Title: Layer codes as partially self-correcting quantum memories (Plenary Talk)

Speakers: Aleksander Kubica

Collection/Series: Year of Quantum Across Canada

Subject: Quantum Information

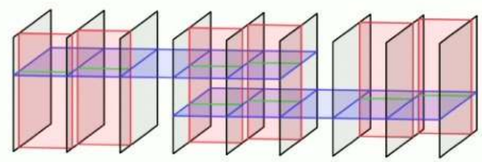
Date: October 09, 2025 - 1:00 PM

URL: https://pirsa.org/25100054

Layer codes as partially self-correcting quantum memories

Aleksander Kubica

Yale University



Perimeter Institute

Oct 9, 2025

Acknowledgements

Libor Caha, TUM

Shin Ho Choe, IQM

Bailey Gu, Yale

Sunny He, MIT

Eugene Tang, Northeastern

- The draft appeared last night — **arXiv:2510.06659**

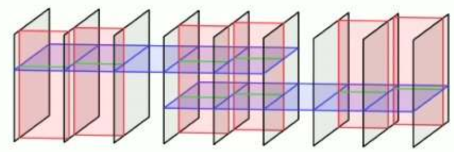
2

Overview

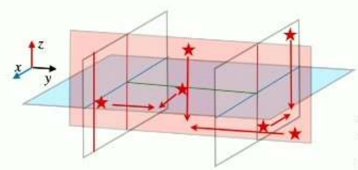
2D	3D	4D
✗	?	✓

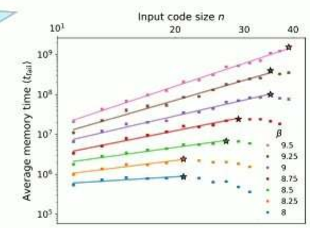
1. Quantum memories & self-correction.

2. Layer codes.



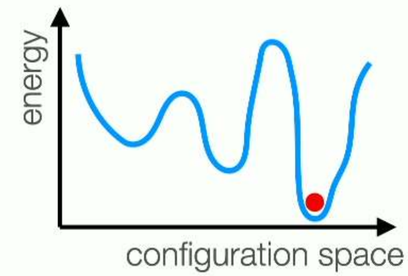
3. Our results.
   - — Random layer codes.
   - — Layer code decoders.
   - — Numerical simulations of memory times.
   - — Proving partial self-correction.





3

Alternative to active QEC



- Storage of classical information in magnetic media.
- Self-correcting quantum memory [Brown+16].
  - Achieved by using a many-body Hamiltonian.
  - Errors suppressed indefinitely.
- Quantum memories must be robust.
  - Finite-temperature effects & local perturbations.
- Criteria for self-correcting quantum memories.
  - Locally embeddable w/ bounded Hamiltonian interactions.
  - Memory time diverges w/ system size at $T > 0$.
  - Efficiently decodable.
  - …

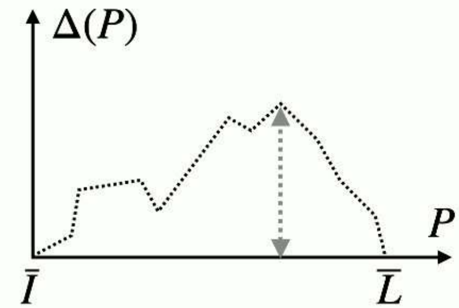
4

Search for quantum memories at $T > 0$

- Unifying language — the stabilizer formalism [Gottesman96].
  — Commuting Pauli Hamiltonians $H = -\sum_i S_i$.

- First, let's rule out a broad class of models.
  — General no-go theorems for energy barrier.

- Energy barrier.
  — Cost of changing the encoded information.
  — Stability in classical models at $T > 0$.
  — Necessary for quantum memories [Temme16].



$$\Delta = \min_{\overline{L}} \max_P \Delta(P)$$

- Beyond commuting Pauli Hamiltonians & energy barrier.
  — Subsystem codes [Poulin05;Bombin15].
  — Other mechanisms: 1-form symmetries [**AK**,Yoshida18;Roberts,Bartlett20].

No-go theorems & limitations

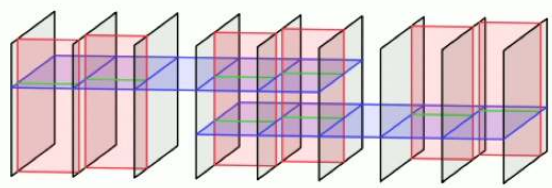
- Existence of self-correcting quantum memories.
  — 2D most restrictive [Alicki+09,…].
  — 3D less restrictive [Haah11;Michnicki14].
  — 4D toric code [Alicki+10].
  — quantum LDPC codes [Hong+25].

2D	3D	4D
✗	?	✓

- Limitations on QEC codes w/ geometrically-local checks in D dimensions.
  — [Bravyi+10;Haah21]: $n \sim L^D$, $k \lesssim L^{D-2}$, $d \lesssim L^{D-1}$.

- In $D = 2$, the surface code is optimal; in $D \geq 3$, the BPTH bounds are saturated by intricate constructions [Portnoy23;Lin+23].

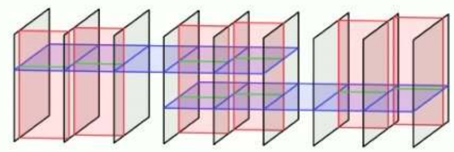
- Layer codes in 3D [Williamson,Baspin24].
  — $n \sim L^3$, $k \sim L$, $d \sim L^2$, $\Delta \sim L$.
  — Self-correction?



Overview

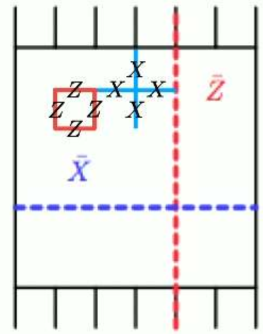
1. Quantum memories & self-correction.

2. Layer codes.



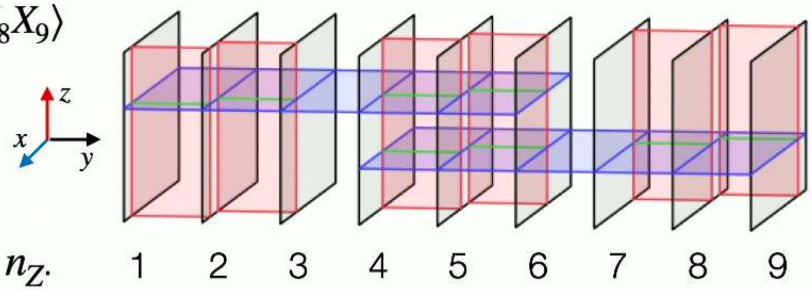
3. Our results.
   — Random layer codes.
   — Layer code decoders.
   — Numerical simulations of memory times.
   — Proving partial self-correction.

Layer codes [Williamson,Baspin24]



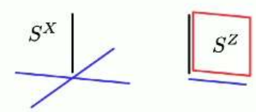
- Two ingredients:
  — 2D surface code,
  — arbitrary CSS stabilizer code.

- Ex: Shor's code.
  $H_Z = \langle Z_1Z_2, Z_2Z_3, Z_4Z_5, Z_5Z_6, Z_7Z_8, Z_8Z_9 \rangle$
  $H_X = \langle X_1X_2X_3X_4X_5X_6, X_4X_5X_6X_7X_8X_9 \rangle$



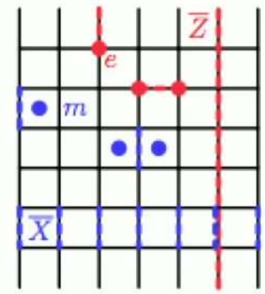
- 2D surface code layers.
  — For each qubit, Z & X check.
  — Form a cuboid of size $n \times n_X \times n_Z$.

- — Coupling of layers specified by H_Z & H_X.
  — Parity checks w/ weight ≤ 6.

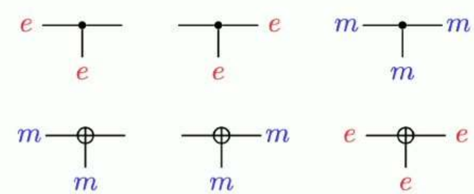


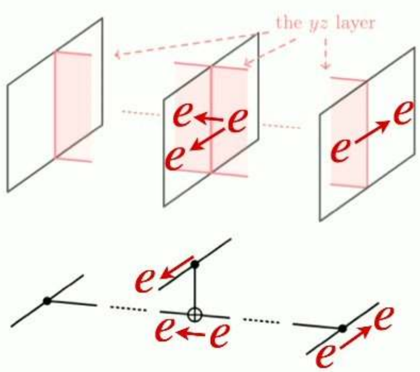
8

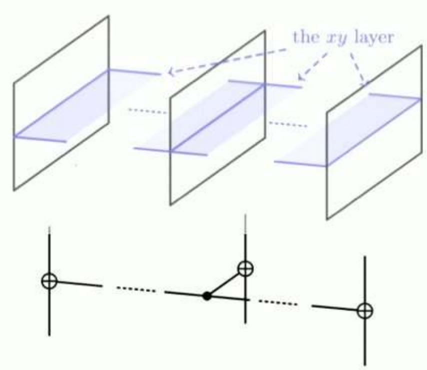
Quasiparticle excitations

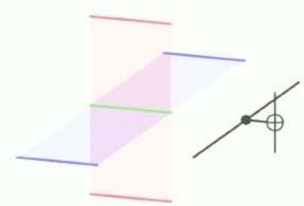


- Commuting Pauli Hamiltonians $H = -\sum_i S_i$.
- 2D surface code: e & m excitations.
  - Fusion rules $e \times e = m \times m = 1$.
- Layer code: e & m excitations.
  - Fusion rules modified along line defects.
  - Excitations split & energetically suppressed.



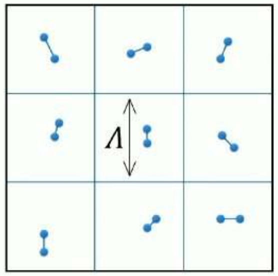




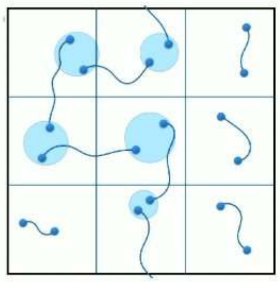


Bad quantum memory — 2D surface code

- Memory time doesn't scale w/ system size at $T = \beta^{-1} > 0$ [Alicki+09].
  — Intuition from the thermal dynamics of excitations.



- Excitation creation.
  — Energy cost $\Delta E = \text{const}$.
  — Average density $\rho \sim \exp(-\beta \Delta E)$.
  — One pair in a square area $\Lambda^2 \sim 1/\rho$.

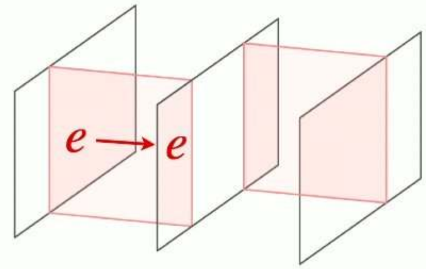


- Excitation diffusion.
  — Diffusion time for a random walk $\tau \sim \Lambda^2$.

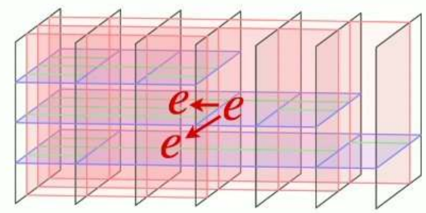
- Excitations start to overlap.
  — Encoded information lost in $t_{\text{mem}} \sim \tau$.
  — Arrhenius' law $t_{\text{mem}} \sim \exp(\beta \Delta E)$.

The puzzle

- Layer codes = coupled 2D surface codes.
  — Bad quantum memories?

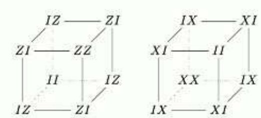


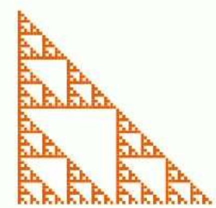
- Freedom of choosing H_Z & H_X.
  — Instead of LDPC, use HDPC codes.



- Many line defects & small patches.
  — Excitations split & energetically suppressed.

- Similar to fractons [Chamon05, Haah11].
  — Good quantum memory?



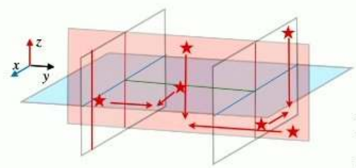


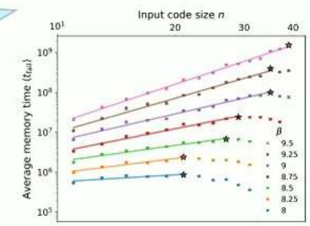
Overview

1. Quantum memories & self-correction.

2. Layer codes.

3. Our results.
    - **Random layer codes.**
    - Layer code decoders.
    - Numerical simulations of memory times.
    - Proving partial self-correction.

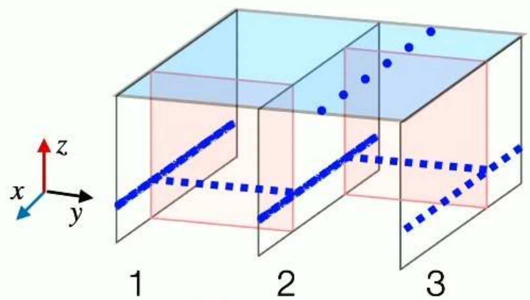




Layer codes for qLDPC & beyond

- Original construction [Williamson,Baspin24].
  - $n_{\text{layer}} \sim n^3$, $k_{\text{layer}} = k$, $d_{\text{layer}} \gtrsim nd/w$, $\Delta_{\text{layer}} \gtrsim \Delta/w^2$
  - Saturating the BPTH bounds for quantum Tanner codes [Leverrier,Zemor22].

- Logical operators via quasiconcatenation.
  - Logical operators of the input code.
  - String operators within gray layers.



$\overline{X} = X_1 X_2 X_3$

- What if input code is not qLDPC?
  - Large w & bounds become trivial.

- Potential issue: a string in a blue layer can be pushed into many strings in gray layers.
  - Reducing the quasiconcated logical operator.

Layer codes from random codes

- A family of random CSS codes $\{\mathscr{C}_n\}$.
  — Get $\mathscr{C}_n$ by drawing uniformly at random $H_X \in \mathbb{F}_2^{n_X \times n}$, $H_Z \in \mathbb{F}_2^{n_Z \times n}$.

- **Thm:** W/ high probability, the layer code from $\mathscr{C}_n$ has:
  (i) distance $d_{\text{layer}} \gtrsim n^2/\log n$,
  (ii) energy barrier $\Delta_{\text{layer}} \gtrsim n/\log n$.

- Random layer codes saturate* the BPTH bounds — no need for good qLDPC codes as in all previous works [Portnoy23;Lin+23;Williamson,Baspin23].

- Practical implications:
  — layer codes of smaller size,
  — amenable to numerical simulations.

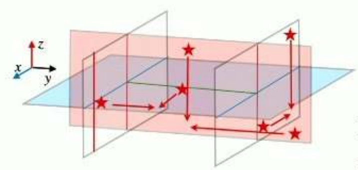
Overview

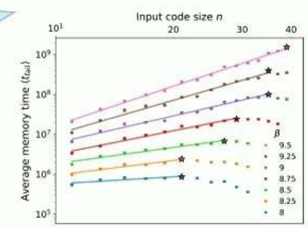
2D 3D 4D

1. Quantum memories & self-correction.

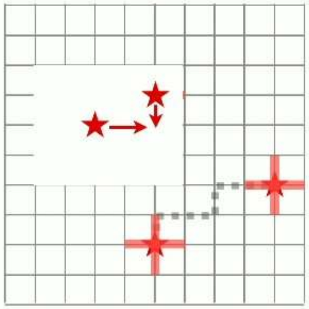
2. Layer codes.

3. Our results.
   - — Random layer codes.
   - **— Layer code decoders.**
   - — Numerical simulations of memory times.
   - — Proving partial self-correction.

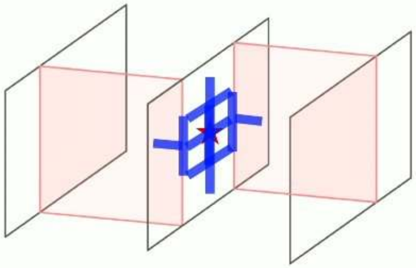




Cluster decoder



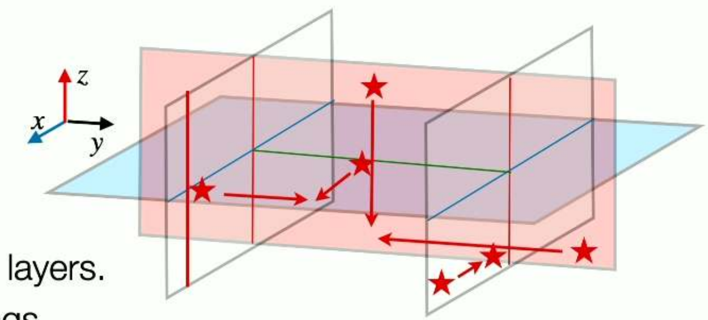
- RG decoders for topological codes & beyond.
  — [Duclos-Cianci,Poulin10;Bravyi,Haah13;Delfosse+21].
- Ex: 2D surface code.
- Initialize clusters & while there are clusters:
  1. grow clusters,
  2. merge overlapping clusters,
  3. remove neutral clusters.



- Adapt to layer codes.
  — Modified condition on neutral clusters.
  — Check by solving a system of equations.
- **Thm:** Cluster decoder has a non-zero threshold for local stochastic noise.

Concatenated decoder

- Min-weight perfect matching [Dennis+02]:
  — match excitations in layers in stages.



- 1. Match in red layers.
  2. Match in gray layers.
  3. Syndrome = parity of excitations in blue layers.
  4. Find a correction and apply vertical strings.
  5. Match arbitrarily in blue layers.

- **Thm:** Concatenated decoder corrects adversarial errors w/ weight $\lesssim d_{\mathrm{dec}} n/w$.

- Correcting errors w/ weight $\lesssim n^2$ for layer codes from quantum Tanner codes.

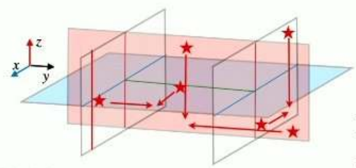
17

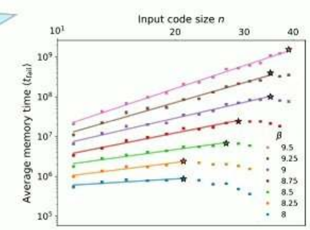
Overview

1. Quantum memories & self-correction.

2. Layer codes.

3. Our results.
   - — Random layer codes.
   - — Layer code decoders.
   - **— Numerical simulations of memory times.**
   - — Proving partial self-correction.

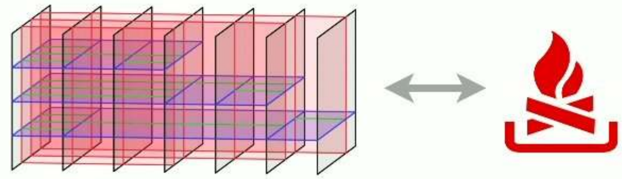




18

How to simulate the memory time

- Standard thermal bath at $T > 0$.
  — Weakly coupled, Markovian, acts locally.



$$H_{\text{tot}} = H_{\text{mem}} + H_{\text{bath}} + \sum_i P_i \otimes f_i$$

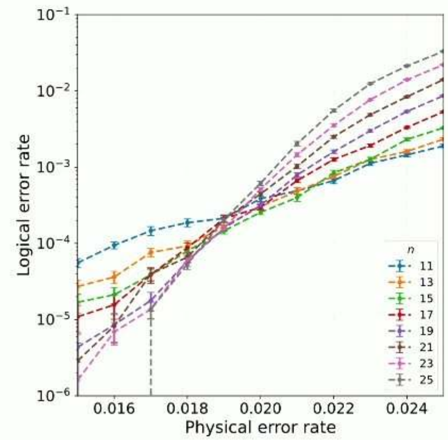
- Thermal evolution of eigenstates of H_{mem}.

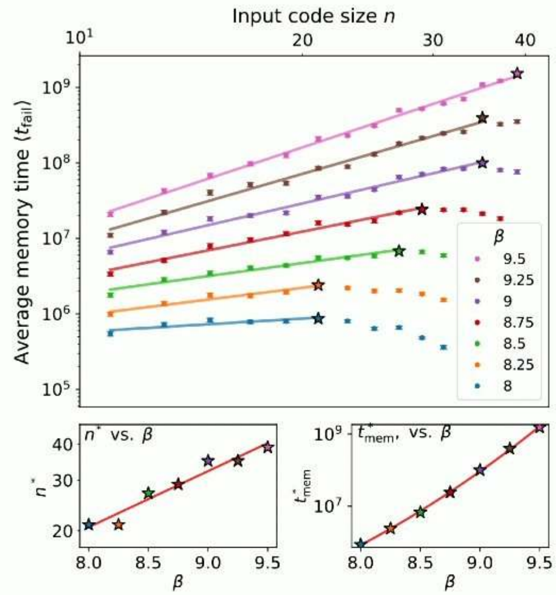
- Monte Carlo simulations [Bortz+75].
  — Initialize $|\psi_0\rangle$ to a ground state & evolve $|\psi_t\rangle \xrightarrow{P_i} |\psi_{t+\delta t}\rangle$.
  — Sequence of discrete errors P_i .
  — $\Pr(P_i)$ determined by $\Delta E = E_{t+\delta t} - E_t$.
  — Time step that passes as P_i is applied $\delta t \sim \text{Exp}(R)$.

19

Numerical simulations

- Challenges w/ layer codes.
  — $n_{\text{layer}} \sim n^3$, so only small input codes!
- Simulations of a threshold.
  — Random layer codes & the cluster decoder.
- Simulations of the memory time.
  — Spin simulations & the cluster decoder.
- Partial self-correction [Bravyi,Haah11].
  — Memory time increases w/ n for $n \leq n^*$.
  — $n^* \approx \exp(a\beta + b)$
  — $t^*_{\text{mem}} \approx \exp(a'\beta^2 + b'\beta + c')$





Defining (partial) self-correction

Defining (partial) self-correction

- $-\{C_n\}$ is a family of stabilizer codes, $H^{(n)}_{\mathrm{mem}}$ is a Hamiltonian associated w/ C_n.
  - At time $t = 0$, $\rho^{(n)}(0)$ is in the ground space of $H^{(n)}_{\mathrm{mem}}$.
  - The evolution of $\rho^{(n)}(t)$ is determined by the Lindblad master equation.
  - Recovering $\rho^{(n)}(0)$ from $\rho^{(n)}(t)$ using a decoder $\Phi(\,\cdot\,) = \sum_\sigma R_\sigma \Pi_\sigma(\,\cdot\,)\Pi_\sigma R_\sigma$.

- **Memory time** $t^{(n)}_{\mathrm{mem}}$.
  - $\forall t < t^{(n)}_{\mathrm{mem}}\, \exists \Phi \forall \rho^{(n)}(0) : \left\| \Phi\left(\rho^{(n)}(t)\right) - \rho^{(n)}(0) \right\|_1 \leq \epsilon.$

- $\{C_n\}$ is:
  - **self-correcting** at inverse temperature β if $t^{(n)}_{\mathrm{mem}}$ diverges w/ n,
  - **partially self-correcting** if $t^{*}_{\mathrm{mem}} = \max_n t^{(n)}_{\mathrm{mem}}$ scales superexponentially in β,
  - **quantum memory** if there exists computationally-efficient Φ.

Partially self-correcting quantum memory

- **Thm:** Let $\{C_n\}$ be a family of qLDPC codes. If $\Delta_n = \Omega(\max(k_n, \log n))$, then $\{C_n\}$ is partially self-correcting w/ $t^*_{\text{mem}} = \exp[\Omega(\Delta_{n^*}\beta)]$ & $n^* = \exp(\beta/2)$.

- Proof sketch.
  — [Bravyi,Haah11]: $\left\| \Phi\left(\rho^{(n)}(t)\right) - \rho^{(n)}(0) \right\|_1 \leq O(t)n2^{k_n}e^{-\beta(\Delta_n - f)/2}$
  — The energy barrier decoder Φ: if σ is a syndrome for some Pauli error E w/ energy barrier smaller than Δ_n, then set the recovery $R_\sigma = E$.

- Partial self-correction is more common than expected, as it arises from a diverging energy barrier & its interplay w/ code parameters!

- **Thm:** Layer codes from quantum Tanner codes or random CSS codes are partially self-correcting (quantum memories w/ the simplified concatenated decoder).

Summary

- Self-correcting quantum memories.
  — Extremely useful for fault-tolerant quantum computers.
  — Maintaining encoded information while equilibrating w/ the environment.

- Exciting progress in 3D.
  — [Portnoy23;Lin+23;Williamson,Baspin24].

2D	3D	4D
✗	?	✓

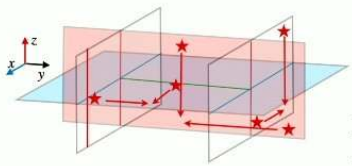
Summary

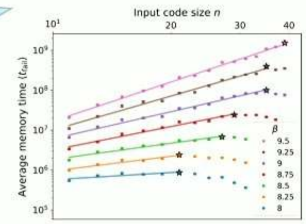
- Self-correcting quantum memories.
  — Extremely useful for fault-tolerant quantum computers.
  — Maintaining encoded information while equilibrating w/ the environment.

- Exciting progress in 3D.
  — [Portnoy23;Lin+23;Williamson,Baspin24].

2D	3D	4D
✗	?	✓

- Our results.
  — Random layer codes.
  — Layer code decoders.
  — Numerical simulations of memory times.
  — Proving partial self-correction.





THANK YOU!

24