Title: Wavefunction branches demand a definition!

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Abstract:

Under unitary evolution, a typical macroscopic quantum system is thought to develop wavefunction branches: a time-dependent decomposition into orthogonal components that (1) form a tree structure forward in time, (2) are approximate eigenstates of quasiclassical macroscopic observables, and (3) exhibit effective collapse of feasibly measurable observables. If they could be defined precisely, wavefunction branches would extend the theory of decoherence beyond the system-environment paradigm and could supplant anthropocentric measurement in the quantum axioms. Furthermore, when such branches have bounded entanglement and can be effectively identified numerically, sampling them would allow asymptotically efficient classical simulation of quantum systems. I consider a promising recent approach to formalizing branches on the lattice by Taylor & McCulloch [Quantum 9, 1670 (2025), arXiv:2308.04494], and compare it to prior work from Weingarten [Found. Phys. 52, 45 (2022), arXiv:2105.04545]. Both proposals are based on quantum complexity and argue that, once created, branches persist for long times due to the generic linear growth of state complexity. Taylor & McCulloch characterize branches by a large difference in the unitary complexity necessary to interfere vs. distinguish them. Weingarten takes branches as the components of the decomposition that minimizes a weighted sum of expected squared complexity and the Shannon entropy of squared norms. I discuss strengths and weaknesses of these approaches, and identify tractable open questions.

89 years of vagueness: Wavefunction branches demand a definition!

Quantum Views **9**, 85 (2025) [arXiv:2506.15663]

C. Jess Riedel

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100 Years of Quantum

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Many names for a constellation of problems

- "Preferred-basis problem"
- "Preferred-(sub)system problem"
- "Set-selection problem"
- "Quantum mereology"
- "Branch-definition problem"
- "Quantum-reality problem"

- Kent's "quantum reality problem" is arguably the most general formulation:
  - What is the sample space for which quantum theory is supposed to determine probabilities?
  - e.g., configurations of beables, events, histories, paths, etc.
- In other words: what is the set of things that can happen?

- I this talk I will sketch a heuristic framework for conceptually organizing historical progress on the reality problem. I categorize by

  1. the (foundational/axiomatic) mathematical structure they *presume*

  2. the (emergent) mathematical structure they *derive*

- Naturally, this framework emphasizes my preferred criteria for an ultimate(-ish) solution:

  - presume only **spatial locality** as axiomatic structure on Hilbert space

  - derive a preferred time-dependent decomposition of the wavefunction of the universe $\psi(t)$ into **branches**

- Historical claim: progress since Birkhoff-von Neumann (1936, 89 years) or Everett (1957, 68 years) has been slow but steady!

  - Admission: I'll focus on work since proto-decoherence ideas of Zeh (1970), but I do think the framing is useful for thinking about earlier stuff, e.g., the generalization from *presumed measurement apparatuse* to *presumed quasiclassical object*

- I'll very briefly describe recent work I find particularly exciting: the quite distinct and incomplete proposals by Weingarten and by Taylor & McCulloch to construct branches from spatial locality using quantum state complexity

Trivial example: Preferred histories → preferred branches

- History "class" operator

$$C_\alpha = \hat{P}_{\alpha_N}(t_N) \cdots \hat{P}_{\alpha_1}(t_1)$$

with decoherence functional

$$D(\alpha, \beta) := \langle \psi | C_\alpha^\dagger C_\beta | \psi \rangle = p_\alpha \delta_{\alpha\beta}$$

- These induce branches

$$|\psi_\alpha\rangle := C_\alpha |\psi\rangle$$

which are orthogonal

$$\langle \psi_\alpha | \psi_\beta \rangle = p_\alpha \delta_{\alpha\beta}$$

- But opposite direction is one-to-many
- Consistent histories "recovers" Everett

Decoherence: subsystem → pointer states

- Crudely, decoherence theory (Zeh, Zurek)

    1. *presumes* some preferred (sub)system $\mathcal{S}$; and

    2. *derives* a preferred set of pointer states $|i\rangle_{\mathcal{S}}$

- Basic decoherence theory does not tell us what the system $\mathcal{S}$ we should focus on

- But the classical limit and desiderata of **predictability** (Zurek, Carroll & Singh) suggest we focus our attention on systems that are **Markovian** and **slow**, properties *determined by the Hamiltonian* $\hat{H}$

Preferred hydro observable → preferred histories

- Consider a fixed observable

$$\hat{A} = \sum_i a_i \Pi_i$$

  that is **hydrodynamic**: it has approximately Markovian equations of motion because

  - it is the mesoscopic average of a density obeying a local continuity equation, so
  - it evolves very slowly compared to microscopic degrees of freedoms which
  - thermalize rapidly compare to evolution of the hydro variable

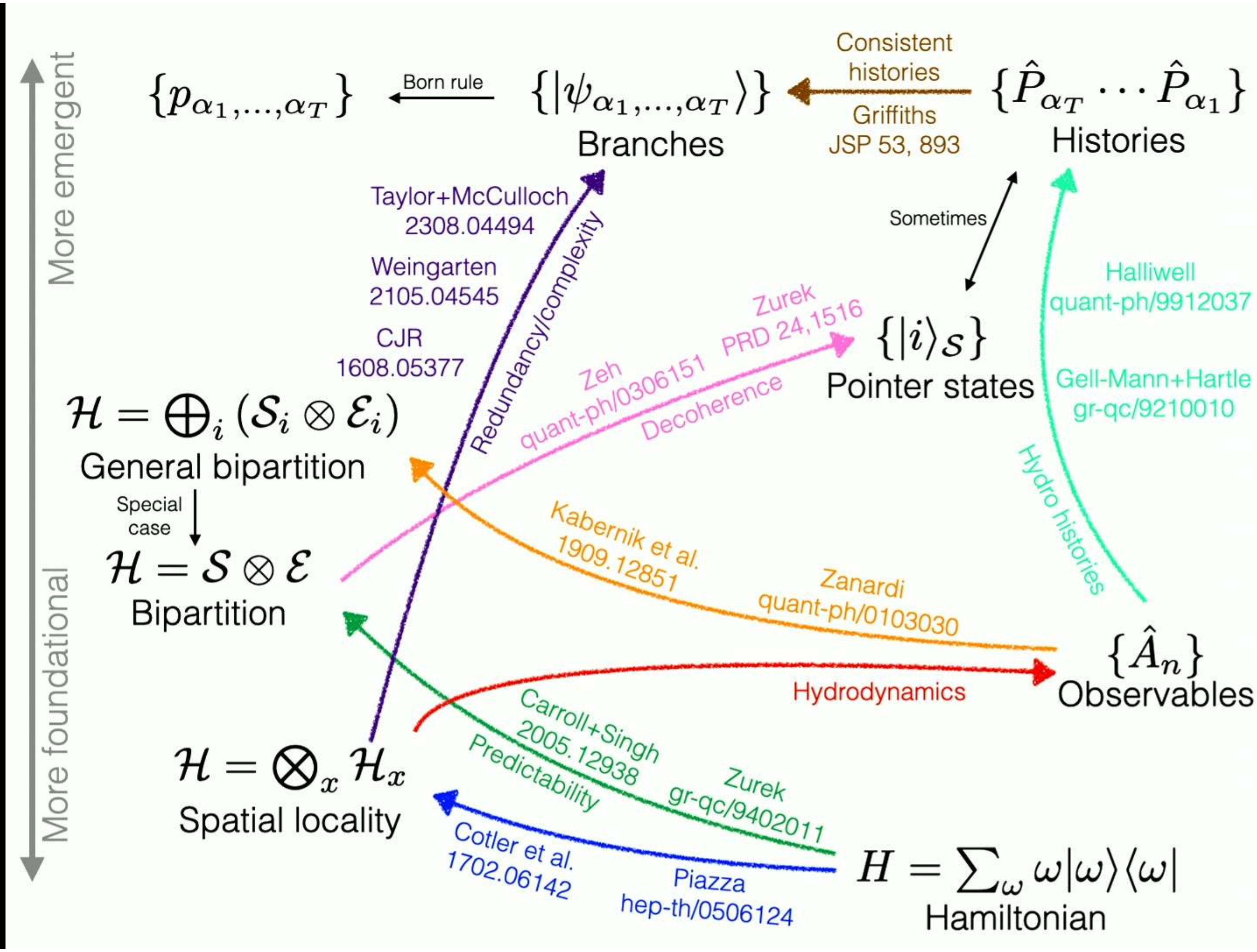
- Then generally (by Halliwell and Gell-Mann & Hartle), for appropriate coarse-graining timescale τ and coarse-graining hydro scale δa, the coarse-grained histories of this observable are consistent:

$$C_\alpha = \hat{P}_{\alpha_N}(t_N) \cdots \hat{P}_{\alpha_1}(t_1) \qquad t_{n+1} - t_n = \tau$$

$$\hat{P}_{\alpha_n} = \sum_{i:|a_i - a(\alpha_n)| < \delta a} \Pi_j$$

$$D(\alpha, \beta) = \langle \psi | C_\alpha^\dagger C_\beta | \psi \rangle = p_\alpha \delta_{\alpha\beta}$$

- Hydro variables are crucially *local* average: picked out by **spatial locality**



More emergent
More foundational
$\{p_{\alpha_1,\ldots,\alpha_T}\}$
Born rule
$\{|\psi_{\alpha_1,\ldots,\alpha_T}\rangle\}$
Branches
Consistent histories
Griffiths
JSP 53, 893
$\{\hat{P}_{\alpha_T}\cdots\hat{P}_{\alpha_1}\}$
Histories
Taylor+McCulloch
2308.04494
Weingarten
2105.04545
CJR
1608.05377
Redundancy/complexity
Sometimes
Halliwell
quant-ph/9912037
Gell-Mann+Hartle
gr-qc/9210010
Hydro histories
Zurek
PRD 24,1516
Zeh
quant-ph/0306151
Decoherence
$\{|i\rangle_{\mathcal{S}}\}$
Pointer states
$\mathcal{H} = \bigoplus_i (\mathcal{S}_i \otimes \mathcal{E}_i)$
General bipartition
Special case
$\mathcal{H} = \mathcal{S} \otimes \mathcal{E}$
Bipartition
Kabernik et al.
1909.12851
Zanardi
quant-ph/0103030
$\{\hat{A}_n\}$
Observables
Hydrodynamics
Carroll+Singh
2005.12938
Predictability
Zurek
gr-qc/9402011
$\mathcal{H} = \bigotimes_x \mathcal{H}_x$
Spatial locality
Cotler et al.
1702.06142
Piazza
hep-th/0506124
$H = \sum_\omega \omega|\omega\rangle\langle\omega|$
Hamiltonian

Complexity for branches

- Both Weingarten and Taylor & McCulloch use quantum state complexity to construct branches
    - (Weingarten uses Nielsen complexity and Taylor & McCulloch use traditional circuit complexity, but I will elide the difference.)
- They rely crucially on some form of the the generic asymptotic linear growth of complexity ("2nd law of quantum complexity"), forms of which have recently been proven rigorously.
    - This is a quantum correlate of the generic linear growth of entropy in classical non-integrable systems
- Definition: Fix a set of "elementary" unitary operations. The **relative complexity** $C_{\mathrm{R}}(\phi, \chi; \Delta)$ between state vectors ϕ and χ is the *minimum* number of elementary operations needed to compose a unitary operator U that maps ϕ and χ up to accuracy $\Delta \in [0, 1]$:

$$|\langle\phi|U|\chi\rangle| \geq \Delta$$

    - On a lattice, the elementary operations are usually N-qubit operators (e.g., N=2), thereby encoding the notion of spatial locality
    - There is yet no principled choice single obvious choice of elementary set in the continuum limit relevant to QFT, but Brown et al. have some results suggesting the renormalization-group ideas dramatically narrow down the plausible candidates

Weingarten branches

- Weingarten propose that the proper branch decomposition of a state ψ is the orthogonal decomposition $\psi = \sum_\alpha \psi_\alpha$ that minimizes a weighted sum of the expected squared complexity of the branches and the Shannon entropy of their squared norms:

$$\begin{aligned} Q(\{\psi_\alpha\}) &:= \sum_\alpha |\psi_\alpha|^2 \left[C(\psi_\alpha, \Omega)^2 - b|\psi_\alpha|^2\right] \\ &= \bar{C}^2(\{\psi_\alpha\}) + b\, S(\{\psi_\alpha\}) \end{aligned}$$

- Here,
  - Ω is the QFT vacuum state
  - $\bar{C}^2(\{\psi_\alpha\}) := \sum_\alpha |\psi_\alpha|^2 C(\psi_\alpha, \Omega)^2$ is the mean squared branch complexity
  - $S(\{\psi_\alpha\}) := -\sum_\alpha |\psi_\alpha|^2 \ln |\psi_\alpha|^2$ is the Shannon entropy of the branch decomposition
  - b is weakly constrained new constant of nature

D. Weingarten, “Macroscopic Reality from Quantum Complexity”, *Foundations of Physics* 52, 45 (2022), arXiv:2105.04545

Taylor-McCulloch branches

- Define **interference complexity** $C_{\mathrm{I}}(\phi, \chi; \Delta)$ and **distinguishability complexity** $C_{\mathrm{D}}(\phi, \chi; \Delta)$ as minimum number of operations to interfere and distinguish, respectively:

$$\frac{|\langle\phi|U|\chi\rangle| + |\langle\chi|U|\phi\rangle|}{2} \geq \Delta \qquad\qquad \frac{|\langle\phi|U|\phi\rangle - \langle\chi|U|\chi\rangle|}{2} \geq \Delta$$

- Taylor & McCulloch propose that a many-body state ψ should be considered to have orthogonal **branches** ϕ and χ when

  1. The state can be expressed as $\psi = \alpha\phi + \beta\chi$ (where $\alpha, \beta \in \mathbb{C}$) and

  2. It's much harder to detect almost any interference between ϕ and χ than it is to distinguish them well:

$$C_{\mathrm{I}}(\phi, \chi; \epsilon) \gg C_{\mathrm{D}}(\phi, \chi; 1 - \epsilon)$$

J. K. Taylor and I. P. McCulloch, "Wavefunction branching: when you can't tell pure states from mixed states" *Quantum* 9, 1670 (2025), arXiv:2308.04494.

The End

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