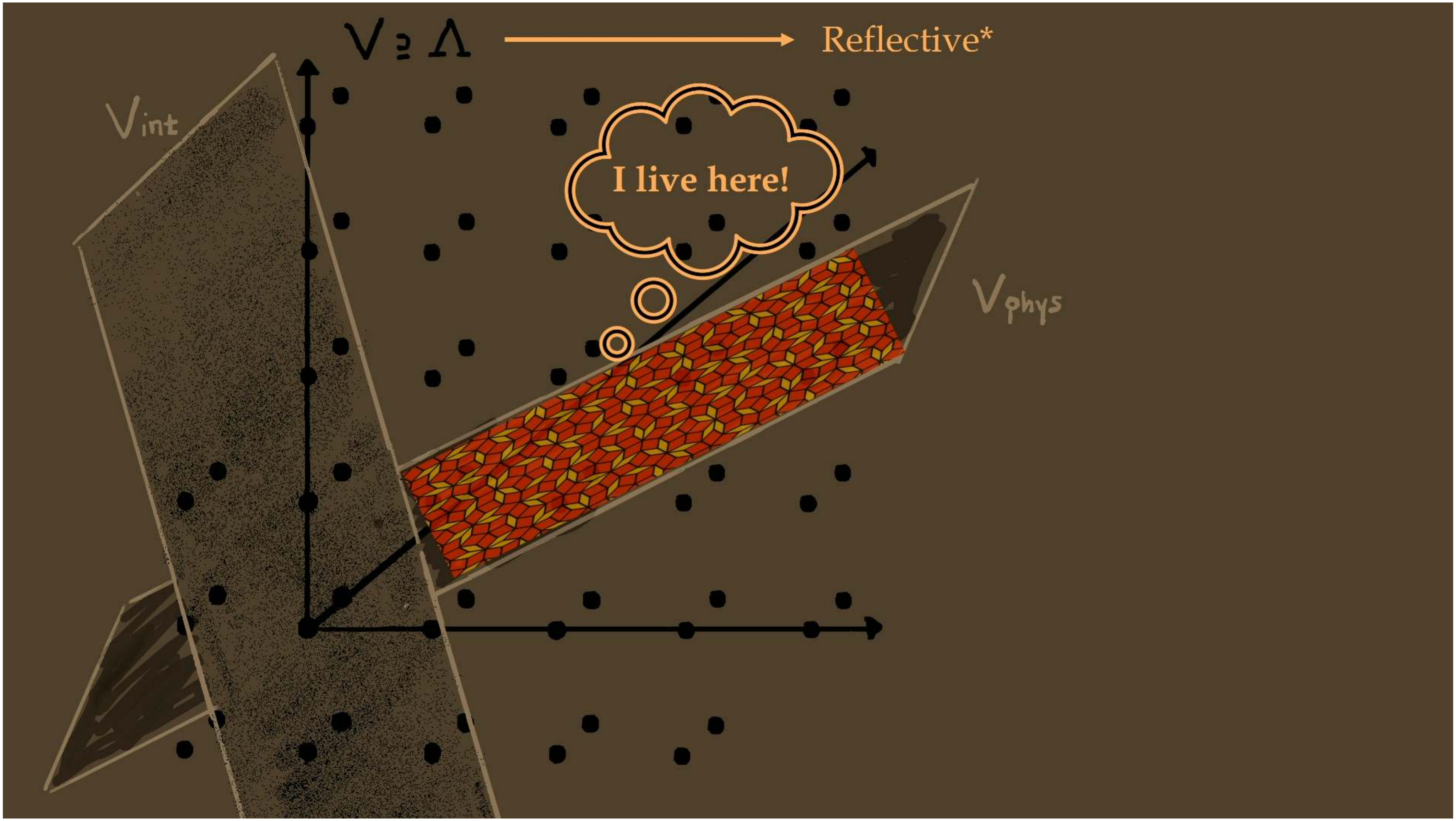

Periodicity *only compatible* with 2-, 3-, 4- or 6-fold rotational symmetry.


Translation + Rotation
"quasi-" = different for each patch


Self-Similarity

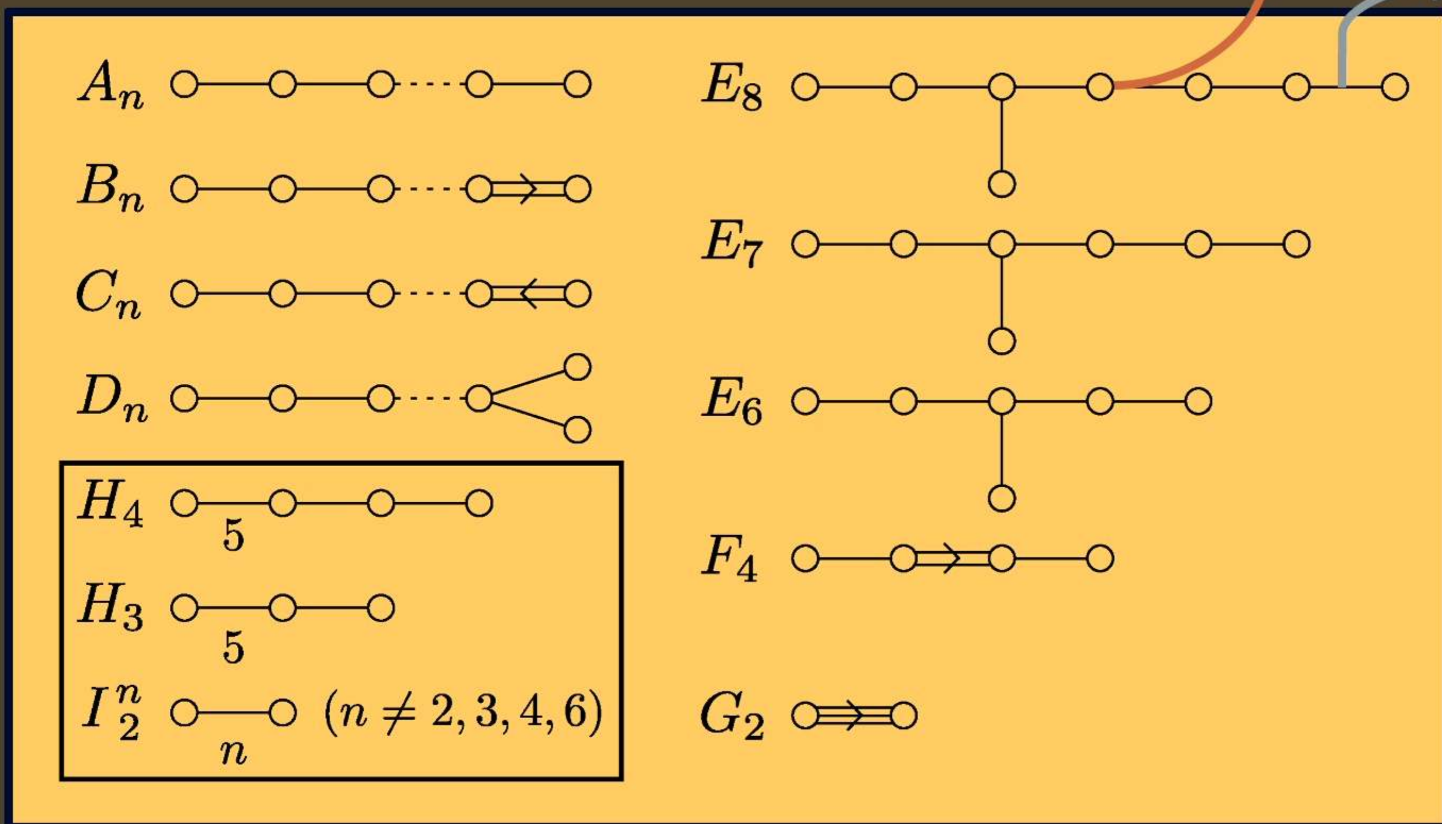
Can we do that in **Lorentzian** Spaces?




V ⊇ Λ
Reflective*
V_int
I live here!
V_phys

Reflection Symmetry is completely classified!

Coxeter-Dynkin Graphs



#nodes = #reflection gens

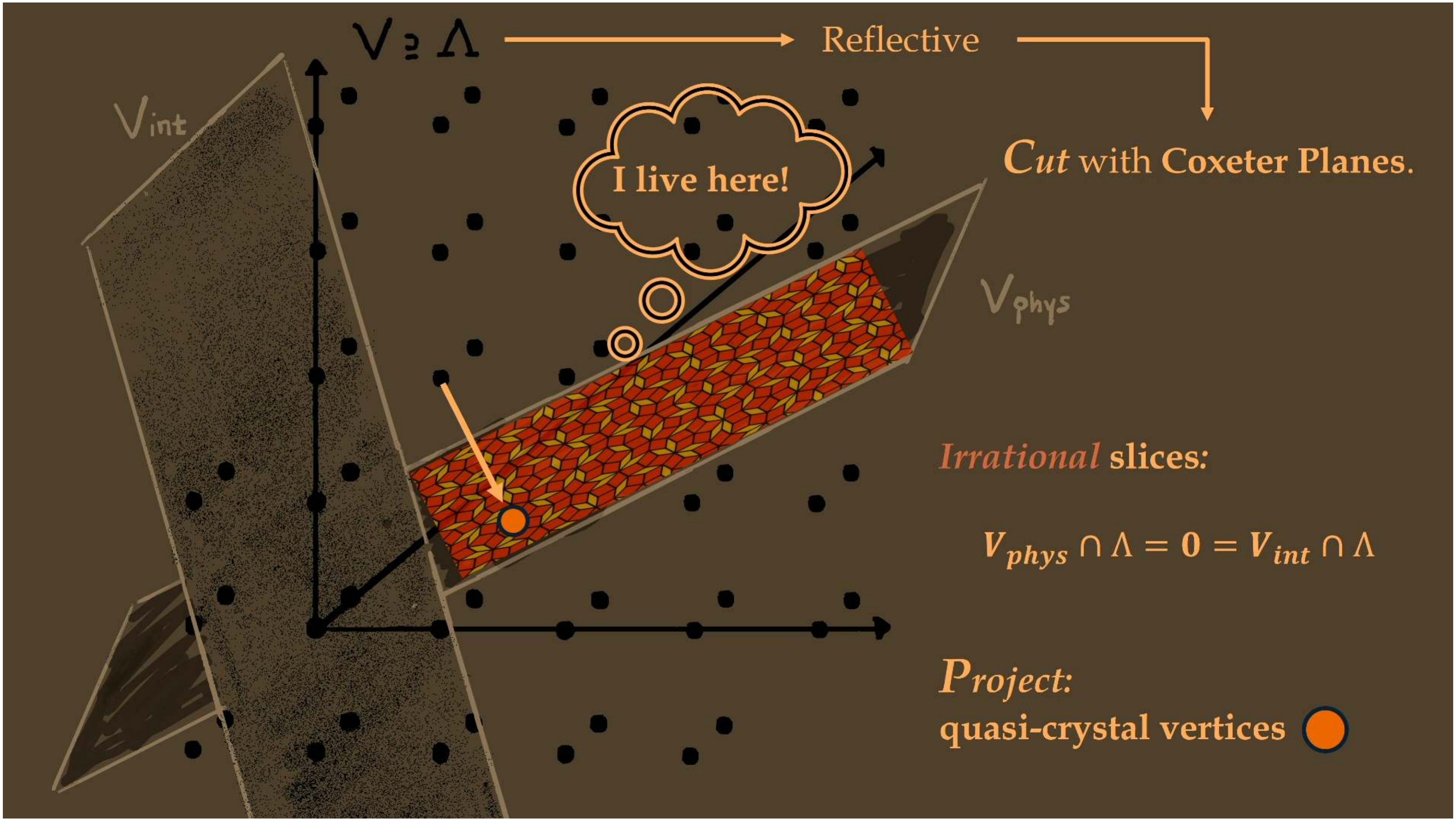
links ~ relative angles

$$R_i^2 = 1 = (R_i R_j)^{m_{ij}},$$
$$m_{ij} \geq 2, \text{ integer}$$

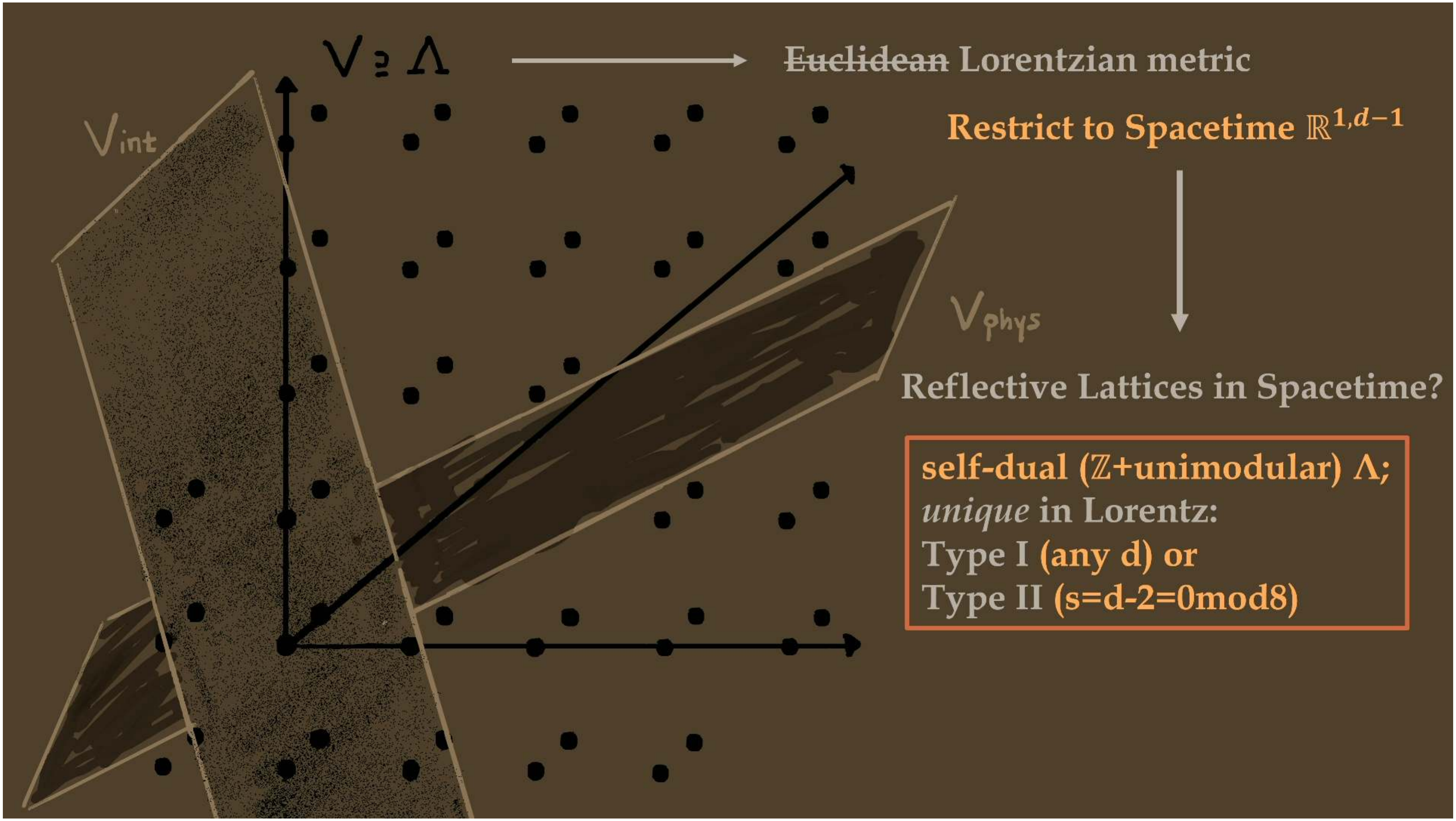
Coxeter group

Coxeter Element C:

$$C = R_1 R_2 \ldots R_m$$



$V \supseteq \Lambda$
Reflective
V_{int}
I live here!
V_{phys}
*C*ut with **Coxeter Planes**.
Irrational **slices:**
$V_{phys} \cap \Lambda = 0 = V_{int} \cap \Lambda$
Project:
quasi-crystal vertices

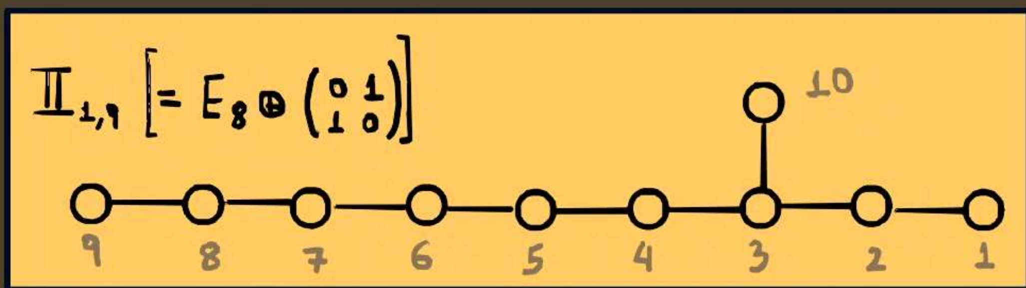


$V \supseteq \Lambda$
~~Euclidean~~ Lorentzian metric
Restrict to Spacetime $\mathbb{R}^{1,d-1}$
V_{int}
V_{phys}
Reflective Lattices in Spacetime?
self-dual ($\mathbb{Z}$+unimodular) Λ;
unique in Lorentz:
Type I (any d) or
Type II (s=d-2=0mod8)

Hyperbolic Coxeter Groups [up to d=10]

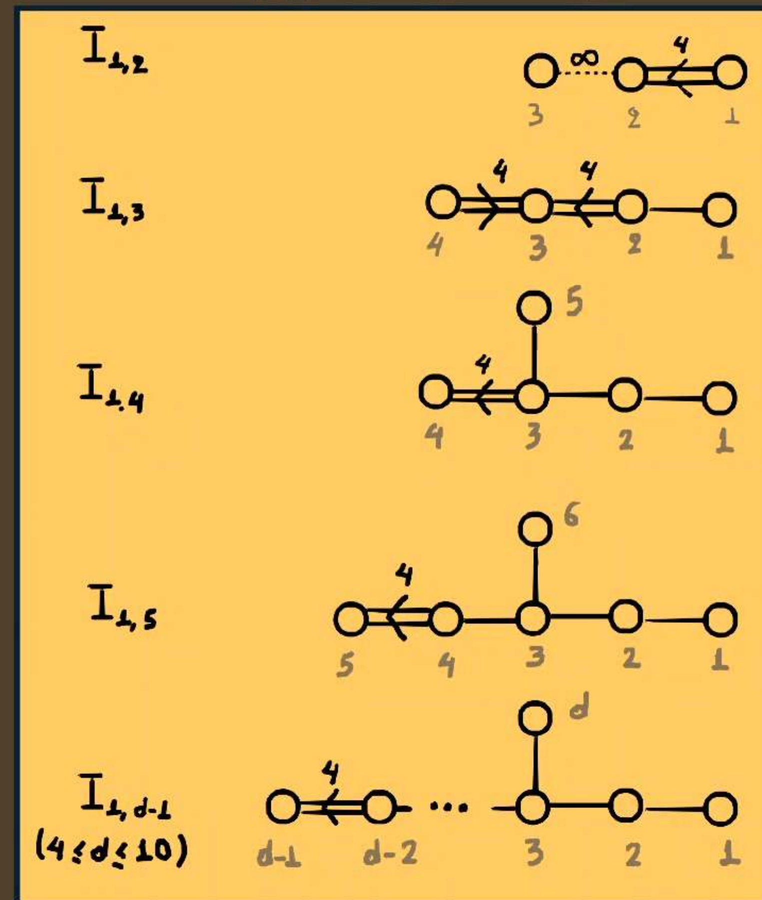
[Vinberg's algorithm; works in the 70s]

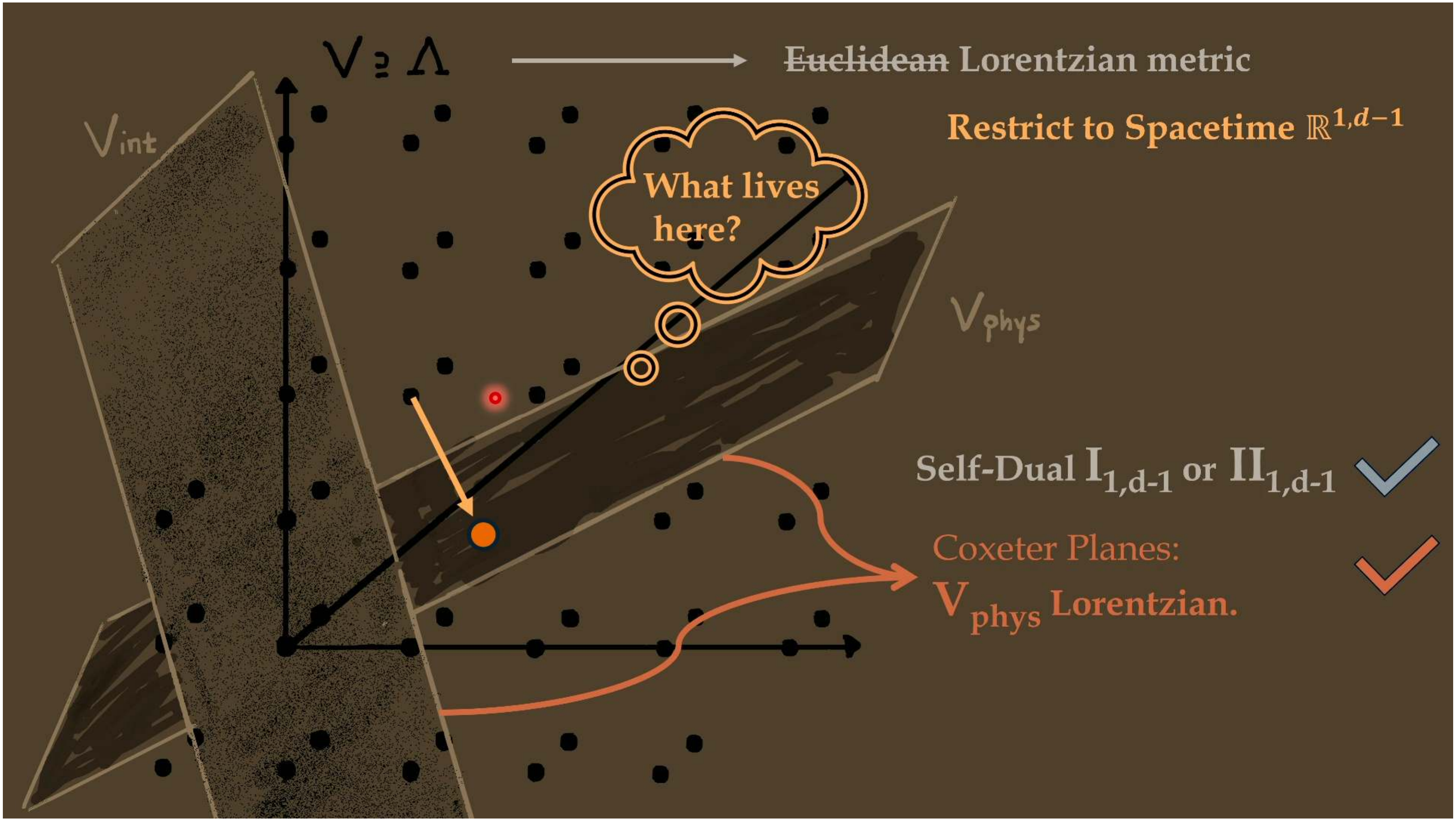
Type II (Even)



(+ more graphs with finite #simple roots up to d=20.)

Type I (Odd)





V ⊇ Λ
Euclidean Lorentzian metric
Restrict to Spacetime $\mathbb{R}^{1,d-1}$
V_int
What lives here?
V_phys
Self-Dual $I_{1,d-1}$ or $II_{1,d-1}$
Coxeter Planes:
V_{phys} Lorentzian.

Spacetime CNP

- **First-ever Example**

(1+1)D Spacetime Quasicrystal from $I_{1,3}$

Coxeter Eigenbasis *is* a NP Frame!

Coxeter element [Point Set] = Point Set

↳ infinite symmetry!
(discrete Lorentz)

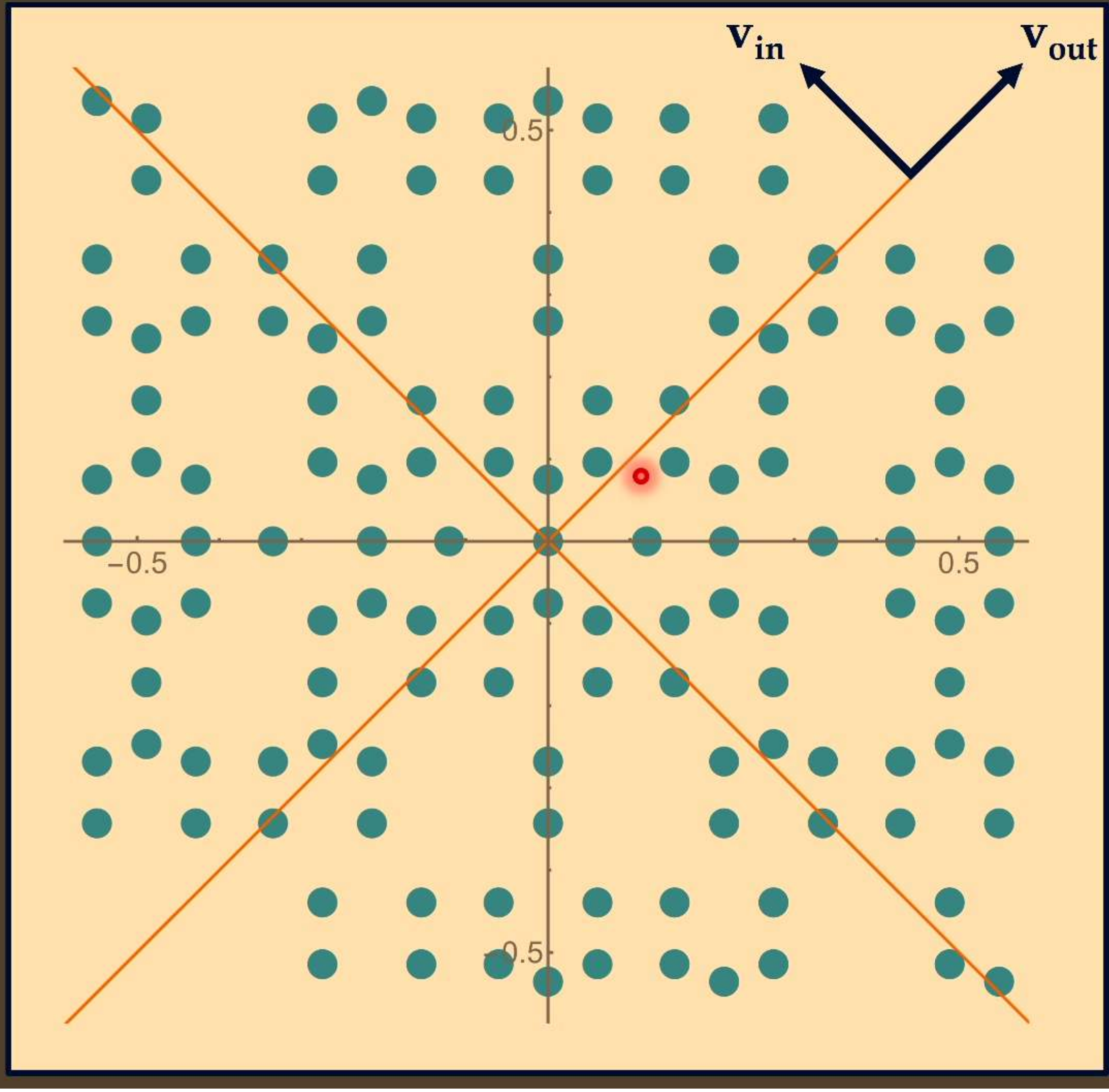
Global Scaling Symmetry.

Irrationality:
No null-separated vertices.

... Causal Sets?

Trade *Randomness* with *Symmetry*!

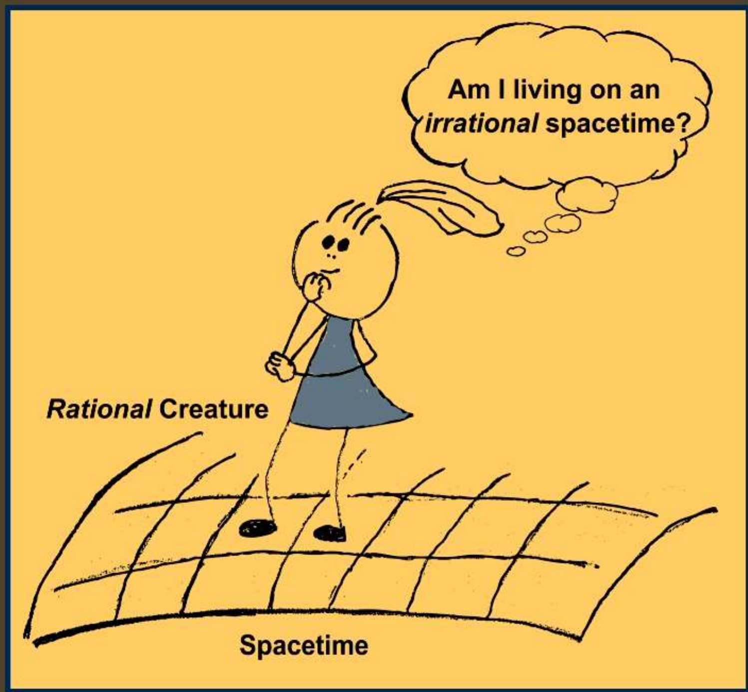
... Q error-correcting codes?



Spacetime CNP

~~First~~ Second-ever Example

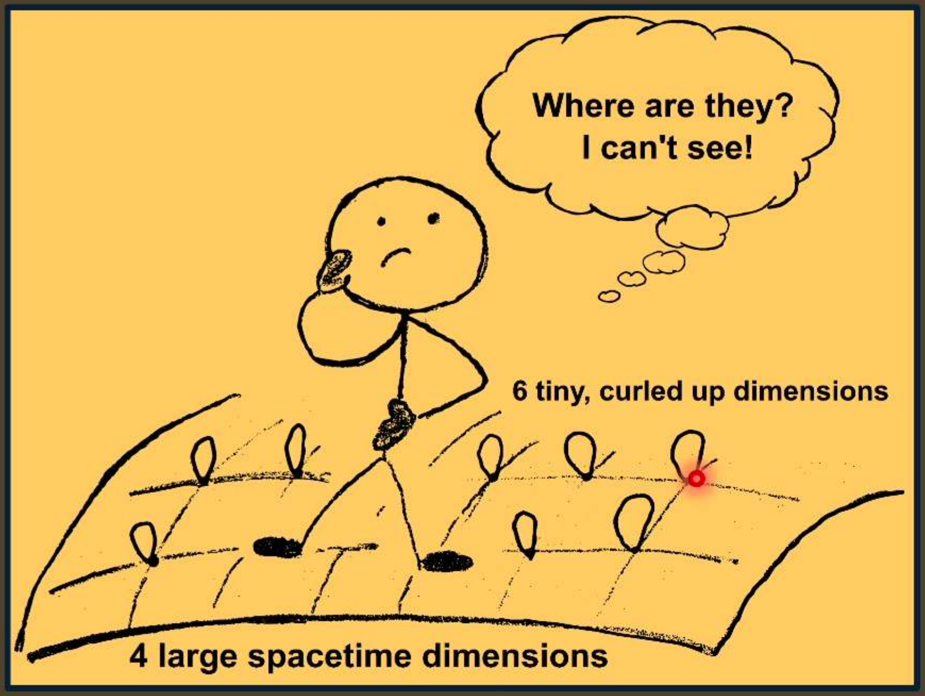
(3+1)D Spacetime Quasicrystal from $II_{1,9}$



1+1 *Rational* Arguments in Favour of an *Irrational* Spacetime

1 Fitting the Universe in a Nutshell

String Theory:
"We *really* live in 10 dimensions."



1 Fitting the Universe in a Nutshell

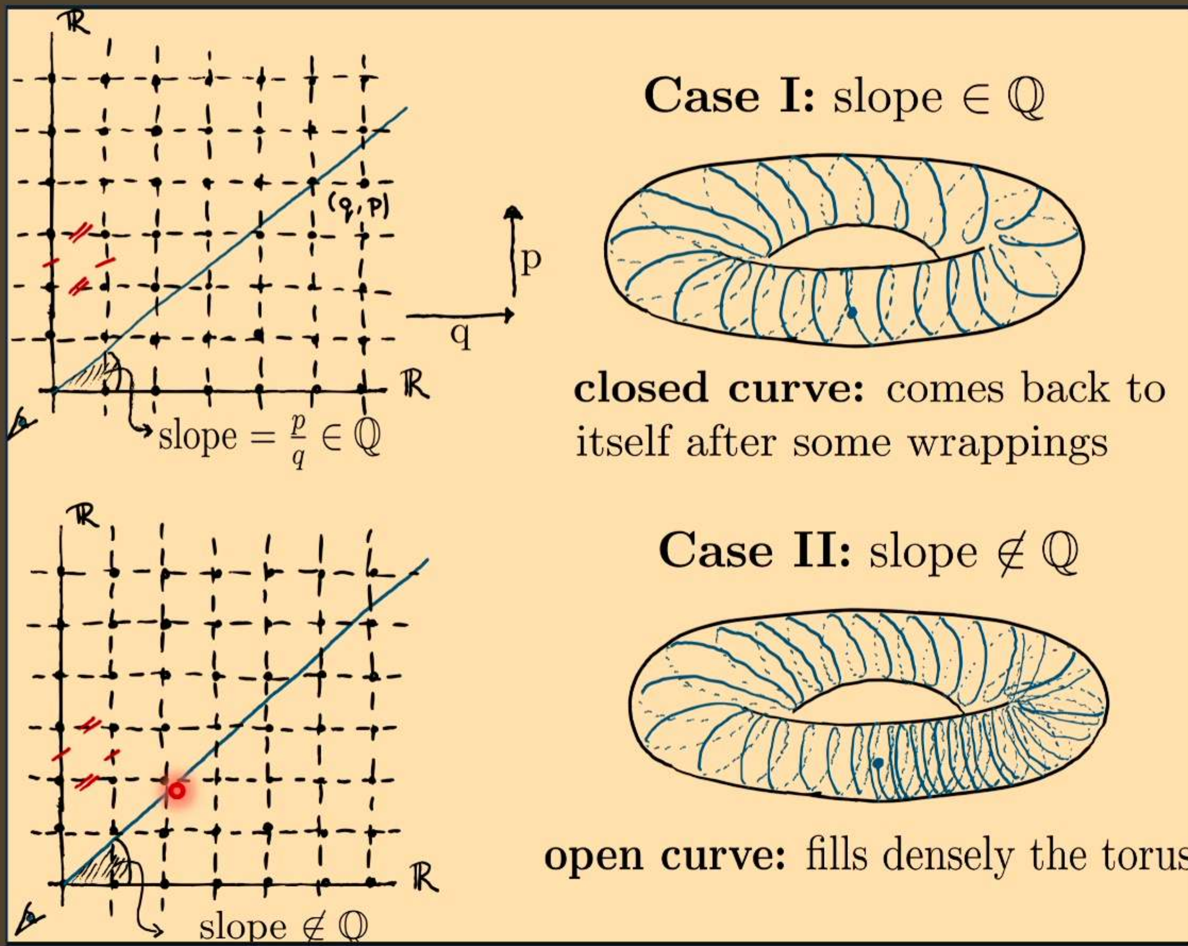
"Toroidal" Compactification

$$\mathbb{T}^{1,9} = \mathbb{R}^{1,9}/\mathrm{II}_{1,9}$$

Moore's "most symmetric" compactification

[Finite in all Directions, hep-th/9305139]

… what about causality violations and CTCs?



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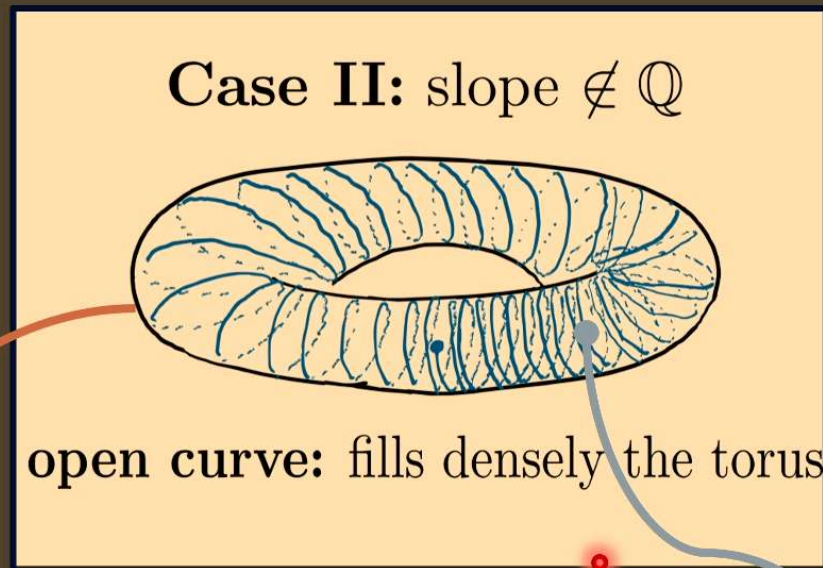


$\mathbb{T}^{1,9}$

4D Spacetime

Enter Irrationality:
no CTCs in 4D!

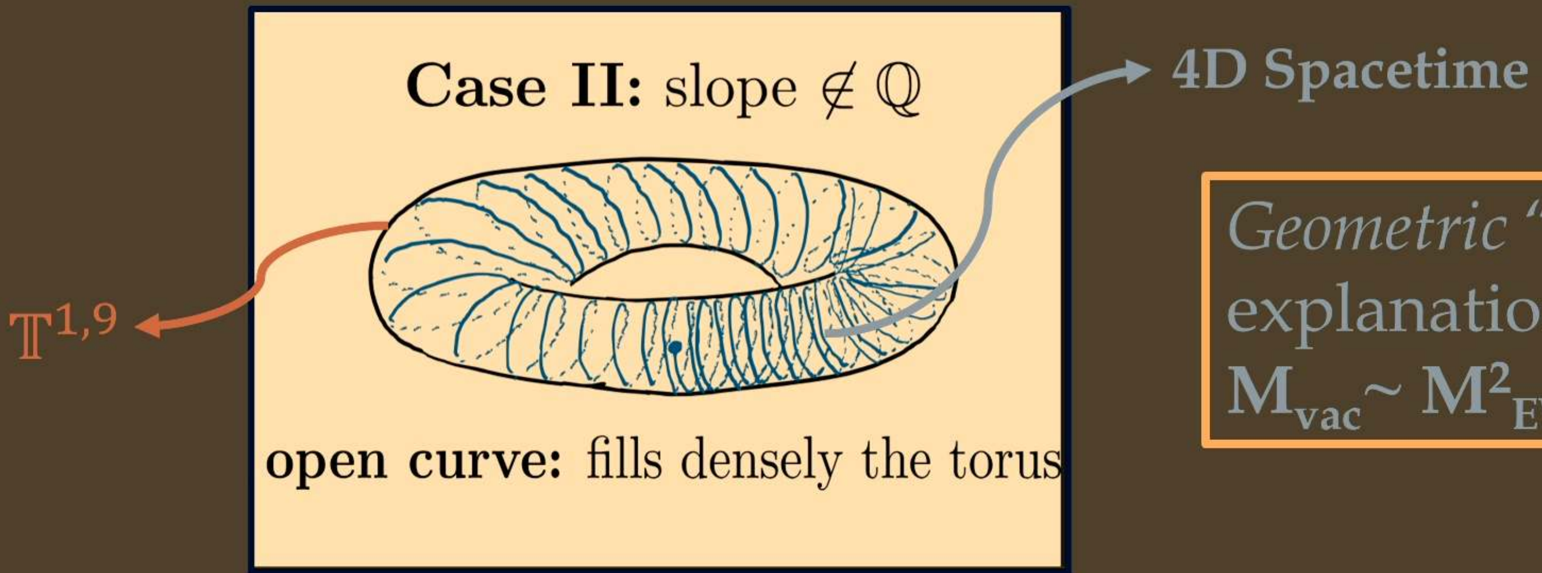
1+1 The Hierarchy Problem

$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}\left[\underbrace{M_{Pl}^2\mathcal{R}}_{\text{Bending}} + \underbrace{M_H^2(h^\dagger h)}_{\text{Higgs}} + \underbrace{M_{vac}^4}_{\text{Stretching}}\right]$$

$M_{Pl} \sim 10^{19}$ GeV

$M_{EW} \sim 10^{2}$ GeV

$M_{vac} \sim 10^{-12}$ GeV



Geometric "seesaw" explanation for $M_{vac} \sim M^2_{EW}/M_{Pl}$

THANK YOU

Is it such an *irrational* idea after all?

Still Feels *Irrational?*
3+1 reasons to think over lunch:

- ✓ **Highly Symmetric** Discrete Point Set in Spacetime with **no Intrinsic Scale.**
- ✓ **Q-Error Correcting** Quasi-Crystalline Spacetime?
- ✓ “Most Symmetric” **String Compactification** Possible.
- ✓ **Geometric Explanation** of the Hierarchy of Scales.