Title: The contextual Heisenberg Microscope

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Abstract:

The Heisenberg microscope provides a powerful mental image of the measurement process of quantum mechanics (QM), attempting to explain the uncertainty relation through an uncontrollable back-action from the measurement device. However, Heisenberg's proposed back-action uses features that are not present in the QM description of the world, and according to Bohr not present in the world. Therefore, Bohr argues, the mental image proposed by Heisenberg should be avoided.

Later developments by Bell and Kochen-Specker shows that a model that contains the features used for the Heisenberg microscope is in principle possible but must necessarily be nonlocal and contextual. In this paper we will re-examine the measurement process within a restriction of QM known as Stabilizer QM, that still exhibits for example Greenberger-Horne-Zeilinger nonlocality and Peres-Mermin contextuality. The re-examination will use a recent extension of stabilizer QM, the Contextual Ontological Model (COM), where the system state gives a complete description of future measurement outcomes reproducing the quantum predictions, including the mentioned phenomena.

We will see that the resulting contextual Heisenberg microscope back-action can be completely described within COM, and that the associated randomness originates in the initial state of the pointer system, exactly as in the original description of the Heisenberg microscope. The presence of contextuality, usually seen as prohibiting ontological models, suggests that the contextual Heisenberg microscope picture could perhaps be enabled in general QM.

The Contextual Heisenberg Microscope

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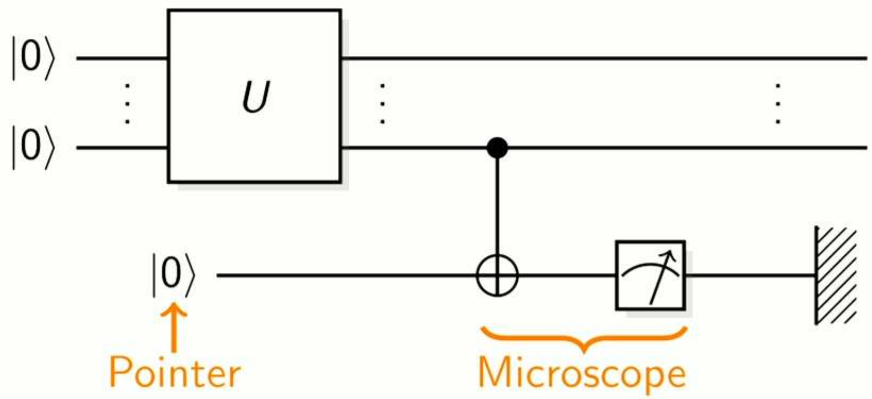
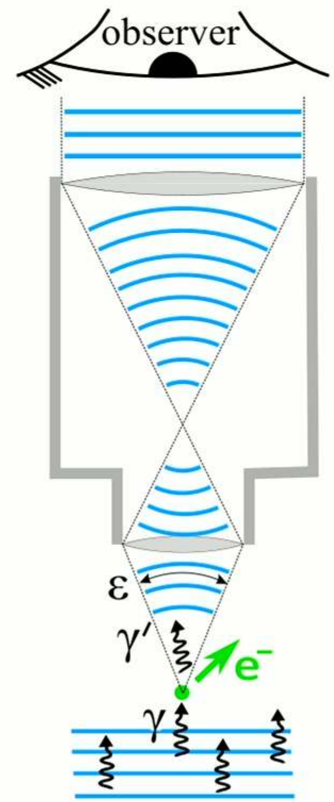


The Heisenberg microscope illustrates the uncertainty relation

- Measuring q causes a change in p. But we're taught that neither q nor p exist to begin with?

As the statistical character of quantum theory is so closely linked to the inexactness of all perceptions, one might be led to the presumption that behind the perceived statistical world there still hides a "real" world in which causality holds. But such speculations seem to us, to say it explicitly, fruitless and senseless. (Heisenberg 1927)

Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. (Bohr 1928)



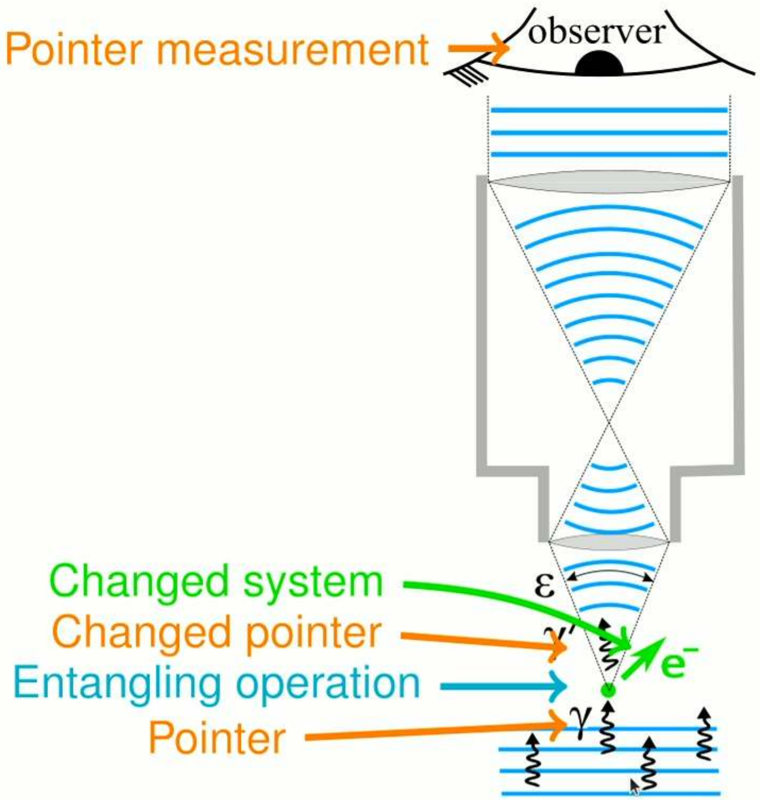
The uncertainty relation for Pauli group measurements on qubits

- For Pauli measurements on qubits we have

$$[X, Y] = 2iZ \quad \Rightarrow \quad \sigma_X \sigma_Y \geq |\langle Z \rangle|$$

- If you can predict the *Z* outcome, *X* and *Y* are maximally unpredictable
- Let us look at the Heisenberg microscope for qubits

The Heisenberg microscope for qubits



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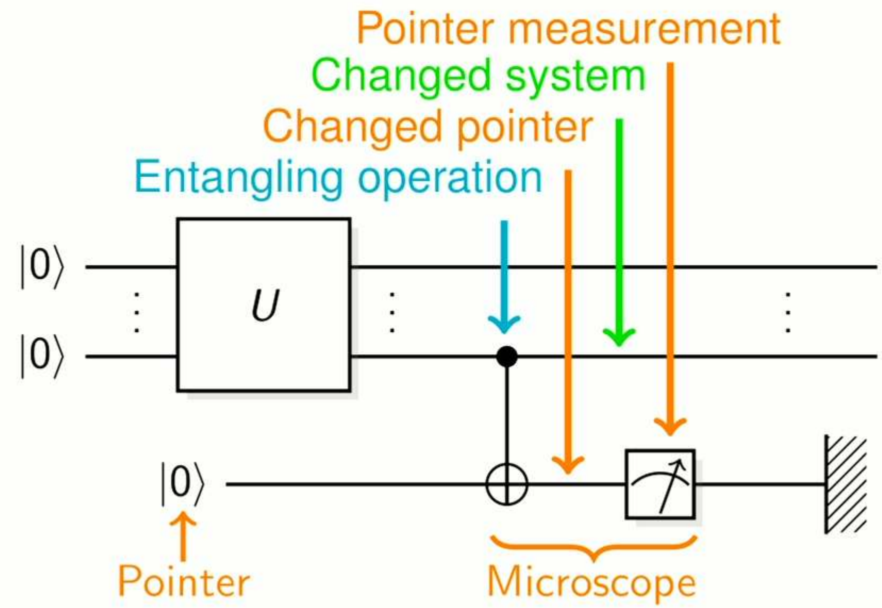

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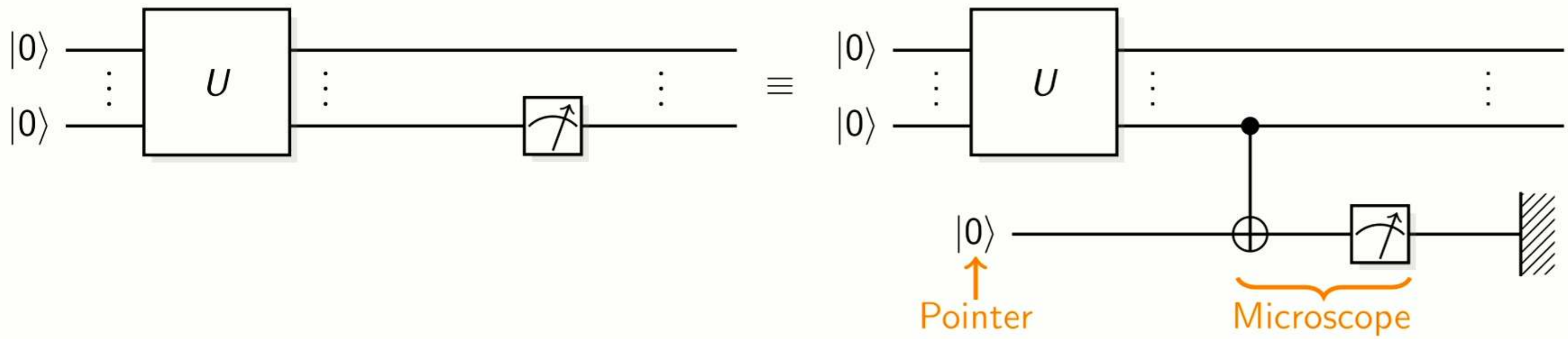
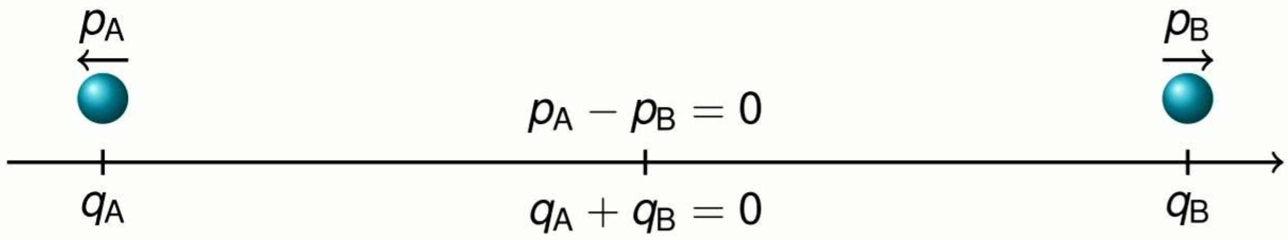


- Observing Z causes a change in X and Y. But we're taught that neither X, Y, nor Z exist to begin with?

Now the quantum postulate implies that any observation of **stabilizer** QM phenomena will involve an interaction with the agency of observation not to be neglected. **But in spite of this an independent reality in the ordinary physical sense can be ascribed to the phenomena as well as to the agencies of observation.**

EPR: An independent reality should have a complete description



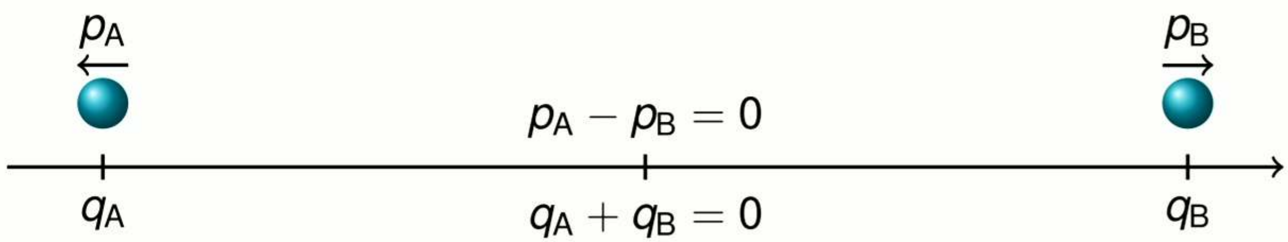
Elements of reality (Einstein, Podolsky, and Rosen, Phys. Rev. 1935)

If, without in any way disturbing a system, we can predict with certainty (i.e., with a probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Completeness (Einstein, Podolsky, and Rosen, Phys. Rev. 1935)

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.*

There is no independent reality within QM



In QM you need to choose, either

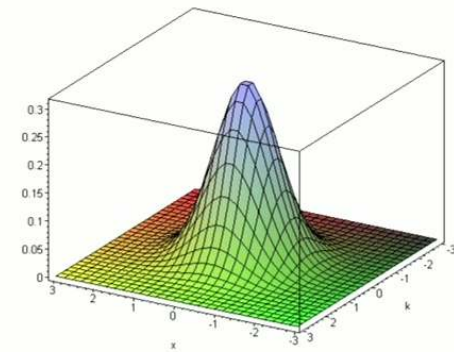
$$|\psi(q_A, q_B)\rangle \text{ or } |\widehat{\psi}(p_A, p_B)\rangle$$

The uncertainty relation prevents q_A and p_A to be well-defined simultaneously

Classical Hamiltonian mechanics gives us the Liouville distribution as a complete description

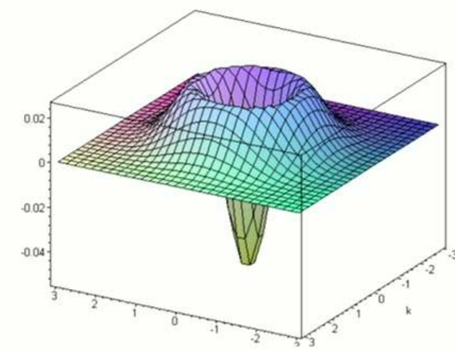
The Liouville distribution $\rho(\boldsymbol{q}, \boldsymbol{p})$ can be read as describing particles with definite $\boldsymbol{q}, \boldsymbol{p}$ that obey

$$\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = +\frac{\partial \mathcal{H}}{\partial \boldsymbol{p}}$$
$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\frac{\partial \mathcal{H}}{\partial \boldsymbol{q}}$$



In QM, the Wigner quasi-probability distribution has positive marginals and total probability 1, but

$$W(\boldsymbol{q}, \boldsymbol{p}) \not\geq 0$$



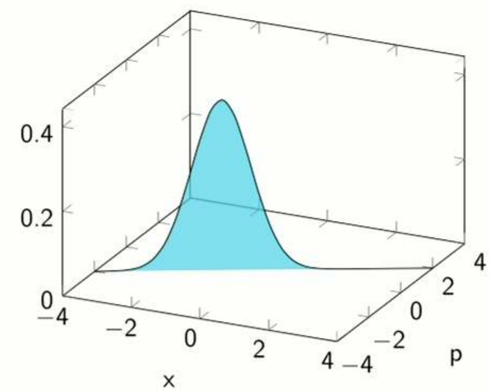
Bohmian mechanics gives us a probability distribution as a complete description

Bohmian mechanics describes particles with definite $\boldsymbol{q}, \boldsymbol{p}$ that obey

$$\frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} = +\frac{\partial(\mathcal{H} + \mathcal{H}_Q)}{\partial \boldsymbol{p}}$$
$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\frac{\partial(\mathcal{H} + \mathcal{H}_Q)}{\partial \boldsymbol{q}}$$

This gives the (Bohm-)Liouville distribution

$$\rho_Q(\boldsymbol{q}, \boldsymbol{p}) = ||\psi(\boldsymbol{q})||^2 d\delta\left(\boldsymbol{p} - \frac{\partial(\arg\psi)}{\partial \boldsymbol{q}}(\boldsymbol{q})\right)$$



Qubits also have a Wigner function on a phase space

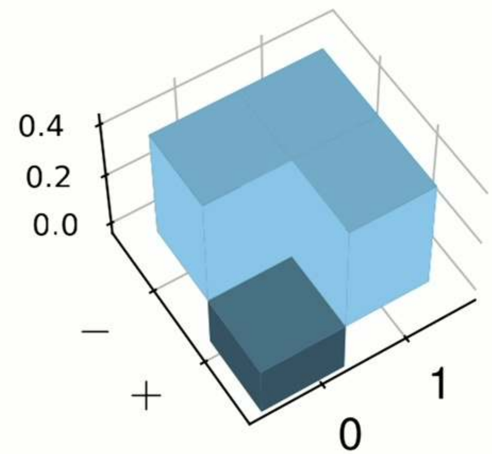
- For qubits there are two binary coordinates, the computational basis and the phase basis, eigenstates of

$$Z|0\rangle = |0\rangle, \quad Z|1\rangle = -|1\rangle$$

$$X|+\rangle = |+\rangle, \quad X|-\rangle = -|-\rangle$$

- The Wigner quasi-probability distribution has positive marginals and total probability 1, but can be negative
- The Hadamard switches between the two coordinates

$$H|0\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big) = |+\rangle, \quad H|1\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big) = |-\rangle$$



The coordinates in the phase space can be expressed as stabilizers

- A quantum state is said to be *stabilized* by an observable if it is unchanged under its action

$$Z|0\rangle = |0\rangle, \quad -Z|1\rangle = |1\rangle$$

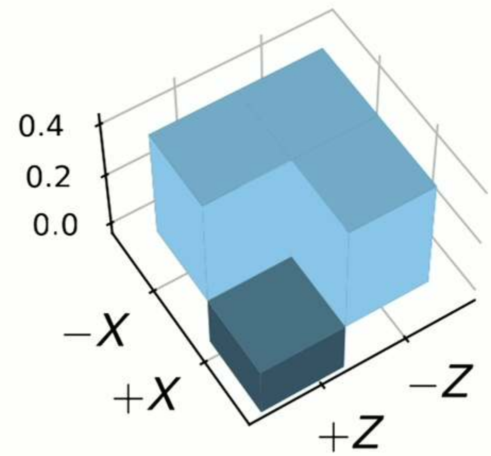
$$X|+\rangle = |+\rangle, \quad -X|-\rangle = |-\rangle$$

- The observables can now be used as coordinate labels
- We can now write stabilizers for two or more systems

$$-ZX|0-\rangle = |0-\rangle \quad \text{and} \quad -\mathbb{I}X|0-\rangle = |0-\rangle$$

- A general stabilizer (Pauli-group observable) can be written

$$M = \pm \bigotimes_{k=1}^{n} i^{x_k z_k} X^{x_k} Z^{z_k},$$



This is known as the *stabilizer formalism*

- A set of joint stabilizers generate a group of stabilizers

$$\langle -ZX, -\mathbb{I}X \rangle = \{\mathbb{I}\mathbb{I},\ Z\mathbb{I},\ -\mathbb{I}X,\ -ZX\}$$

- A large enough set stabilizes a unique state, n qubits need n independent generators
- The stabilizer formalism *is* stabilizer QM (with different bookkeeping)
- Some (but not all) superpositions are stabilizer states, and some (but not all) entangled states are stabilizer states

$$\langle -ZX, -\mathbb{I}X \rangle \;\sim\; |0-\rangle$$

$$\langle -ZZ, -XX \rangle \;\sim\; \tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = \tfrac{1}{\sqrt{2}}(|+-\rangle - |-+\rangle)$$

In Stabilizer QM some predictions can be made with probability 1

- The Stabilizer State Tableau Representation (SSTR) replaces the quantum state with its stabilizer generators

$$\langle M_k \rangle_k = \langle -ZX, -\mathbb{I}X \rangle \sim |0-\rangle$$

- For measurements that commute with the stabilizers, SSTR gives probability 1 predictions

$$M = v(M) \prod_{k=1}^{n} M_k^{m_k}$$

- In our example $M_1 = -ZX$, $M_2 = -IX$, and

$$ZX = \underbrace{(-1)}_{v(ZX)} \underbrace{(-ZX)^1}_{M_1^{m_1}} \times \underbrace{(-\mathbb{I}X)^0}_{M_2^{m_2}}, \qquad Z\mathbb{I} = \underbrace{(+1)}_{v(ZI)} \underbrace{(-ZX)^1}_{M_1^{m_1}} \times \underbrace{(-\mathbb{I}X)^1}_{M_2^{m_2}}$$

Calculations can be simplified by using “destabilizers”

- To calculate m_k SSTR uses a set of "destabilizers" C_k

$$M = v(M) \prod_{k=1}^{n} M_k^{m_k}, \quad m_k = M \cdot C_k \bmod 2$$

- The symplectic product checks if two Pauli group elements commute

$$M \cdot M' = \sum_{k=1}^{n} (x_k z_k' - z_k x_k') = 0 \bmod 2 \qquad \Leftrightarrow \qquad [M, M'] = 0$$

- Pick destabilizers C_k that commute with all stabilizers M_j except one

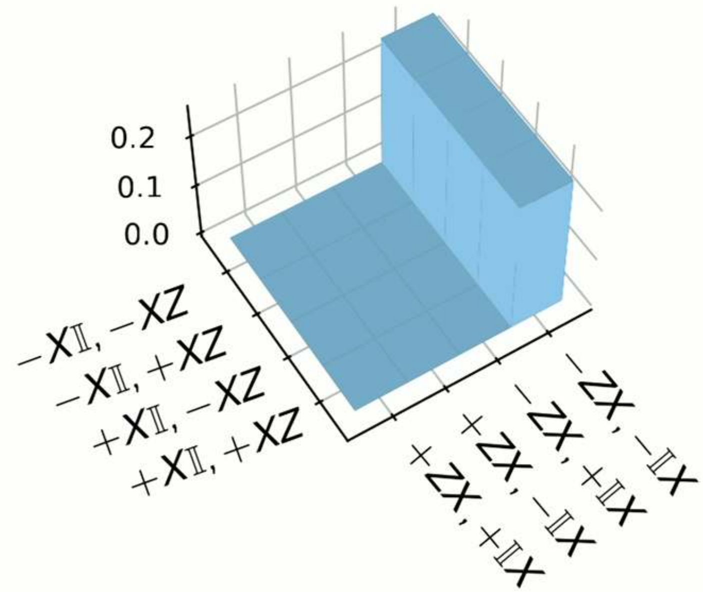
$$C_k \cdot M_j = \delta_{kj} \bmod 2$$

The stabilizers and destabilizers can be used as phase space coordinates

- SSTR state $\{M_1, M_2; C_1, C_2\} = \{-ZX, -\mathbb{I}X; X\mathbb{I}, XZ\}$
- If M commutes with all M_k let $m_k = M \cdot C_k$ and

$$M = v(M) \prod_{k=1}^{n} M_k^{m_k},$$

  where $v(M)$ is the *pre-existing* outcome (don't change the state)
- Otherwise,
  - A: Generate a random outcome $v(M)$ ("quantum randomness")
  - B: Store $v(M)M$ in the M_k, making sure to maintain the commutation relations



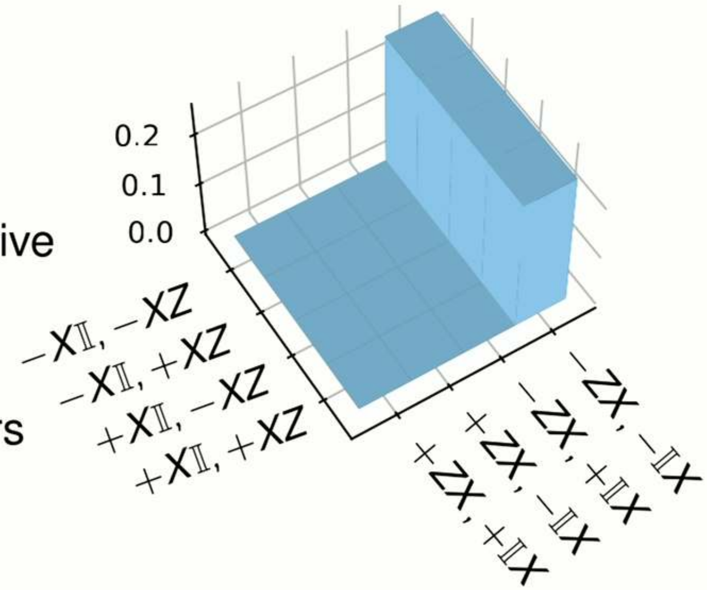
The SSTR state constitutes a symplectic basis

- Observation 1: Our $2n$ elements form a *symplectic basis*,e.g.,

$$\{-ZX, -\mathbb{I}X; X\mathbb{I}, XZ\}$$

- Observation 2: The outcome distribution gives a positive distribution on phase space in these coordinates
- Observation 3: The phase space distribution corresponds to randomized phases of the destabilizers

$$\{-ZX, -\mathbb{I}X; \pm X\mathbb{I}, \pm XZ\}$$



The Contextual Ontological Model (COM) state is a single point in phase space

- COM state $\{M_1, M_2; C_1, C_2\} = \{-ZX, \mathbb{I}X; \pm X\mathbb{I}, \pm XZ\}$
- This is a symplectic basis for Pauli-group elements, so for all M let $m_k = M \cdot C_k$, $c_k = M \cdot M_k$, and

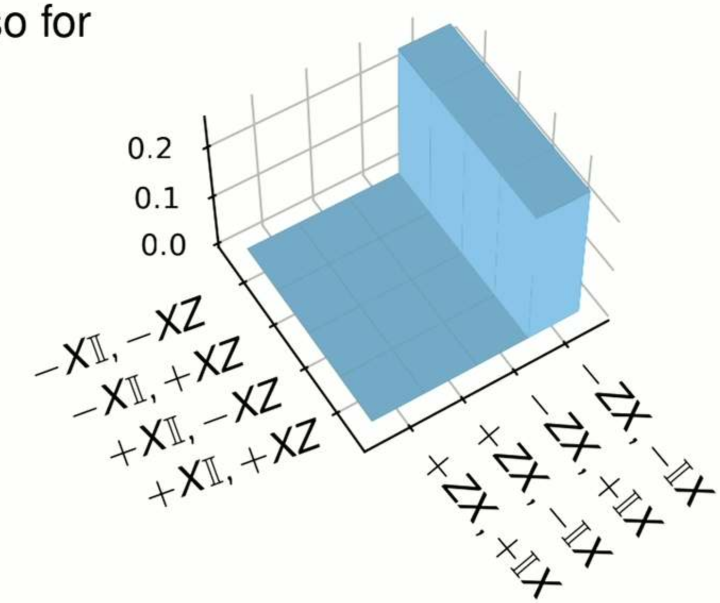
$$M = v(M) i^w \prod_{k=1}^{n} M_k^{m_k} \prod_{k=1}^{n} C_k^{c_k}$$

where $v(M)$ is the *pre-existing* outcome (and $w = 1$ if the two products don't commute)

A: Reveal $v(M)$

B: Store $v(M)M$ in the M_k, making sure to maintain a symplectic basis

C: Randomize the sign of the corresponding C_k (post-measurement "Quantum randomness")



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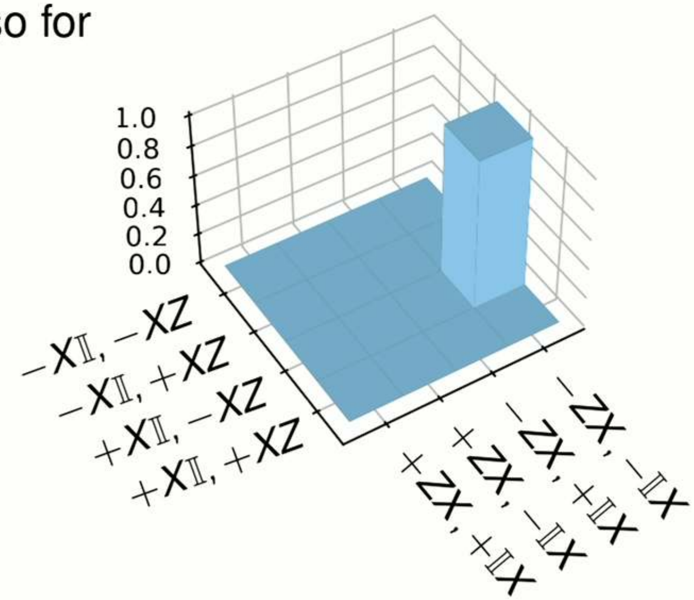
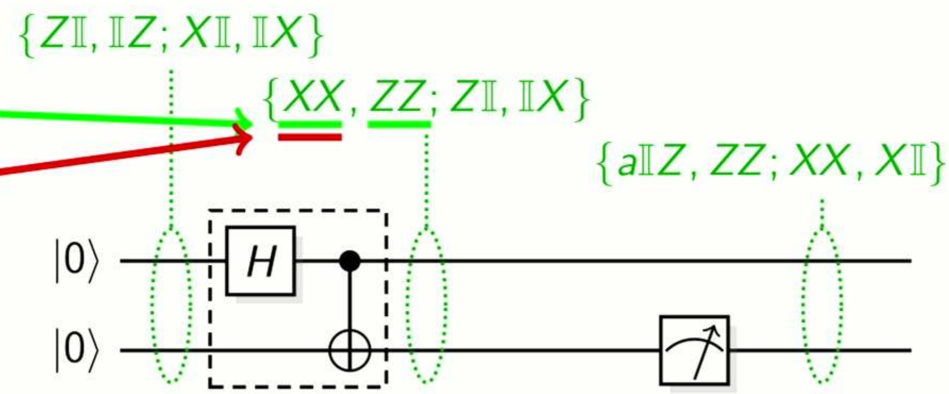


Example: SSTR measurement of Pauli Z on half a Bell state

- The dashed "Bell state generator" outputs $|00\rangle + |11\rangle$, stabilized by XX and ZZ
- The $\mathbb{I}Z$ observable does not commute with XX so the outcome $v(\mathbb{I}Z) = a = \pm 1$ is random
- State update stores the new stabilizer $a\mathbb{I}Z$ in place of XX



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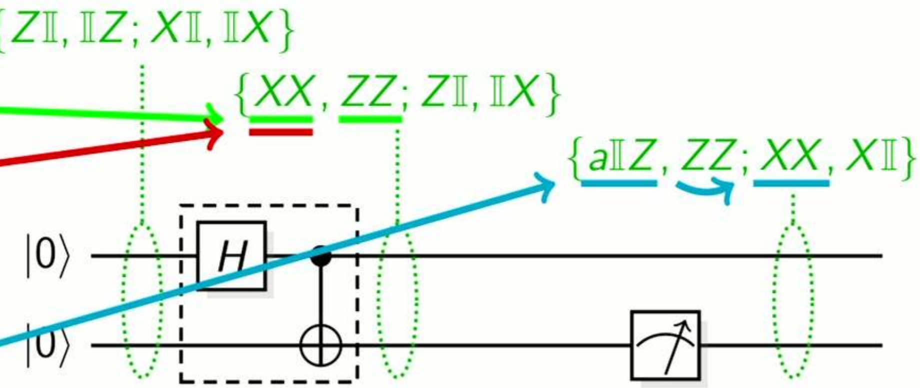
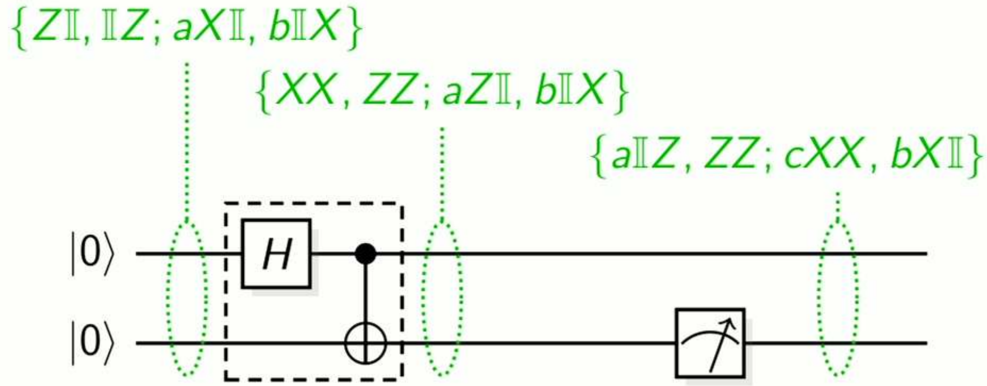


Example: COM measurement of Pauli Z on half a Bell state

- The COM state destabilizers have random initial a and b
- The observable $\mathbb{I}Z = v(\mathbb{I}Z)M_2C_1 = ZZ \times aZ\mathbb{I}$ so the outcome $v(\mathbb{I}Z) = a$ with probability 1
- State update stores the new stabilizer $a\mathbb{I}Z$ in place of XX
- The phase of XX is randomized to cXX



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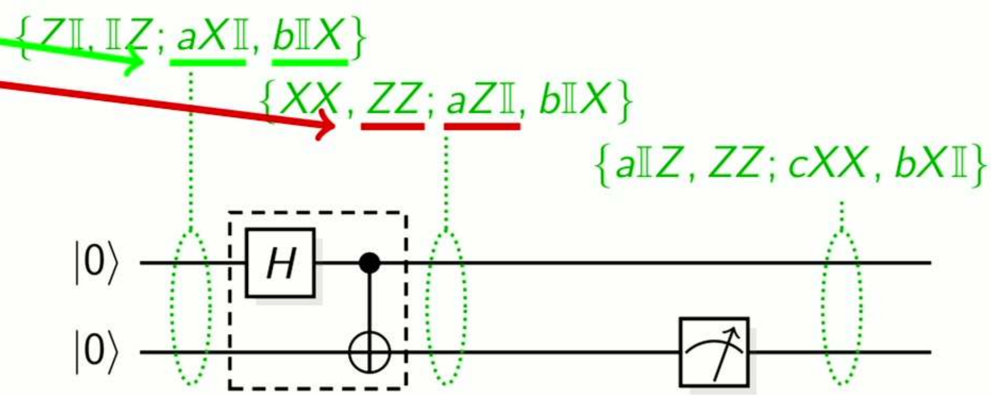

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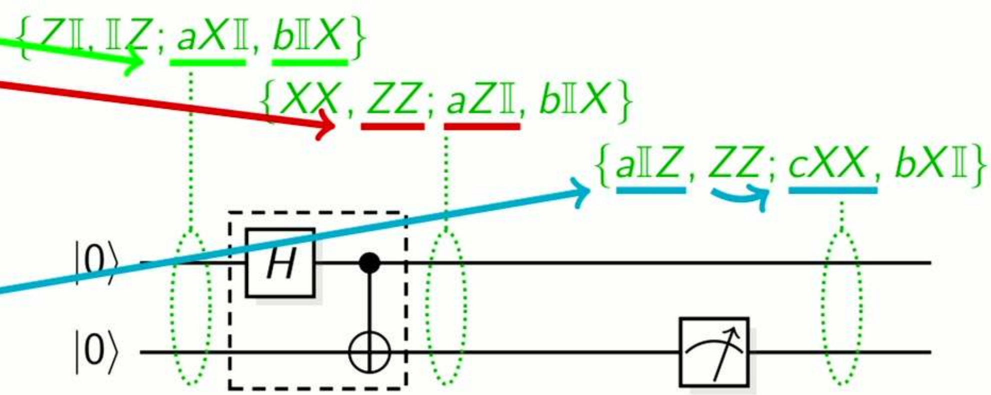
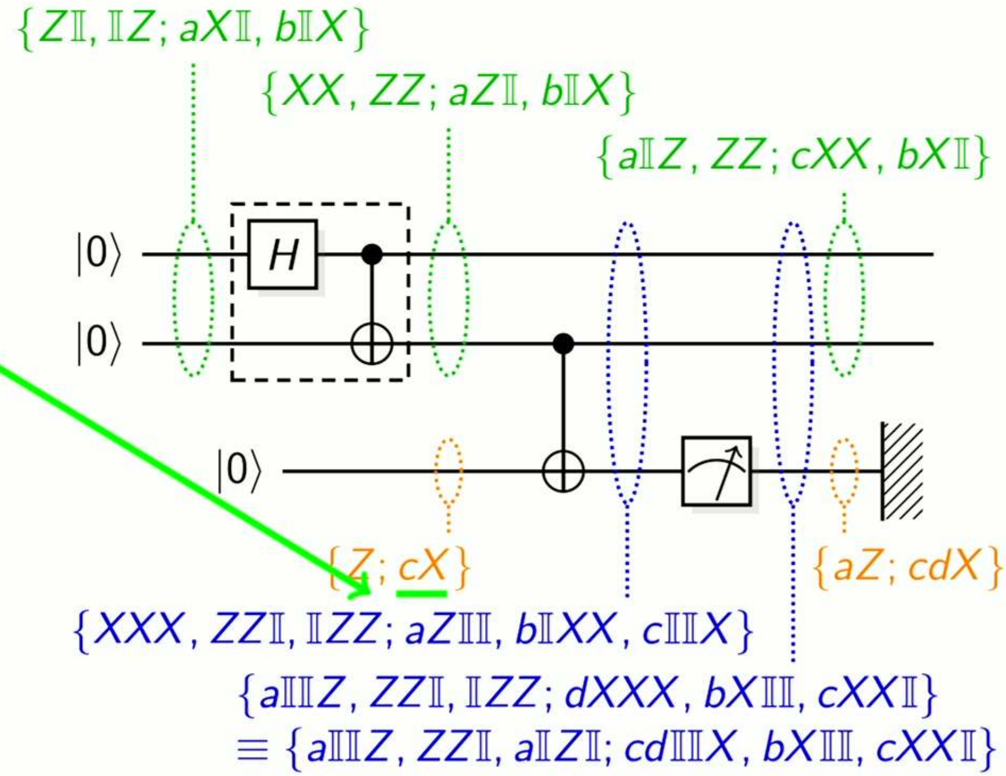


Example: COM Heisenberg microscope on Pauli Z on half a Bell state

- The additional system destabilizer has random initial c
- The observable $\mathbb{I}\mathbb{I}Z = v(\mathbb{I}\mathbb{I}Z)M_2 M_3 C_1$ $= ZZ\mathbb{I} \times \mathbb{I}ZZ \times aZ\mathbb{I}$ so the outcome $v(\mathbb{I}Z) = a$ with probability 1
- State update stores the new stabilizer $a\mathbb{I}\mathbb{I}Z$ in place of XXX
- The phase of XXI is deterministically changed to c



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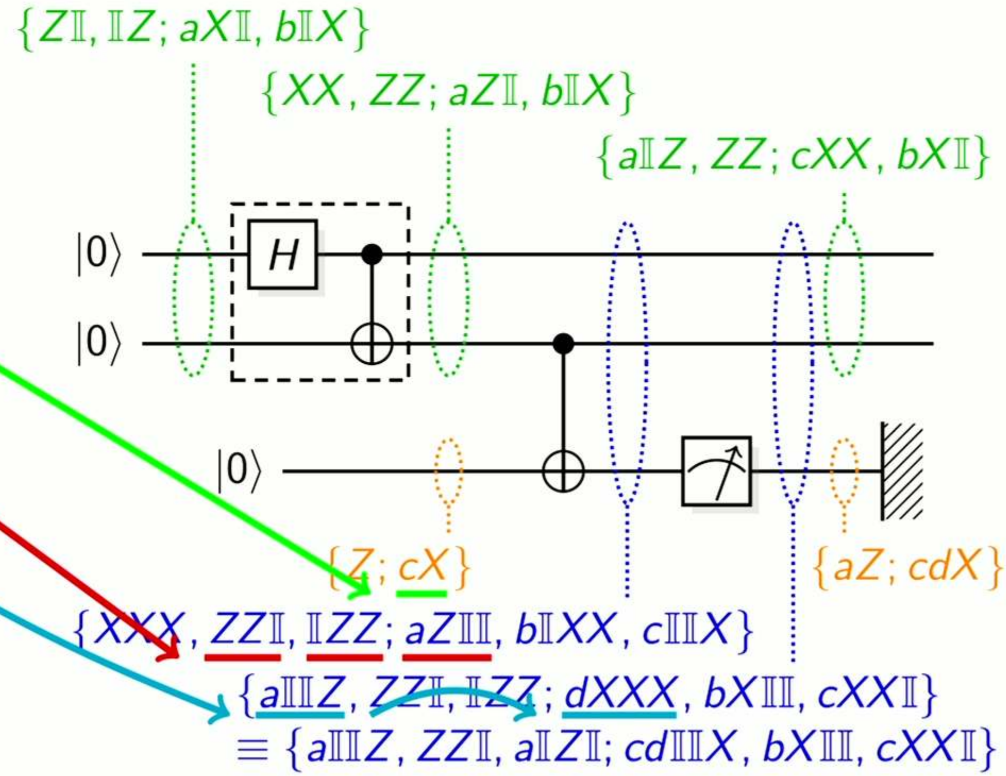

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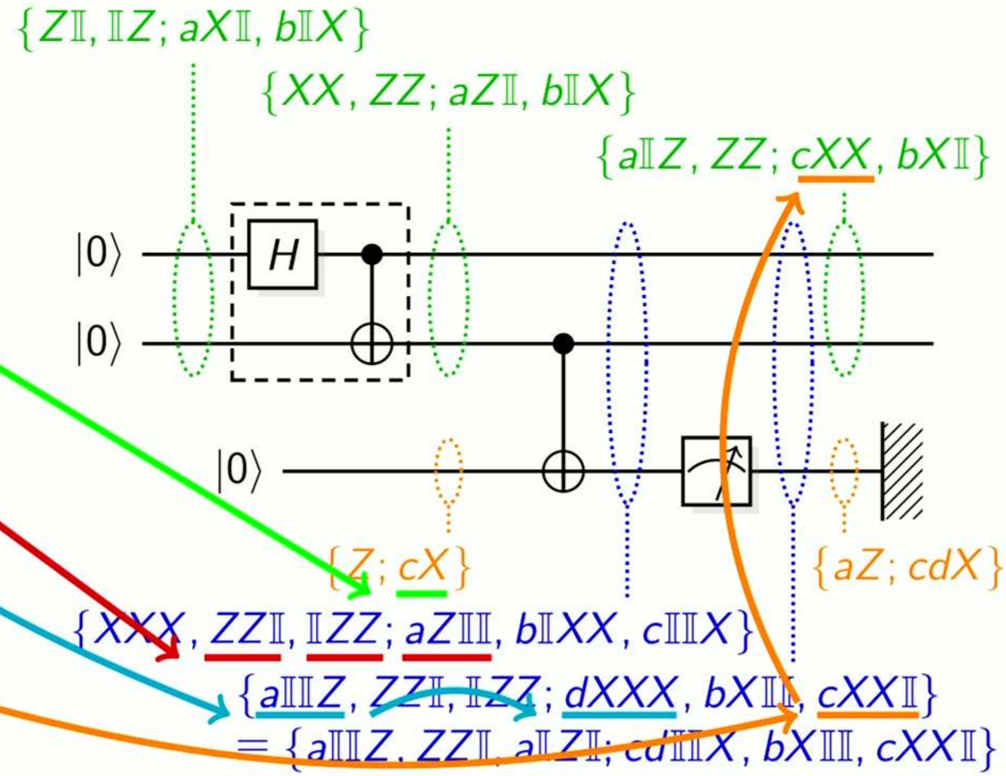
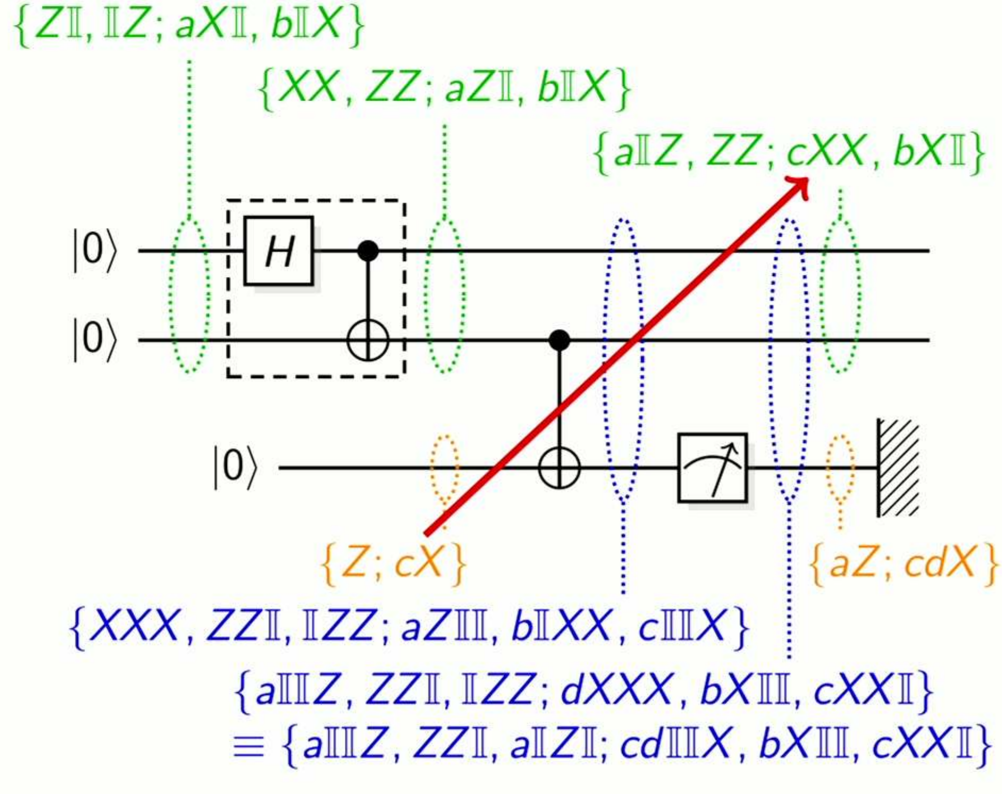


The COM measurement update rule is deterministic

- The phase changes, but in a well-defined manner
- No randomness
- . . . except our ignorance of the initial pointer state

Now the quantum postulate implies that any observation of **stabilizer** QM phenomena will involve an interaction with the agency of observation not to be neglected. **But in spite of this an independent reality in the ordinary physical sense can be ascribed to the phenomena as well as to the agencies of observation.**



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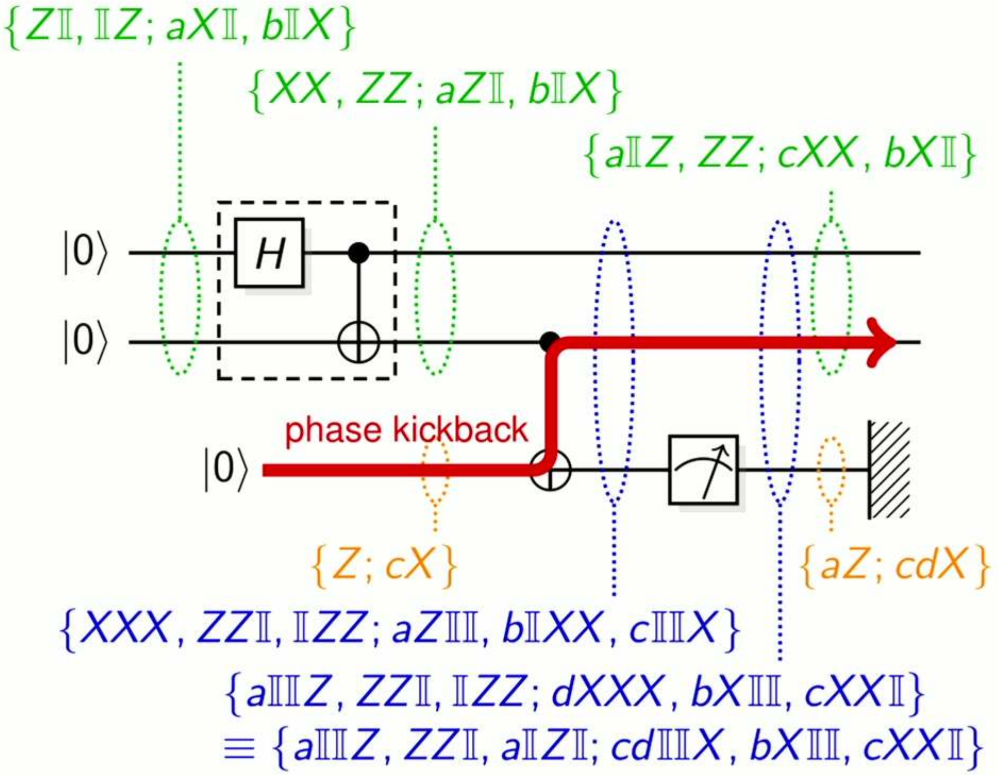
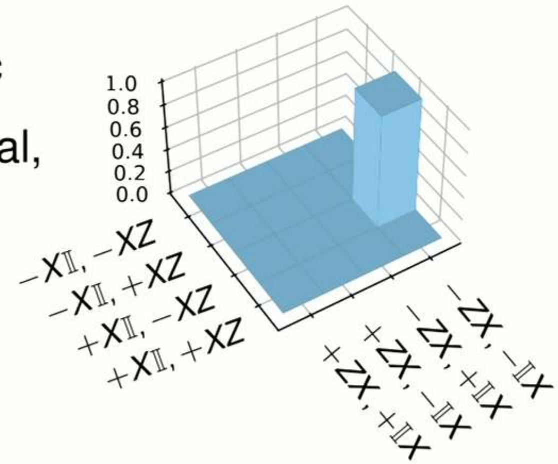


The Contextual Ontological Model is definitely **not** Bohmian mechanics for qubits

	Bohmian mechanics	Contextual ontological model
Deterministic initialization	✘(postulate)	✘($\equiv$measurement)
Deterministic evolution	✔	✔
Deterministic measurement update	✔	✔ (was ✘)
ψ-ontic	✔	✔
Nonlocal	✔	✔
Reveals pre-existing value	✘	✔
EPR complete	✘	✔
Canonical coordinate	Position	Last measured

Conclusions

- The Contextual Ontological Model (PRL 2022) is a phase space description that
  - Is EPR-complete, outcome-deterministic, and ψ-ontic
  - Contains stabilizer QM, is KS-contextual, Bell-nonlocal, but signal-local
  - Has a deterministic measurement update rule, see *The contextual Heisenberg microscope* arXiv:2504.20816



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