Title: What are the `axioms' of General Relativity? Ask Quantum Gravity!

Speakers: Fay Dowker

Collection/Series: Quantum Gravity

Subject: Quantum Gravity

Date: October 02, 2025 - 6:30 PM

URL: https://pirsa.org/25100108

Abstract:

The good books on GR agree on certain aspects of the theory that we could consider to be its "core" features. Beyond the core, however, we do not know exactly what GR should allow or not allow physically. For example, should we consider spacetimes with closed causal loops as part of physical GR or not? Are spacetimes with certain types of boundaries or singularities part of GR? Most people expect that the answer to these sorts of questions will be answered by quantum gravity when we have it. We can start to address them within different quantum gravity approaches even at their current partial state of their development. Within the framework of the path integral for quantum gravity and a Feynmanian heuristic for the recovery of the classical approximation, I will sketch out how answers may arise. I will use the example of causal set quantum gravity but I suggest that the ideas are applicable to other path integral approaches.

What are the axioms of GR? Ask Quantum Gravity!

Fay Dowker

Theoretical Physics Group
Imperial College London

Perimeter Institute October 2025

With
Dionigi Benincasa, Michel Buck, Josh Chevalier, L Glaser, Ian Jubb, Roger Liu, Daniel Lloyd-Jones, Bernhard Schmitzer, Rafael Sorkin, Sumati Surya

Plan

- The Feynmanian heuristic
- The gravitational path integral and what recovery of GR looks like
- The causal set approach to quantum gravity
- The action of a causal set
- Two conjectures about the action of manifold-like causal sets.
- Can we rule out timelike boundaries of spacetime?

On recovering classical physics from the Path Integral

(Feynman *et al.*, 1965):

> *"Suppose that for all paths, [the action] S is very large compared to $\hbar$. One path contributes a certain amplitude. For a nearby path, the phase is quite different, because with an enormous S even a small change in S means a completely different phase - because $\hbar$ is so tiny. So nearby paths will normally cancel their effects out in taking the sum - except for one region, and that is when a path and a nearby path all give the same phase in the first approximation (more precisely, the same action within $\hbar$). Only those paths will be the important ones."*

- This is a heuristic for how classical physics is recovered from quantum physics: the paths that dominate the sum are all **close to** the classical trajectory. Such paths are not themselves the classical trajectory but **they are all approximated by it**.
- We predict the classical trajectory up to quantum fluctuations i.e. the common properties of the paths that contribute.
- This heuristic explains why there is an action principle for classical mechanics.

Path Integral Quantum Gravity

$$\text{“}Z = \sum_{M \in \mathcal{M}} \int_{g \in \mathcal{G}} [dg]\, e^{iS[g]}\ \text{”}$$

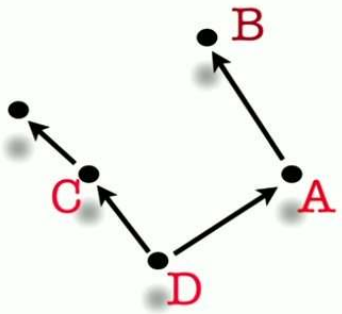
is a statement of intent (Loll), a hope for a gravitational sum-over-histories (SOH).

- Basing quantum theory fundamentally on the SOH is work in progress (Hartle, Sorkin).
- In the meantime, we can use the Feynman heuristic. We ask: which (classes of) histories dominate and which are suppressed in the SOH?
- For GR to emerge from path integral quantum gravity á la Feynman we need:
  Suppression of class of all “bad” histories
  Domination of class of histories approximated by GR solution(s)
- One aspect of the cosmological constant problem in quantum gravity from the path integral perspective is that “bad histories threaten to dominate”. (More on this later.)

I will use causal set quantum gravity to illustrate how these issues are playing out in a specific path integral approach.

3 pillars of Causal Set approach to quantum gravity

1. **Discreteness:** physical, fundamental atomicity of spacetime at the Planck scale.
2. **Order:** a more primitive organising principle for physics even than space and time: the spacetime causal order--of all continuum spacetime structure--survives in the deep theory.



Causal set = discrete order = transitive directed acyclic **graph**

10^{240}

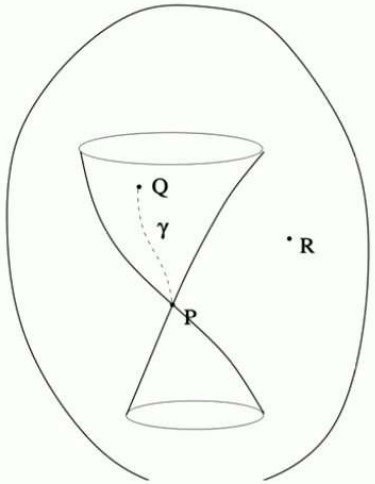
3. **Path integral:**

Finite!

$$\text{“ } Z(V) = \sum_M \int \mathcal{D}g\ e^{iS[g]} \longrightarrow Z(N) = \sum_{\text{causal sets of cardinality N}} e^{iS(C)} \text{”}$$

In the Feynmanian scheme of GR recovery, we can already predict that spacetime cannot have closed causal curves because there are no causal sets approximated by spacetimes with ccc's.

Causal Order is central to GR



GR textbooks are full of Penrose diagrams

Black holes are the epitome of GR and black hole uniqueness theorems, no-hair theorems, Hawking's area theorem, follow from the **causal** nature of the event horizon.

Theorem (Kronheimer-Penrose-Hawking-Malament): The spacetime causal order gives the topology (incl. dimension) the differential structure and 9/10 of the metric (in 3+1)

In other words: Order + Volume = Geometry

For a **discrete** order (causal set) the **counting measure** can provide the volume:

Order + Number = Geometry (conjecture)

The Discrete-Continuum Correspondence for causal sets :

A causal set C recovers (M,g) if C **faithfully embeds** in (M,g) at Planck density: i.e. there exists an embedding $C \rightarrow M$ such that

(i) Number $\approx$ Volume in large, physically nice regions and

(ii) The order of C equals the causal order of the embedded points in (M,g)

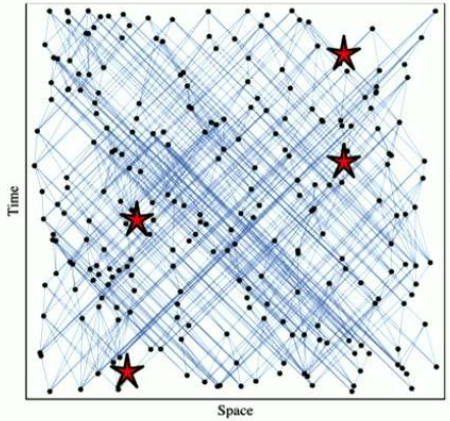
We can produce such causal sets by a Poisson process "sprinkling" into (M,g)

Evidence that this holds correspondence holds water:

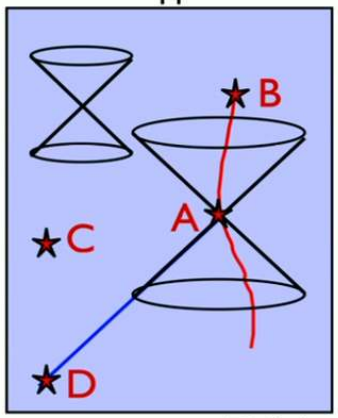
- Kronheimer-Penrose-Hawking-Malament theorem
- Meyer-Sorkin theorem: At infinite density, a faithfully embedded causal set $\longrightarrow$ (M,g) with probability one.
- Direct evidence that can read out geometry from a sprinkled causal set e.g. dimension (Myrheim-Meyer), spatial topology (Major, Rideout&Surya) geodesic proper time (Myrheim, Brightwell&Gregory, Bachmat)

Every nice Lorentzian geom is an approximation to some causal set

Causal set (atoms)



Continuum approximation (fluid)



Finite!

$$\text{“ } Z(V) = \sum_M \int \mathcal{D}g\ e^{iS[g]} \longrightarrow Z(N) = \sum_{\text{causal sets of cardinality N}} e^{iS(C)} \text{”}$$

The sum over all causets includes all nice Lorentzian geometries (many times), all nice topologies (incl dimensions) and a **lot** more…….

Causal set actions (Dionigi Benincasa, FD & L Glaser)

1-parameter family of bilocal actions. Parameter corresponds to spacetime dimension. C is a *finite* causet.

$$\text{The 2d BDG action is} \qquad S^{(2)}[C] \equiv 2\Big[N - 2N_1 + 4N_2 - 2N_3\Big]$$

where N is the cardinality of C and N_m is the number of *order intervals* of cardinality $m+1$ in C. [BB]

$$\text{The 4d BDG action is} \qquad S^{(4)}[C] \equiv \zeta_4\Big[N - N_1 + 9N_2 - 16N_3 + 8N_4\Big]$$

where $\zeta_4 = -\frac{4}{\sqrt{6}}(\frac{l}{l_p})^2$ is a dimensionless number of order 1, l is a fundamental length and $l_p = \sqrt{8\pi G}$ is the (cosmologists') Planck length.

- *Sprinkling* into a d-dimensional spacetime (M, g) at density $\rho = l^{-d}$ turns these causet actions into random variables, $\mathbf{S}_l^{(2)}(g)$ and $\mathbf{S}_l^{(4)}(g)$
- How is the random variable, $\mathbf{S}_l^{(d')}(g)$, related to the continuum Einstein Hilbert action of $S_{EH}[g]$?

Behaviour of Causal set actions I: towards a solution of the Cosmological Constant problem

The action has to be special. It must suppress the bad histories which dominate entropically in the SOH. The number of causal sets of cardinality N grows like $2^{N^2/4}$ and the vast majority of those are the Kleitman-Rothschild 3-layer orders. [BB]

- The action of a KR order equals the number of links: N_1
- Remarkably, one can show (Loomis & Carlip, 2018; Carlip *et al.*, 2022)

$$\lim_{N\to\infty} \sum_{\text{KR orders}} e^{i\alpha S[C]} = 0$$

  and the limit is approached exponentially fast, for any choice of BDG action (for a range of possible α).
- k-layer orders are also suppressed, for any fixed k.
- So a large class of bad histories (the most numerous) are suppressed in the SOH
- Much more to do

Behaviour of Causal set actions II: Sprinklings

If $d' \neq d$ the typical value of S for a sprinkling is nonlocal $\sim N^{2-2/d'}$ for $d > 2$ and $N \log N$ for $d = 2$.

If $d' = d$ then:

Conjecture 1: for (M, g) globally hyperbolic and fixed, as $l \to 0$:

$$\lim_{l\to 0} \langle \boldsymbol{S}_l^{(d)}[g] \rangle = \frac{1}{l_p^{d-2}} \int_M d^d x \sqrt{-g} \frac{R}{2} + \frac{1}{l_p^{d-2}} \int_J d^{d-2} x \coth(\Theta(x))$$

where J is the "joint", the intersection of the past and future boundaries and Θ is the Lorentzian angle between the past and future boundaries in the co-dimension 2 plane normal to both [BB].

- No Gibbons-Hawking-York b.t. Have to add it by hand.
- Verified for flat causal diamonds in any dimension: limit of mean action is the joint term which is just its (co-dimension 2) volume (Buck *et al.*, 2015).
- Verified for special examples of conformally flat space and in causal intervals in Riemann normal neighbourhoods to first order in curvature (FD; Machet & Wang)
- Evidence from deSitter simulations (D. Benincasa, D, Rideout unpublished)
- Evidence for joint term from examples and simulations in flat space (FD, Daniel Lloyd-Jones, Roger Liu)

Behaviour of Causal set actions III

Note that for the conjecture to hold need very effective cancellation:

- d=4: $\langle S \rangle \sim N^{1/2}$, dominant terms $\sim N^{3/2}$
- d: $\langle S \rangle \sim N^{1-2/d}$, dominant terms $\sim N^{2-2/d}$

Conjecture 2: for (M, g) with **timelike** boundary Σ, as $l \to 0$:

$$\lim_{l \to 0} \langle \boldsymbol{S}_l^{(d)}[g] \rangle \, l = \frac{a_d}{l_p^{d-2}} \int_\Sigma d^{d-1}x\sqrt{-h}$$

where a_d is a dimension dependent constant of order 1. We have numerical evidence from flat space examples of this conjecture and numerically derived value ranges for a_d. (FD, Liu & Lloyd-Jones).

This timelike boundary term $\langle \boldsymbol{S}_l^{(d)}(g) \rangle = O(l^{-1})$ and dominates over any EH or joint term in the limit.

Assuming the conjectures hold: Potential Implications

Why do boundaries of space not exist?

- Take action $d = 4$. Suppose all the non-manifoldlike causal sets are suppressed (à la Carlip, et al)
- All with dimension $\neq 4$ will be suppressed (no cancellations between terms of action means a small change in C will change $S[C]$ by order 1).
- For globally hyperbolic causal sets, the action is close to the EH action* and a Feynmanian argument implies suppression of non-solutions compared to solutions.
- Comparing causal sets with and without boundary, of realistic cosmological scales:

$$\frac{1}{\hbar}S(C_{edge}) \sim 10^{180}, \quad \frac{1}{\hbar}S(C_{no\ edge}) \sim 10^{120}$$

The large ratio of the magnitudes of their actions indicates that the measure of the neighbourhood of C_{edge} over which the action is approximately constant is much smaller than that of $C_{no\ edge}$, leading to the suppression of M_{edge} in the gravitational path integral.

Additional discussion points.

- *Fluctations of the action around the mean can't be ignored.
- The BDG action is real and has no imaginary contribution even when there's topology change in the approximating manifold. Analytic continuation of the parameters of the action?