Title: Lecture - Quantum Field Theory I (Core), PHYS 601

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Subject: Quantum Fields and Strings

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Lecture Eleven

$$F(x,\mu,g_R)=g_R+g_R^2\,\alpha \ln \frac{\mu}{x}$$

renormalization group $<$ invariant $\langle x | x \rangle = \text{const}$

$U_1 U_2 = U_3 \leftarrow$ same group

in the same theory — doing experiment at different energy scale

$$
\begin{tikzcd}
(g_R, \mu) \arrow[r, "U_1"] \arrow[rr, bend right, "U_3"'] & (g_R', \mu') \arrow[r, "U_2"] & (g_R'', \mu'')
\end{tikzcd}
$$

$\langle x|x\rangle$ = const

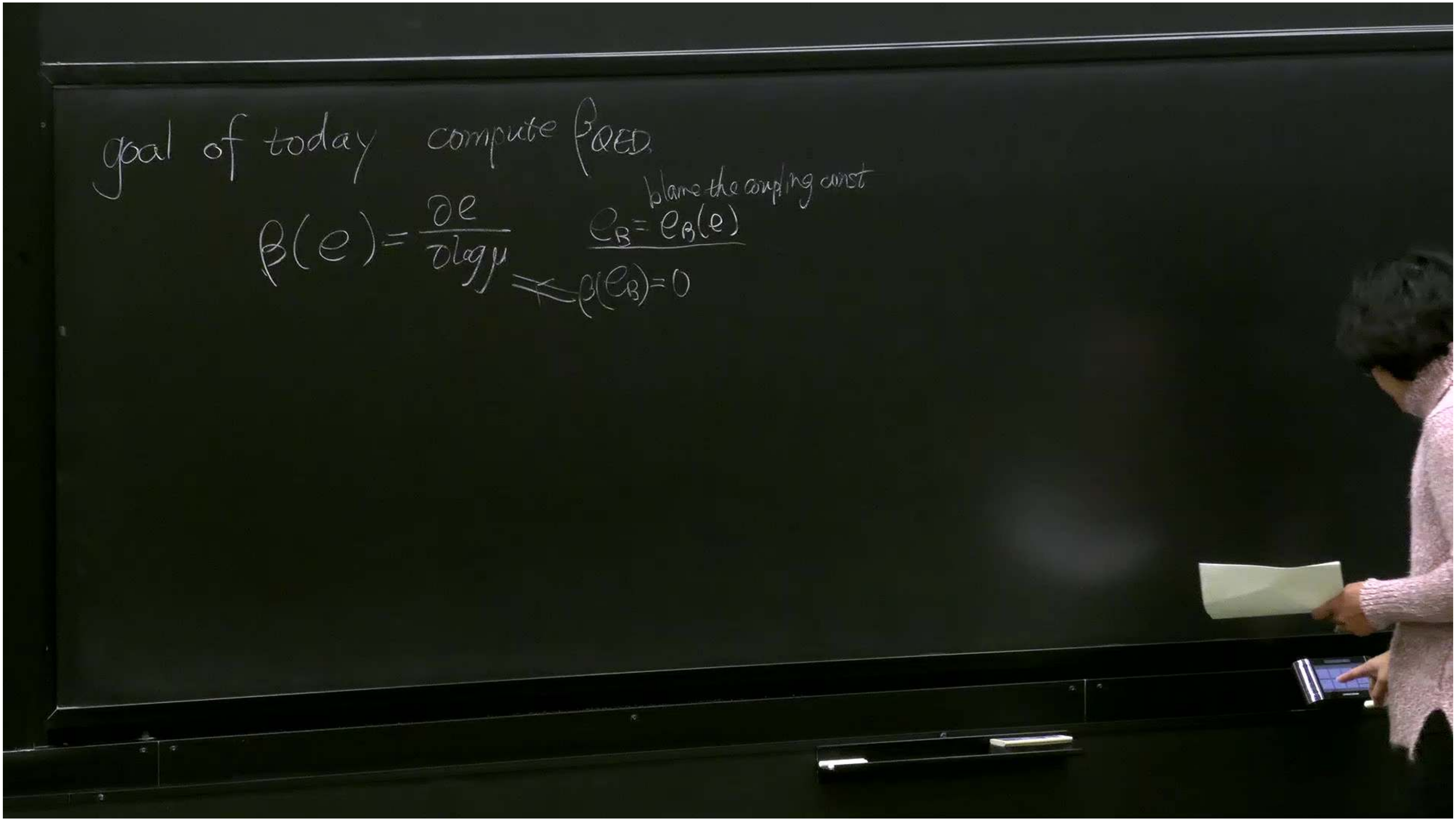
same group

...rent at different energy scale

$) \xrightarrow{\mu_2} (g_R'', \mu'')$

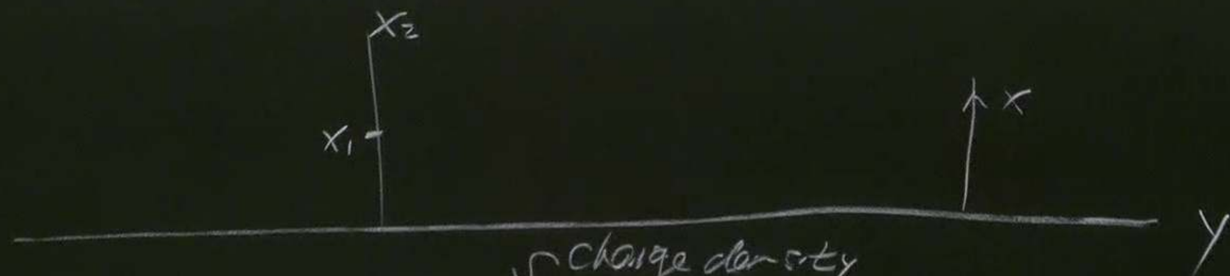
$$\underbrace{F(g_R;\mu,\mu'')}_{U_3} = g_R'' = F\Big(\overbrace{g_R' = F(g_R,\mu,\mu')}^{U_2}, \mu', \mu''\Big)$$

check


goal of today compute β_{QED}
blame the coupling const
$\beta(e) = \frac{\partial e}{\partial \log \mu}$
$e_B = e_B(e)$
$\beta(e_B) = 0$

technique. dimensional regularization

EM



charge density

$$V(x) = \int_{-\infty}^{\infty} \frac{\lambda \, dy}{\sqrt{x^2+y^2}} = \lambda \ln\left|y+\sqrt{x^2+y^2}\right| \Big|_{-\infty}^{+\infty}$$

observation of the integral

$$V(x_1) = V(x_2)$$

test charge will mo

physics

V(

$$V(x) = \int_{-L}^{L} \frac{\lambda\, dy}{\sqrt{x^2+y^2}} = \ln \frac{L+\sqrt{L^2+x^2}}{-L+\sqrt{L^2+x^2}}$$

$$V(x_1) - V(x_2)\Big|_{L\to\infty} = \ln \frac{x_2^2}{x_1^2}$$

$y \to y + c$ symmetry is broken

other idea

$\int dy = \int$

ll move

$\sqrt{\ell^2+x^2}$

$\sqrt{\ell^2+x^2}$

$\frac{X_2^2}{X_1^2}$

...ry is broken.

other idea

$\int dy = \int dy^d = \int \Omega_d\, y^{d-1} dy \quad \Omega_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}$

$V(x) = \Omega_d \int_0^\infty \left(\frac{y}{\ell}\right)^{d-1} \frac{dy}{\sqrt{x^2+y^2}}$

$d = 1-\epsilon$ regulator ↑ forced to introduce a scale

Maths

$= \left(\frac{\ell}{x}\right)^\epsilon \frac{\Gamma(\frac{\epsilon}{2})}{\pi^{\epsilon/2}}$

Maths

$\lim_{\epsilon\to 0} V(x_1) - V(x_2) = \ln \frac{X_2^2}{X_1^2}$

$V(x) = \frac{2}{\epsilon} + \text{finite \#s} + \ln \frac{\ell^2}{x^2} +$ renormalization scale

$V_{ren}(x) = \ln \frac{\ell^2}{x^2}$

goal of today compute β_{QED}.

$$\beta(e) = \frac{\partial e}{\partial \log \mu}$$

blame the coupling const

$$\underline{e_B = e_B(e)}$$

$$\Longleftarrow \beta(e_B) = 0$$

photon propagator (QFT II propaganda)

scalar case $\mathcal{L}_{KG} = \phi \underline{(\partial^2 + m^2)} \phi$

$-p^2 + m^2 \xrightarrow{\text{inverse}} \frac{i}{p^2 - m^2}$ propagator.

Catch

photon case $\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$

$= \frac{1}{2} A_\mu (\partial^2 \eta^{\mu\nu} - \partial^\mu \partial^\nu) A_\nu +$

$-p^2 \eta^{\mu\nu} + p^\mu p^\nu$ matrix invert it $\to$ photon propagator

$(-p^2 \eta^{\mu\nu} + p^\mu p^\nu) p_\mu = 0$ zero mode pagator

Catch: we haven't fixed the gauge

gauge fix $\partial_\mu A^\mu = 0$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2$$

$$-p^2\eta^{\mu\nu} + p^\mu p^\nu\left(1-\frac{1}{\xi}\right)$$ can be inverted: $$\frac{-i}{p^2+i\epsilon}\left(\eta^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2}\right)$$

$\xi = 1$ Feynman gauge