Title: Lecture - Quantum Field Theory I (Core), PHYS 601

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Collection/Series: Quantum Field Theory I (Core), Phys 601, October 8 - November 7, 2025

Subject: Quantum Fields and Strings

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Quantum Field Theory (QFT) I

Outline part 1 3.5 weeks Dirac fields/electron and interactions with Hi

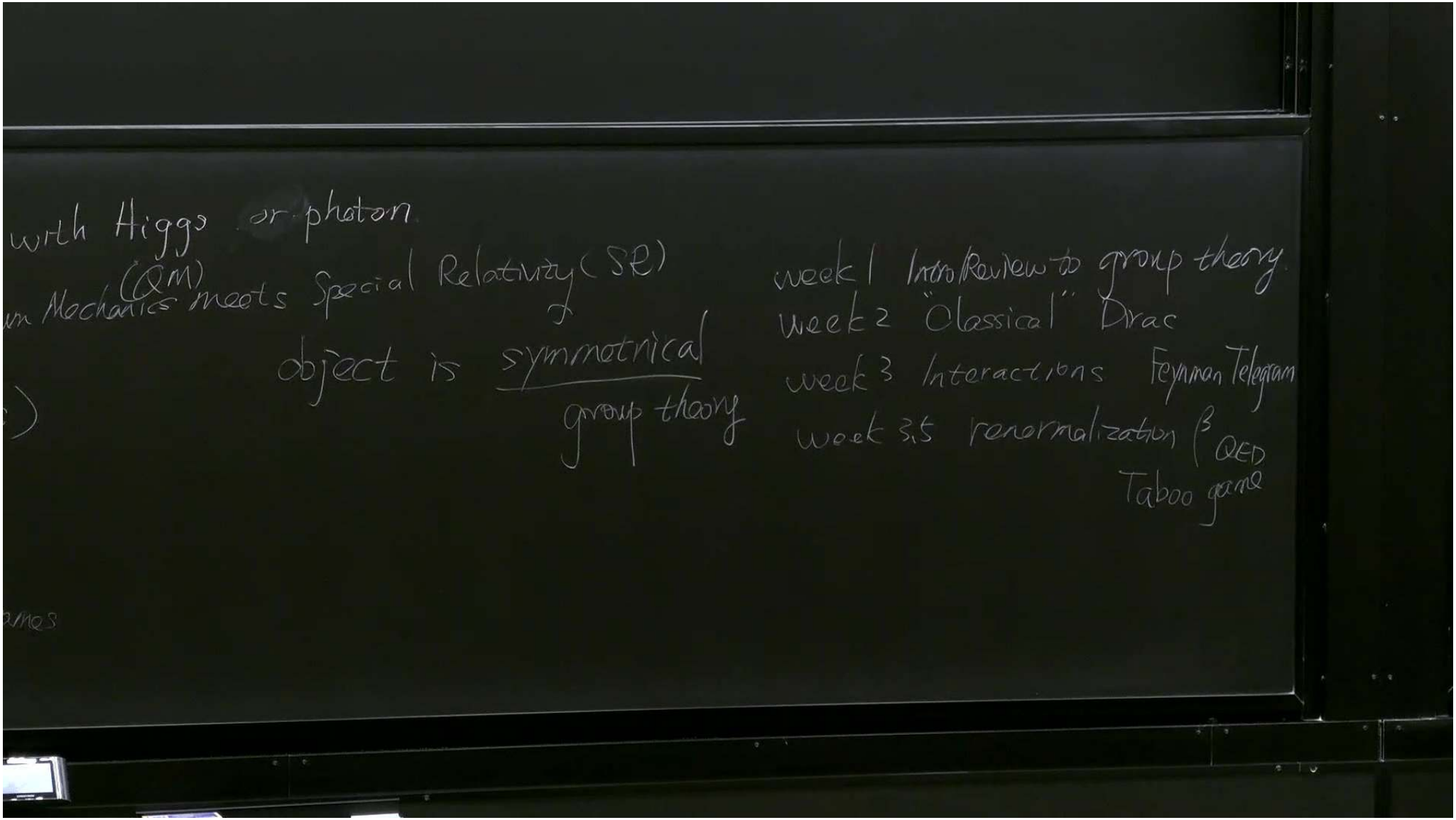
part 2 1.5 weeks Who ordered fields? When Quantum Mechanic

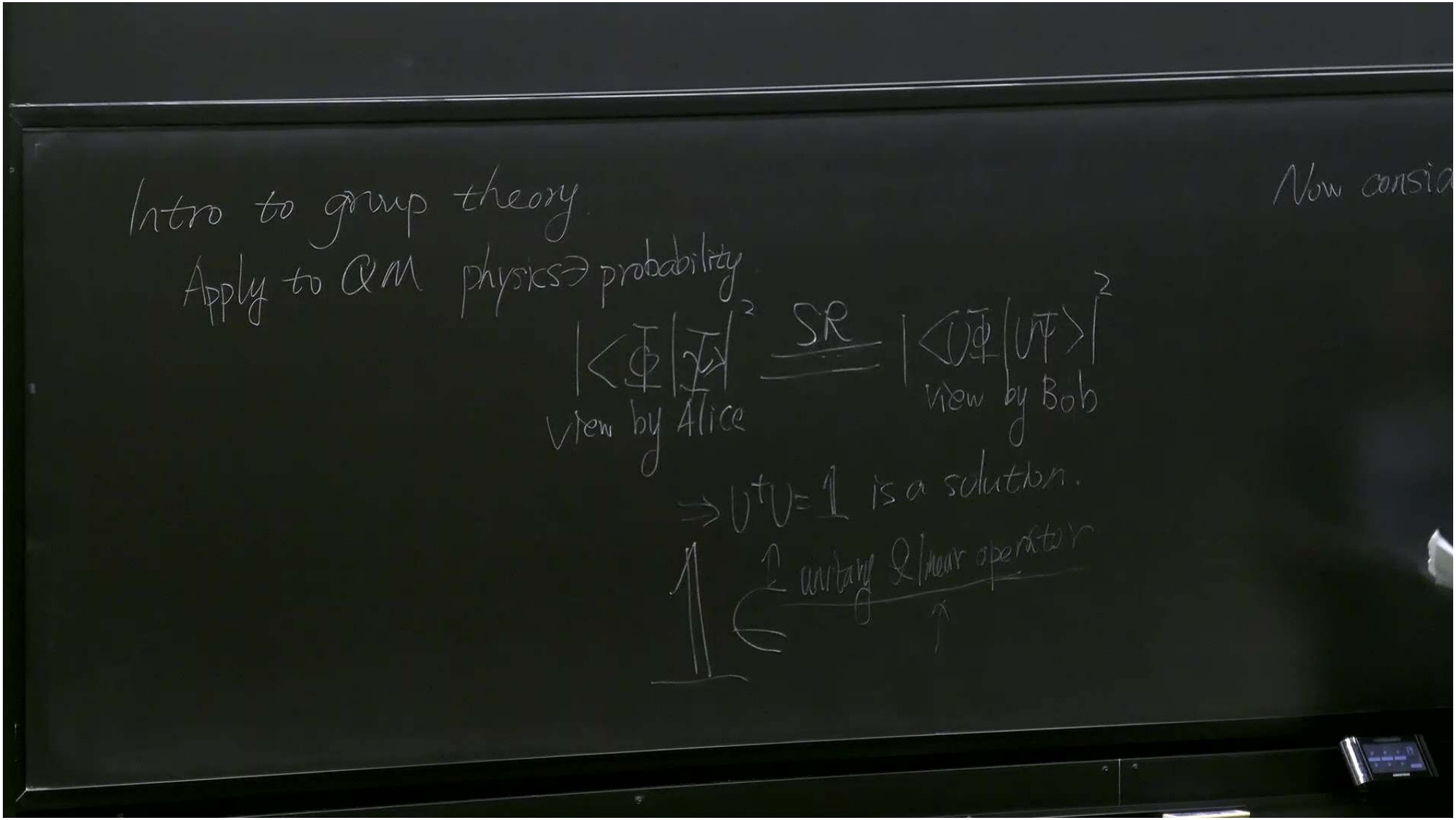
part 1 motivation electrons - fermions (Dirac fields)

Relativistic Quantum theory

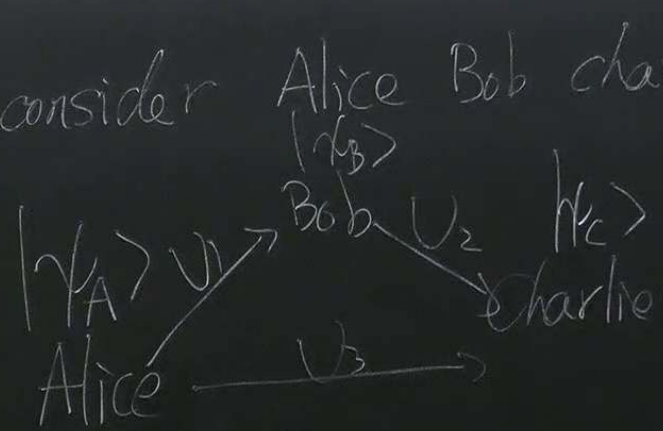
All physics are the same in all inertial frames

invariant


with Higgs or photon
(QM)
n Mechanics meets Special Relativity (SR)
object is symmetrical
group theory
week 1 Intro/Review to group theory
week 2 "Classical" Dirac
week 3 Interactions Feynman Telegram
week 3.5 renormalization β QED
Taboo game


Intro to group theory
Apply to QM physics → probability
$|\langle\Phi|\Psi\rangle|^2 \overset{SR}{=\!=} |\langle U\Phi|U\Psi\rangle|^2$
view by Alice
view by Bob
$\Rightarrow U^\dagger U = \mathbb{1}$ is a solution.
$\mathbb{1} \in$ unitary & linear operator
Now consi

Now consider Alice Bob charlie



$|\psi_C\rangle$ direct $U_3|\psi_A\rangle$

indirect $U_2|\psi_B\rangle = U_2 U_1 |\psi_A\rangle$

$U_3 = U_2 U_1$

consecutive transformations of the symmetrical kind is still symmetrical

group theory

Group

The set of the symmetric transformations form a group

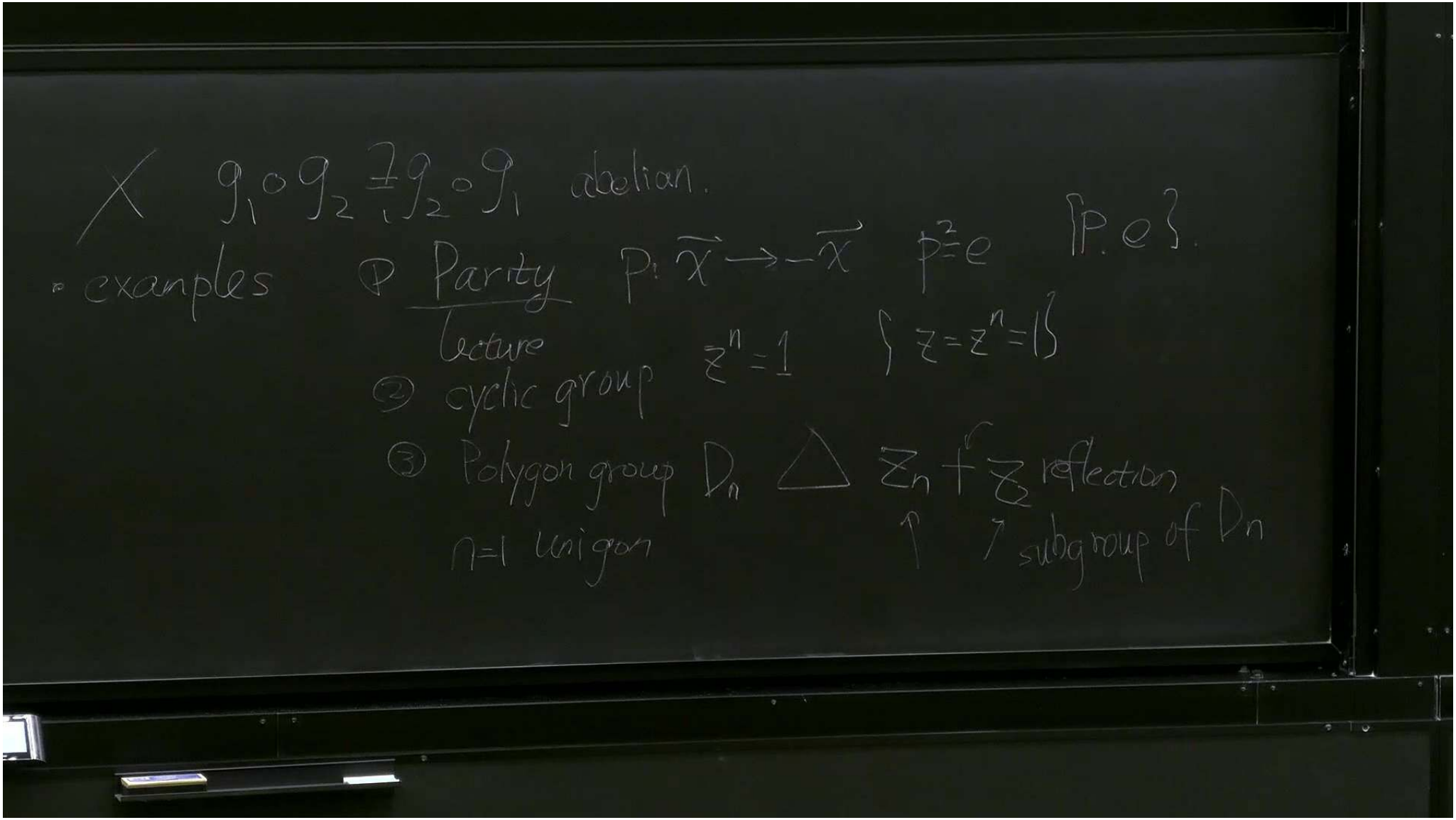
- def abstractly. it is a set with a binary operator $\circ$

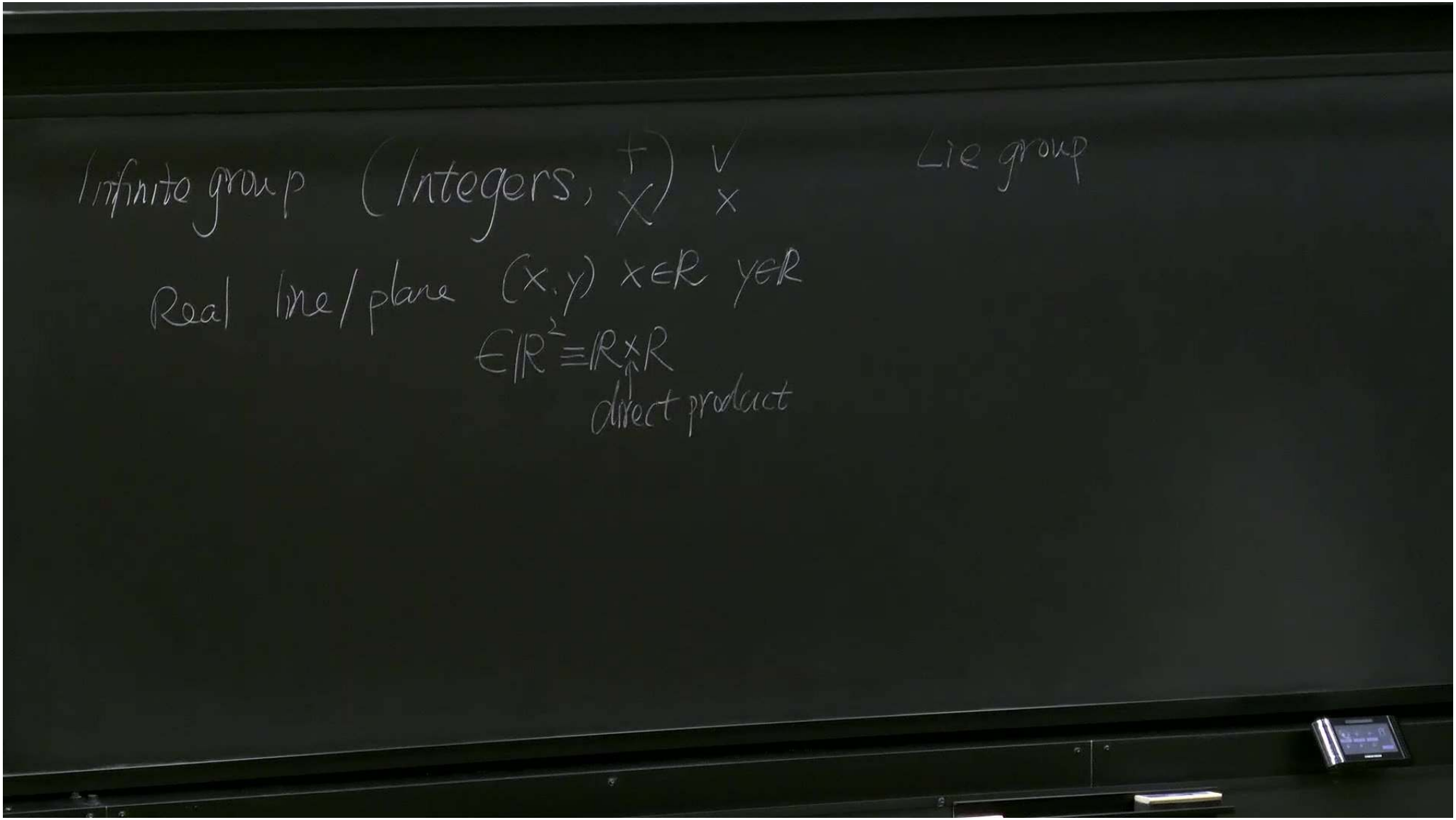
① closed $g_1 \circ g_2 = g_3$

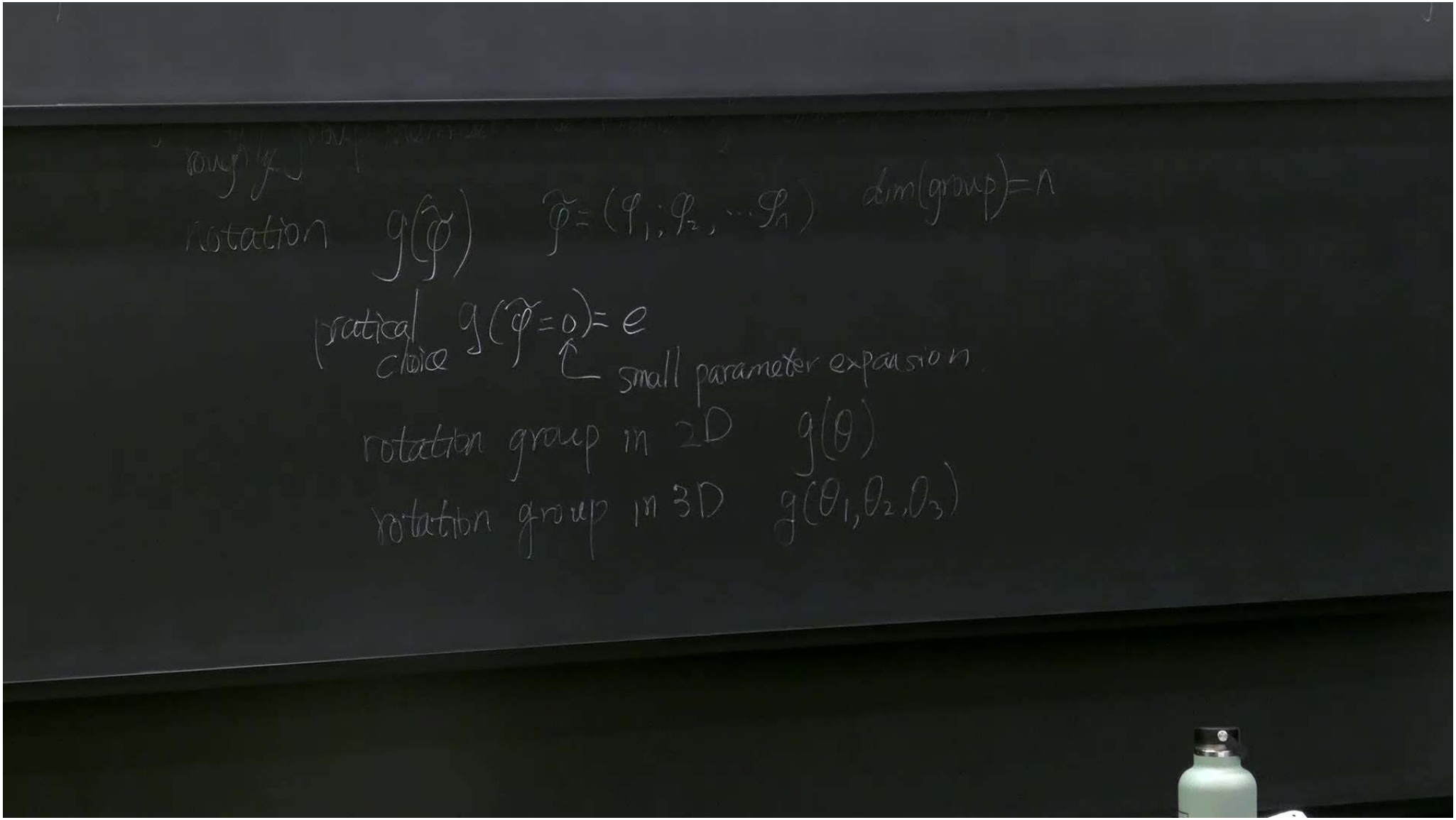
② Identity. $\exists e \in G$ $e \circ g = g \circ e = g$

③ Inverse $\forall g. \exists g^{-1}$ $g \circ g^{-1} = g^{-1} \circ g = e$

④ associativity


$g_1 \circ g_2 \overset{?}{=} g_2 \circ g_1$ abelian.
examples
① Parity $P: \vec{x} \to -\vec{x}$ $P^2 = e$ $\{P, e\}$.
lecture
② cyclic group $z^n = 1$ $\{z = z^n = 1\}$
③ Polygon group D_n $\triangle$ $Z_n + Z_2$ reflection
$n=1$ unigon
subgroup of D_n


Infinite group (Integers, +) V
×
Lie group
Real line / plane (x, y) x∈R y∈R
∈ ℝ² ≡ ℝ × ℝ
direct product


rotation $g(\vec{\varphi})$ $\vec{\varphi} = (\varphi_1, \varphi_2, \cdots \varphi_n)$ dim(group) = n
practical choice $g(\vec{\varphi} = 0) = e$
small parameter expansion
rotation group in 2D $g(\theta)$
rotation group in 3D $g(\theta_1, \theta_2, \theta_3)$

systematically study rotation group Euclidean space

$R^T R = \mathbb{1}$

expand vector $|x\rangle = \sum_{i=1}^{3} x_i |e_i\rangle$

define scalar product $\langle x|y\rangle = \sum_{i=1}^{3} x_i y_i$

the norm $\langle x|x\rangle = \sum_{i=1}^{3} x_i x_i$

The rotation $R: |x\rangle \longrightarrow |x'\rangle = R|x\rangle$

$\langle x|x\rangle = \langle x'|x'\rangle$

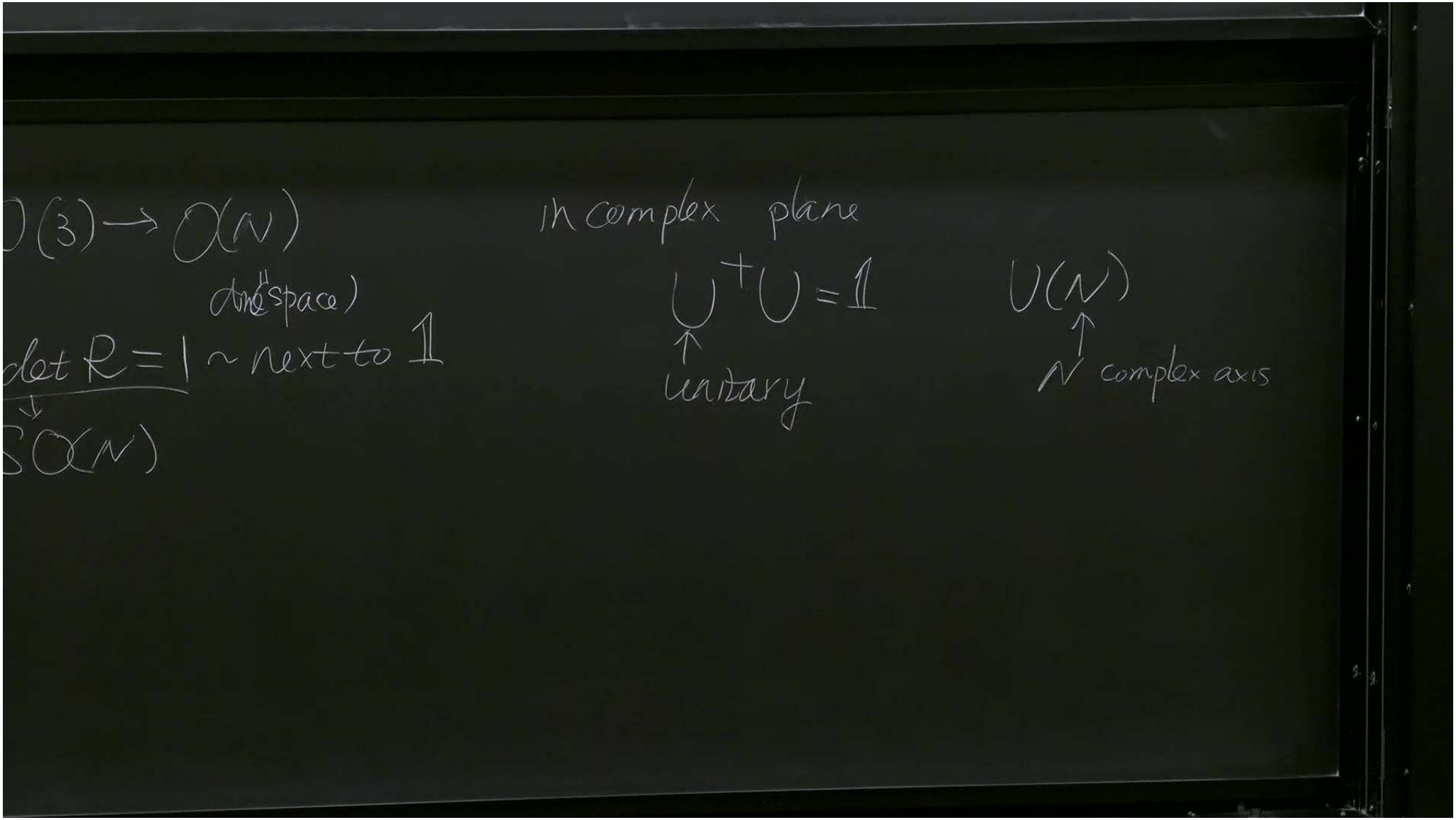
$R^T R = \mathbb{1}$ $O(3) \to O(N)$ in complex plane

(and space)

$(\det R)^2 = 1$ $\det R = 1$ ~ next to $\mathbb{1}$

$\downarrow$

$SO(N)$

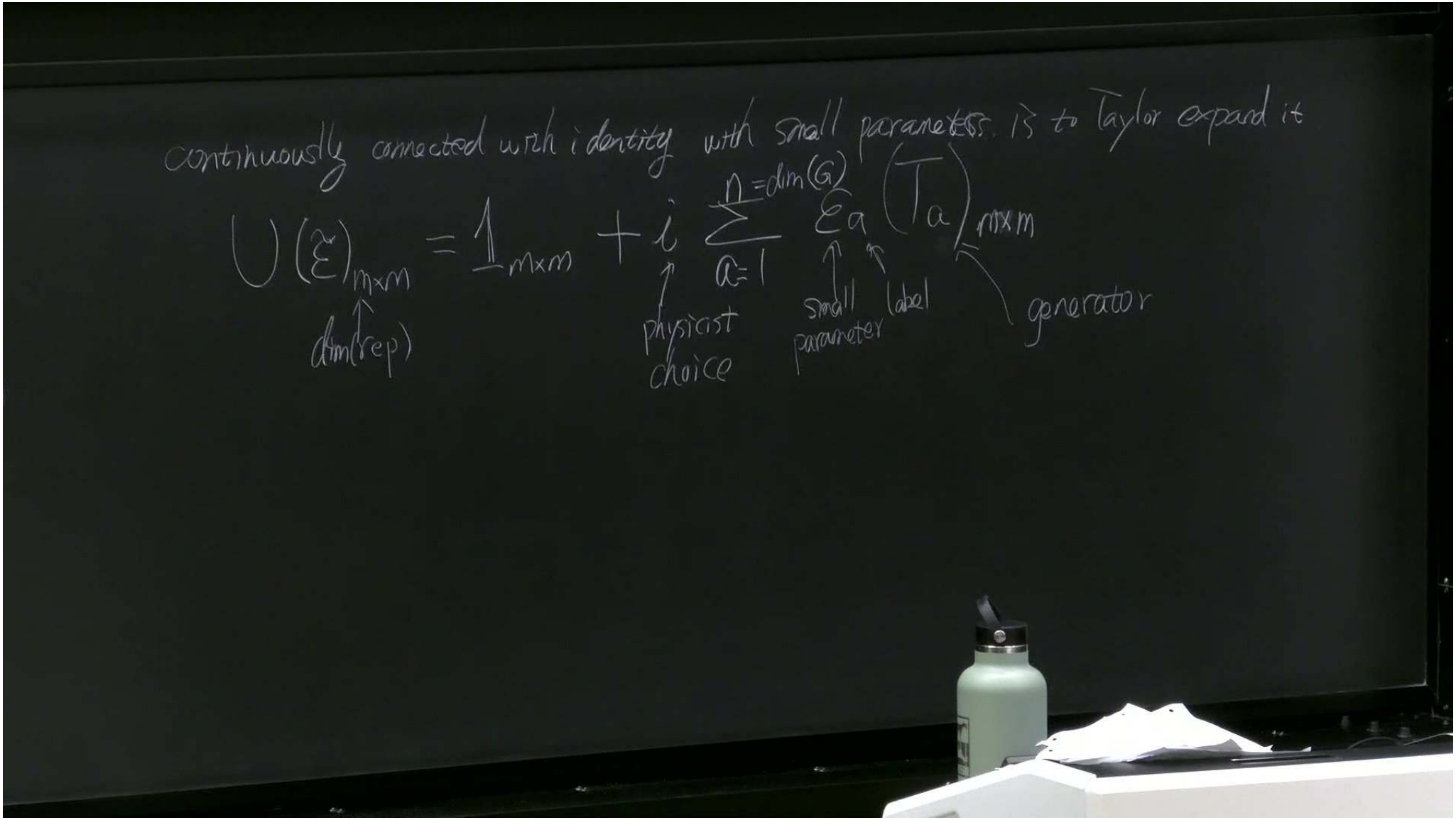

O(3) → O(N)
(the space)
det R = 1 ~ next to 𝟙
SO(N)
in complex plane
U† U = 𝟙
unitary
U(N)
N complex axis

How to study a group: matrix representation.

$g_1 \circ g_2 = g_3$

$D(g_1)_{m\times m}$ $D(g_2)_{m\times m}$ $=$ $D(g_3)_{m\times m}$

dim(rep) = many choices for given group with unique dim(G)


continuously connected with identity with small parameters. is to Taylor expand it
$U(\xi)_{m\times m} = \mathbb{1}_{m\times m} + i \sum_{a=1}^{n=\dim(G)} \varepsilon_a (T_a)_{m\times m}$
dim(rep)
physicist choice
small parameter
label
generator