Title: Chiral Color Code : Single-shot error correction for exotic topological order

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Abstract:

We present a family of simple three-dimensional stabilizer codes, called the chiral color codes, that realize fermionic and chiral topological orders. In the qubit case, the code realizes the topological phase of a single copy of the fermionic toric code. For qudit systems with local dimension d, the model features a chiral parameter α and realizes 3D topological phases characterized by $\mathbb{Z}^\alpha_d$ anyon theories with anomalous chiral surface topological order. On closed manifolds, the code has a unique ground state after removing bulk transparent fermions or bosons. Furthermore, we prove that the bulk is short-range entangled (for odd d, coprime α) by constructing an explicit local quantum channel that prepares the ground state. The chiral color codes are constructed within the gauge color code, and hence inherit its fault-tolerant features: they admit single-shot error-correction and allow code switching to other stabilizer color codes. These properties position the chiral color codes as particularly useful platforms for realizing and manipulating fermions and chiral anyons.

Chiral Color Code:
Single-shot error correction for exotic topological order

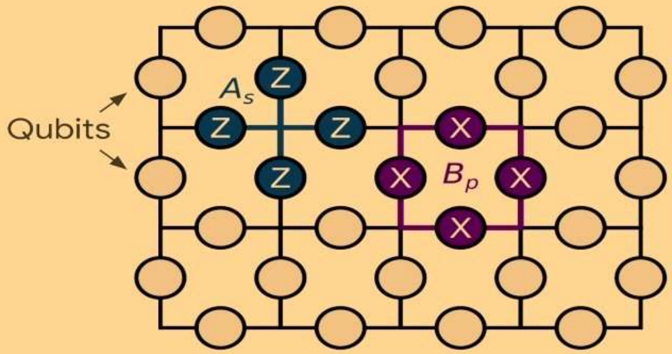
Dongjin Lee and Beni Yoshida (Perimeter Institute)

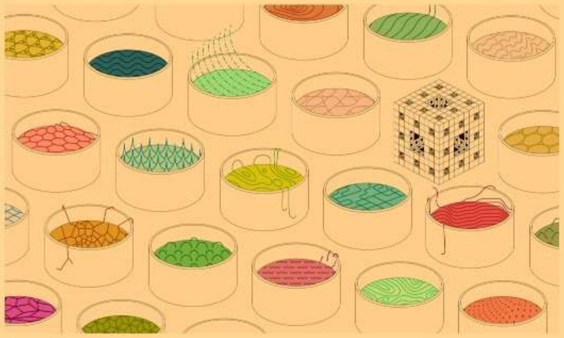
arXiv:2509.18324

Year of Quantum Across Canada
October.08 2025

Introduction

Quantum error correction and quantum matter are deeply interconnected.





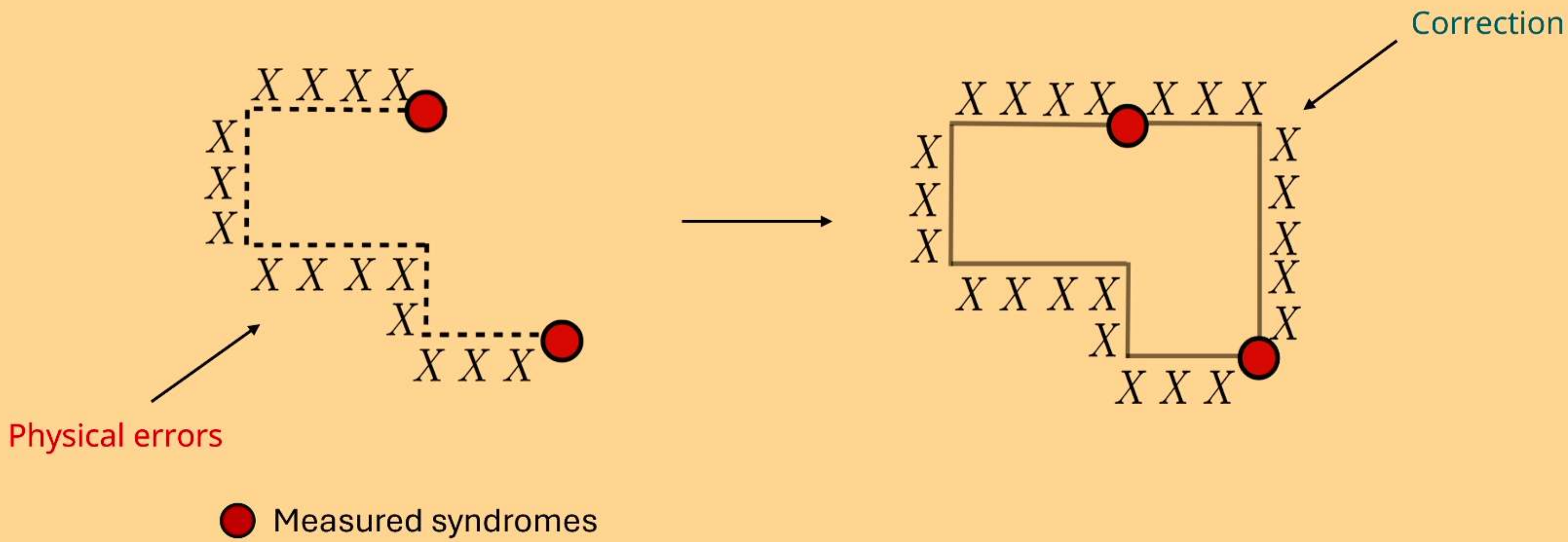
[from quanta magazine]

Topological quantum memory		Topological order
Logical gates	⟷	SPT phases
LDPC code		Spin glass order
⋮		⋮

Introduction

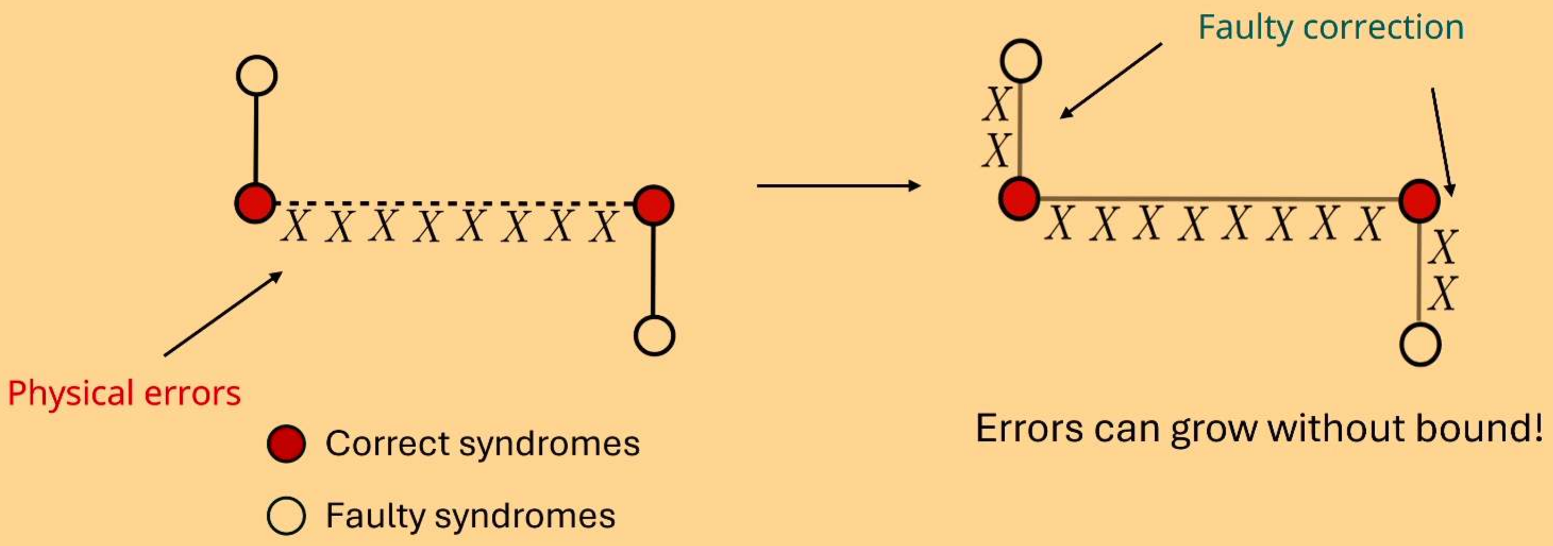
Why?

- One reason is that physical errors are assumed to be **local** in well-controlled settings.



- But these are not the only possible errors : we also need to consider **syndrome measurement errors**.

Syndrome measurement errors



- To obtain a threshold, we need to **repeat** the measurements a number of times that <u>scales with the system size</u>. [Dennis, et. al. (2001)]

Solution: Single-shot error correction

[Bombin (2014), Brown, Nickerson, Browne (2015)]

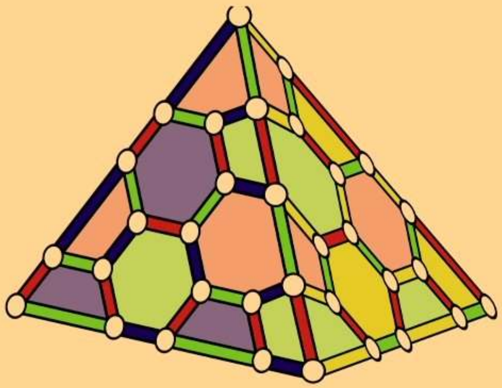
: We can decode a topological code within the framework of <u>subsystem code error correction</u>.

- The first example: <u>3D gauge color code</u>

⟶ Can decode a 3D color code by a single shot.

- Allows fault-tolerant code switching between 3D and 2D color code.

⟶ Fault-tolerant universal quantum computation!



- A 3D toric code and its $\mathbb{Z}_d$ extensions can also be decoded by a single-shot error correction.

[Vasmer, Kubica (2021)] [Stahl (2023)] [Bridgeman, Kubica, Vasmer (2023)]

Can we decode topological codes <u>beyond toric codes</u> using single-shot error correction?

This talk

In this talk, we show how to realize <u>various topological orders</u> in 3D gauge color code.

- For qubit system, we can realize a **3D fermionic toric code.**
- For qudit system, we can realize **Walker-Wang models with $\mathbb{Z}_d^\alpha$ anyon theories.**

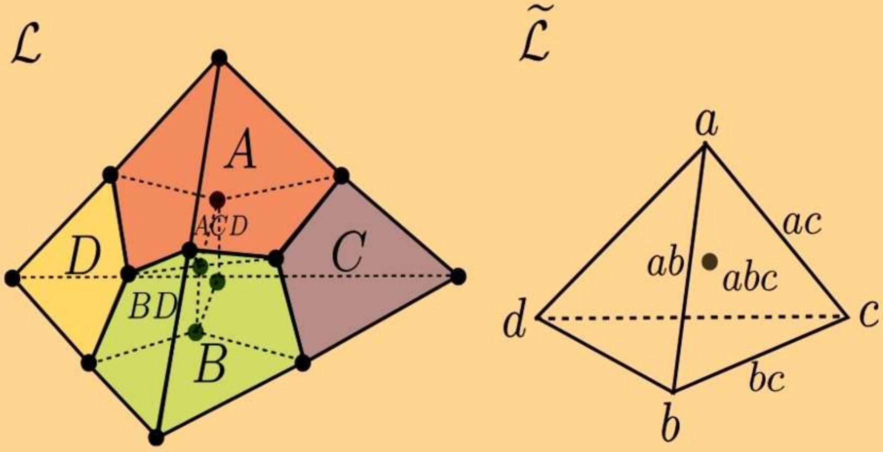
↓

In principle, we can realize **<u>all abelian anyon theories</u>** by condensation!

Review: 3D Gauge color code

Notation

- We consider a four-valent and four-colorable lattice.
- We denote four colors by $\{A, B, C, D\}$. In a dual lattice, we use lower-case alphabet $\{a, b, c, d\}$.
- We denote a set of i cells Δ_i. We assign a color on each cell using the colors of neighboring 3 cells.



Review: 3D Gauge color code

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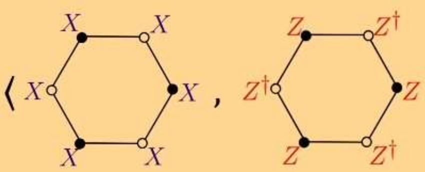
Definition: (3D gauge color code)

$$G_{GC} = \langle X(f), Z(f) \rangle \ , \ f \in \Delta_2$$

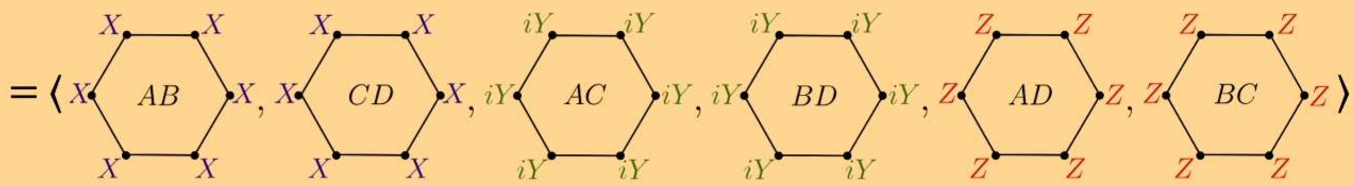
$= \langle$  $\rangle$

$$S_{3DCC} = \langle X(c), Z(f) : f \in \Delta_2, c \in \Delta_3 \rangle \subset G_{GC}$$
$$S_{2DCC} = \langle X(f), Z(f) : f \in \Delta_2(\partial L) \rangle \subset G_{GC}$$

Model: XYZ color code ($\boldsymbol{d} = \mathbf{2}$)

Definition: (XYZ color code)

$$S_{XYZ-CC} = \langle X(f_{AB}), X(f_{CD}), iY(f_{AC}), iY(f_{BD}), Z(f_{AD}), Z(f_{BC}) \rangle \ , \ f \in \Delta_2$$

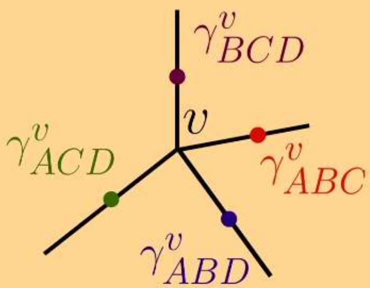


- $S_{XYZ-CC} \subset G_{GC}$
- Different Pauli operators for different colors.
- $k = b_2$ for a closed manifold ⟷ $k = 3b_2$ for the usual 3D color code.

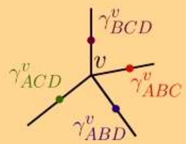
Majorana code

The code realizes the phase of **3D fermionic toric code.** [Levin, Wen (2003)]

- Can check this using Majorana operators. ($\gamma_i^\dagger = \gamma_i$, $\{\gamma_i, \gamma_j\} = 2\delta_{ij}$)



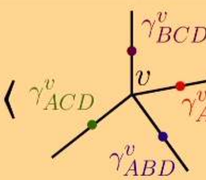
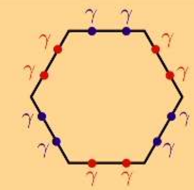
$\dim \mathcal{H}_{\text{local}} = 2^2$



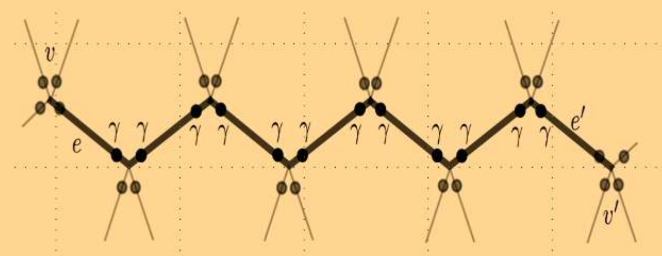
$= I$

$\longrightarrow$

$\dim \mathcal{H}_{\text{local}} \to 2$

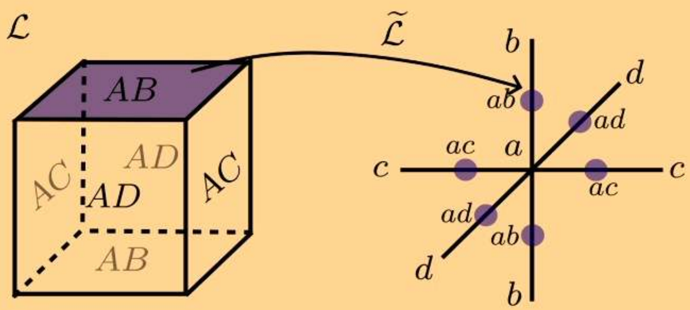
$S_{XYZ-CC} \longleftrightarrow S_{\text{Majorana}} = \langle$  ,  $\rangle$

- Thus, the XYZ color code contains fermionic string operators.



Excitations

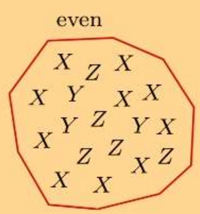
• The code has a local meta check at each volume.



$$\prod_{f \in C_A} S(f) = I$$

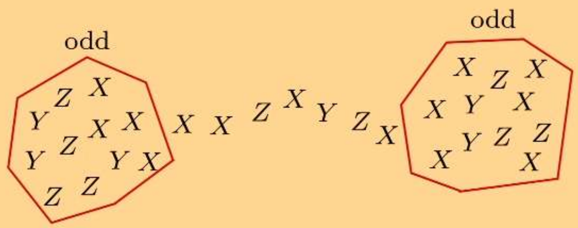
• Thus, <u>all excitations form $\mathbb{Z}_2$ loops</u> in the dual lattice.

1) Even weight:



→ **Bosonic** excitations induced by a membrane operator.

2) Odd weight:



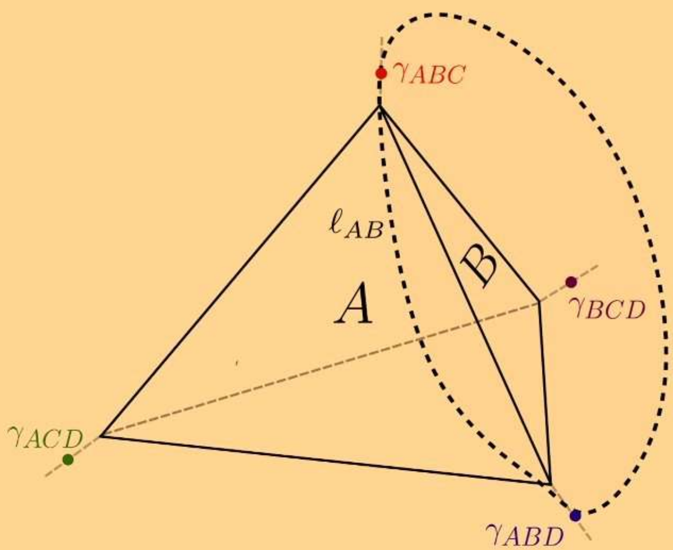
→ **Fermionic** excitations induced by a string operator.

Single-shot error correction with Majorana corners

- On manifolds without defects (closed or open), the gauge color code does not encode any logical qubits.

  ⟶ Single-shot state preparation

- On a <u>tetrahedral lattice</u>, gauge color code encodes 1 logical qubit.



XYZ color code encodes a logical qubit though **Majorana zero modes.**

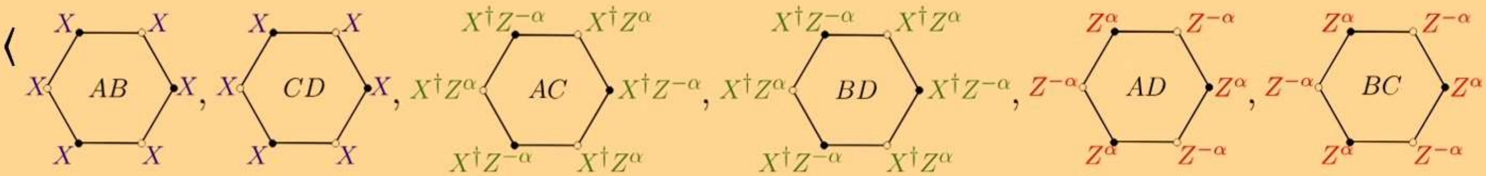
$$\bar{X} \sim O_{l_{AB}},\ O_{l_{CD}} \qquad i\bar{Y} \sim O_{l_{AC}}, O_{l_{BD}} \qquad \bar{Z} \sim O_{l_{AD}}\ ,\ O_{l_{BC}}$$

Model: Chiral color code ($d > 2$)

We can generalize the construction for **qudit systems.**

Definition: (Chiral color code)

$$S_{\text{chiral}-CC} = \langle X(f_{AB}), X(f_{CD}), X^{-1}Z^{-\alpha}(f^{AC}), X^{-1}Z^{-\alpha}(f_{BD}), Z^{\alpha}(f_{AD}), Z^{\alpha}(f^{BC}) \rangle$$

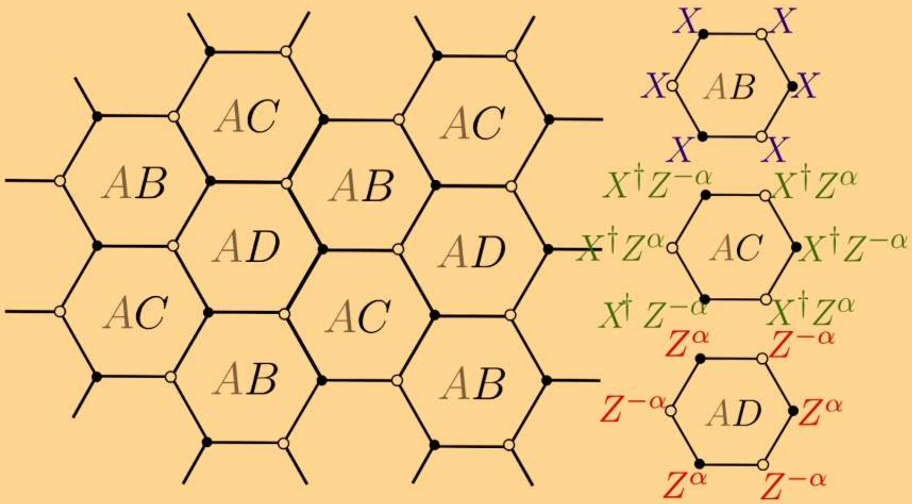
$= \langle$  $\rangle$

- $S_{\text{chiral}-CC} \subset G_{GC}$
- When d **is odd**, the bulk ground state is **short-range entangled**.
- When $d = 0$ **(mod 4)**, the bulk realizes a **bosonic toric code.**

  $d = 2$ **(mod 4)**, the bulk realizes a **fermionic toric code.**
- Color boundary supports **chiral anyon theory.**

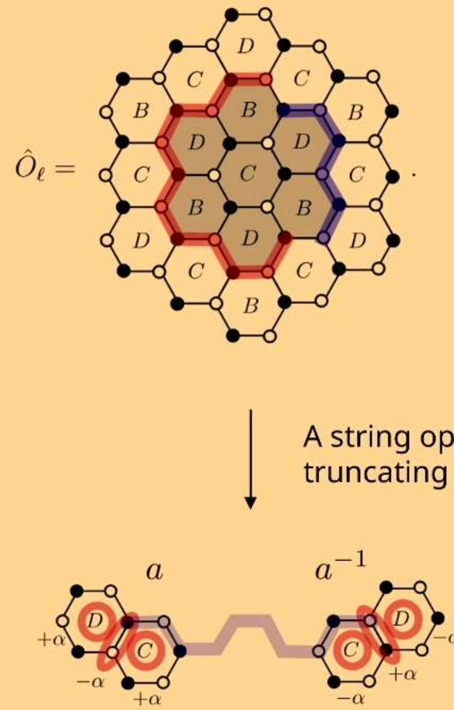
Color boundary with $\mathbb{Z}_d^\alpha$ anyons

A colored boundary



$$\mathcal{A}_d^{\text{toric}} \equiv \{e^i m^j\}_{i,j=0,\cdots,d-1}$$

$$\mathcal{A}_d^{(\alpha)} \equiv \{a^i\}_{i=0,\cdots,d-1} \subset \mathcal{A}_d^{\text{toric}} \ , \ a \equiv em^\alpha$$



A string operator constructed by truncating boundary stabilizers.

$\mathbb{Z}_d^\alpha$ anyon braiding and self statistics.

Conclusion

- There exist simple 3D stabilizer models that realize fermionic or chiral topological order within the 3D gauge color code.
- We can fault-tolerantly process quantum information carried by fermionic or chiral degrees of freedom.

Future direction

- Hidden symmetries in the model (fault-tolerant logical gates)?
- Generalize construction for non-abelian anyon?
- Advantages from hybrid quantum computation using bosonic/fermionic/chiral degrees of freedom?