Title: Varieties of Rigour: Charting the Landscape of Reformulation Programmes in 1950 Quantum Field Theory

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Abstract:

Historians and philosophers of quantum field theory (QFT) face a distinctive challenge: the existence of multiple, mathematically and conceptually divergent formulations of the theory. Discussions of this issue in the extant literature often contrast the mainstream perturbative formalism—empirically powerful but mathematically dubious—with axiomatic QFT—mathematically rigorous but largely detached from empirical predictions. This paper complicates this dichotomy by drawing attention to a parallel tradition, beginning in the 1950s, that sought to render perturbative QFT itself more rigorous. One way to think about this is that there is considerable diversity even within the "mathematical QFT" camp. In order to account for this, I suggest that the axiomatic QFT and causal perturbation theory traditions can be understood as adopting distinct ideals of mathematical rigour: a global and a local conception, respectively.

Varieties of Rigour: The Landscape of Reformulation Programmes in 1950 Quantum Field Theory

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Varieties of QFT

There are many approaches to formulating quantum field theory (QFT).

This raises the question, if we are interested in the philosophical foundations of the theory, which version should we focus on?

Axiomatic QFT vs Conventional QFT

Philosophers have distinguished two versions of QFT in particular:

Axiomatic QFT	Conventional QFT
Rigorous	Non-rigorous
No Realistic Models	Predictively Successful
Non-perturbative	Perturbative

In the debate between Doreen Fraser (2011) and David Wallace (2011) this is presented as an underdetermination problem, with D. Fraser arguing that the underdetermination ought to be broken in favour of axiomatic QFT and Wallace in favour of conventional/cutoff QFT.

Sophistocating this Picture with History

In this talk I point out that axiomatic QFT is not the only mathematically orientated reformulation of QFT: we also find rigorous reformulations of perturbative QFT, exemplified by the causal perturbation theory approach.

I look at how these different strands of "mathematical QFT" developed in the 1950s.

Why the 1950s?

In the 1950s there were renewed concerns about the consistency of QFT leading to a rise of reformulation programs aimed at improved mathematical rigour: axiomatic QFT and causal perturbation theory both go back to this period.

Examining these programs historically illuminates the different methodological strategies motivating them.

Varieties of Rigour

Claim: Axiomatic QFT and causal perturbation theory were animated by different conceptions of the project of making physics mathematically rigorous:

1. Axiomatic QFT by a global conception of rigour
2. Causal perturbation theory by a local conception of rigour


James Fraser

QFT in the 1930s: The Ultraviolet Divergence Crisis

Most of the empirical predictions of QFT are derived from a perturbative expansion in the interaction coupling:

$$F(\alpha) = a_0 + a_1\alpha + \boxed{a_2}\,\alpha^2 + ...$$

Ultraviolet divergences appear in the coefficients, however. E.g., momentum-space integrals like:

$$\int^{\Lambda} d^4q \frac{1}{(q^2+m^2)^2} \sim ln\Lambda.$$

This led to worries about the consistency of the QFT framework in the 30s.

QFT in the 1940s: Perturbative Renormalization

In the late 40s renormalization techniques were developed which systematically removed the infinities from the series coefficients leading to extremely accurate empirical predictions.

Roughly speaking, the divergences are absorbed into an infinite "bare coupling", with the series being re-expressed in terms of a finite "renormalized coupling".

QFT in the 1950s: Renewed Crisis

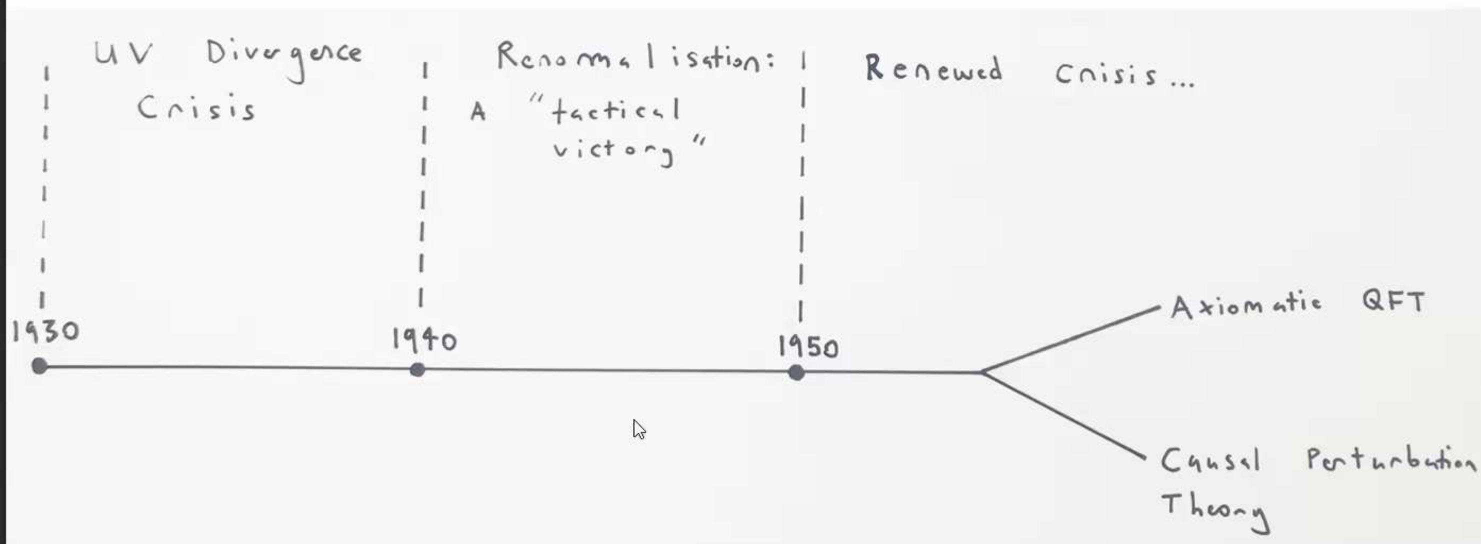
We can see the 1950s as a period of critical reflection on what had really been achieved with the invention of renormalization. The following problems arose:

1. The mathematical rigour of the renormalization procedure was questioned. As a result, its physical interpretation was contentious.
2. Even after the ultraviolet divergences had been dealt with, it was argued in the 1950s that QFT perturbation series do not converge.
3. New non-perturbative reasons to worry about the consistency of QFT were developed, e.g. the Landau pole problem.

Mathematical QFT

Mathematical QFT: Faced with this atmosphere of confusion, many theorists argued that higher standards of mathematical rigour were needed to decisively answer the key foundational questions about QFT.

Two Strands of Mathematical QFT



Axiomatic QFT

Axiomatic QFT attempted to move beyond the perturbation series and provide an entirely non-perturbative formulation of the theory.

Strategy:

1. Specify a class of basic objects.
2. Write down axioms that they can be expected to satisfy.
3. Derive the consequences of the axioms.

Wightman

Haag

Algebraic QFT

In the case of the Haag-Kaster (1964) axiomatisation of QFT, The basic objects are C*-algebras of observables associated with regions of Minkowski space-time, $\mathfrak{A}(\mathcal{O})$.


Haag

We then write down axioms like:

1. **Isotony**. If we have a smaller region $\mathcal{O}_1$ contained in a larger region $\mathcal{O}_2$, then $\mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$.
2. **Micro-causality**. If $\mathcal{O}_1$ and $\mathcal{O}_2$ are spacelike separated regions then $[\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = \{0\}$.

Results and Open Issues

Successes:

1. Rigorous proofs of structural results like the spin-statistics and CPT theorems.
2. Characterisation of inequivalent Hilbert space representations in the limit of infinite degrees of freedom.

However, constructing four-dimensional interacting models of the Wightman or Haag-Kastler axioms remains an open problem.

Wightman

Haag

Causal Perturbation Theory

Another parallel program focused on the mathematical structure of the series expansion itself.

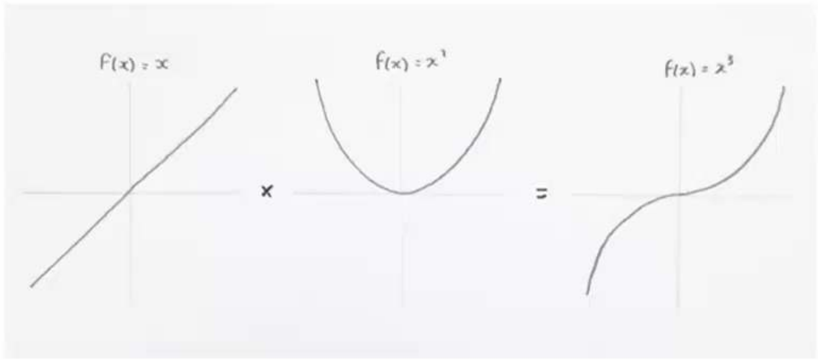
What came to be known as causal perturbation theory:

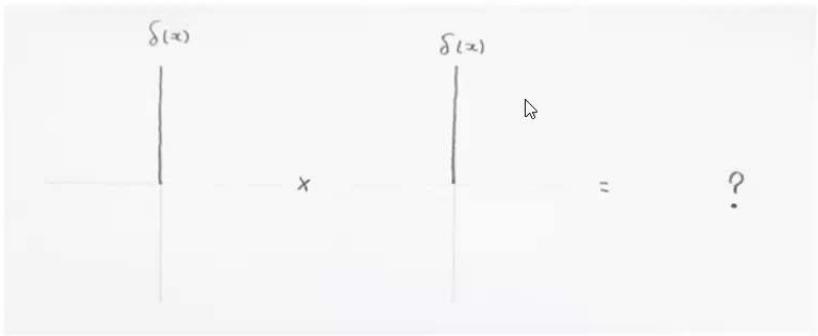
1. Developed an alternative derivation of the series expansion based on using a causality condition to iteratively construct each term.
2. Developed a distribution theoretic analysis of perturbative ultraviolet divergences and renormalization.

Stueckelberg

Bogoluibov

Distribution Theoretic Renormalization





A new diagnosis of the ultraviolet divergences problem: infinities occur in the conventional formulation because singular distributions are naively multiplied together.

Distribution Theoretic Renormalization

Key result: we can provide a definition for the products of distributions appearing in QFT perturbative expansions without ever writing down or manipulating ill-defined integrals:

1. Start by defining the products on a restricted space of test functions which vanish at coincident points—this is what you get from the causality condition.
2. Extend this product to the full space of test functions. This extension exists but it is not unique.

This construction method was developed an improved by later mathematical physicists from the 1970s onwards (Epstien and Glaser, 1973).

Results and Open Issues

Successes:

1. Gives a more rigorous reconstruction of renormalized perturbation theory.
2. Clarifies the interpretative significance of the ultraviolet divergence problem - it doesn't show that QFT breaks down at high energies!

However, causal perturbation theory does not address the issues associated with the sum of the series and so leaves the consistency of QFT (existence of exact solutions) open.

1. Motivation
2. Crisis in 1950s QFT
3. Axiomatic QFT
4. Causal Perturbation Theory
5. Varities of Rigour

Accounting for the Diversity of Mathematical QFT

Question: How did the overarching goal of improving the mathematical rigour of QFT lead to such different research programs?

One answer: axiomatic QFT and causal perturbation theory add additional methodological commitments to their pursuit of improved rigour.

AxQFT: Rigour + Reject perturbation theory

CPT: Rigour + Develop perturbation theory

Wightman on Perturbation Theory

"[A]re the renormalized series convergent or divergent? Although there is even yet no proof in quantum electrodynamics, it is generally believed on the basis of a study of simpler cases that they are divergent... Thus at this stage in the development, which occurred in the early 1950s, the key problem was to find some non-perturbative approach to the solutions of quantum field theory." (Wightman, 1976, 195)

The axiomatic QFT program attempted to move away from perturbation theory entirely and develop a new non-perturbative language for relativistic quantum theory.

Bogoliubov on Perturbation Theory

"[Perturbative QFT] best corresponds to the actual modern state of field theory, where up to the present various formal expansions in powers of the smallness of the interaction cannot be removed and where all fundamental results were obtained with the help of these expansions" (Bogoliubov 1955, 237)

Causal perturbation theory is more conservative: it tries to shore up the mathematical rigour of the existing framework used to describe interactions.

Global vs Local Conceptions of Mathematical Rigour

But, why did axiomatic QFT and causal perturbation theory have different views of perturbation theory?

Suggestion: we can explain this by viewing these programs as being animated by different conceptions of rigour in physics:

1. Axiomatic QFT by a global conception of rigour.
2. Causal perturbation theory by a local conception of rigour.

Global Conception of Rigour

- Inspired by Hilbert's Axiomatic Program in Mathematics, and by Hilbert's sixth problem - the axiomatization of physics - in particular.
- Rigour is established by starting from well-defined objects and building up using well-defined operations. If we can recover the key results of an area of physics in this way then all of its problems with mathematical clarity are resolved at once, and consistency is guaranteed.

"Let me comment on the general significance of the axiomatic method in the historical evolution just described. I think it accords with the attitudes expressed by Hilbert..." (Wightman 1976, 220)

Local Conception of Rigour

Suggestion: there is another way of thinking about rigour.

- Start with physical theories as they have been formulated in the physics literature and try to resolve areas of mathematical unclarity in a piecemeal fashion.
- Allows for some mathematical problems to be bracketed for the purposes of addressing another (so overall consistency is not necessarily guaranteed). E.g., causal perturbation theory improves the mathematical rigour of the perturbative renormalization proceedure, but it does not resolve the problem of the large-order divergence of the series.

Local Conception of Rigour

"We now know how to formulate a perturbation theory in which any fixed approximation is free from divergences. This certainly represents only a very partial success, but it is of great practical value... [However] we do not have even the first intuitive idea of how to sum the whole series. Thus, the question whether in general quantum electrodynamics even exists as a theory, free of internal contradictions, remains open" (Bogoliubov, 1965, 31-32)

Conclusions

Why does this matter?

For philosophers: philosophers of science have arguably tended to prioritise a global conception of rigour. It may help them understand physics practice to also consider the local notion.

For historians: some philosophical reflection on big concepts like rigour may help to map the landscape of reformulation programs that we find in post-war relativistic quantum theory.

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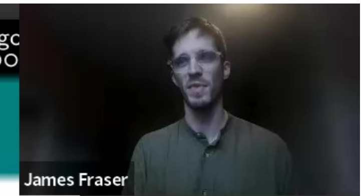

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