Title: Lecture - Statistical Physics (Core), PHYS 602

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Subject: Condensed Matter, Other

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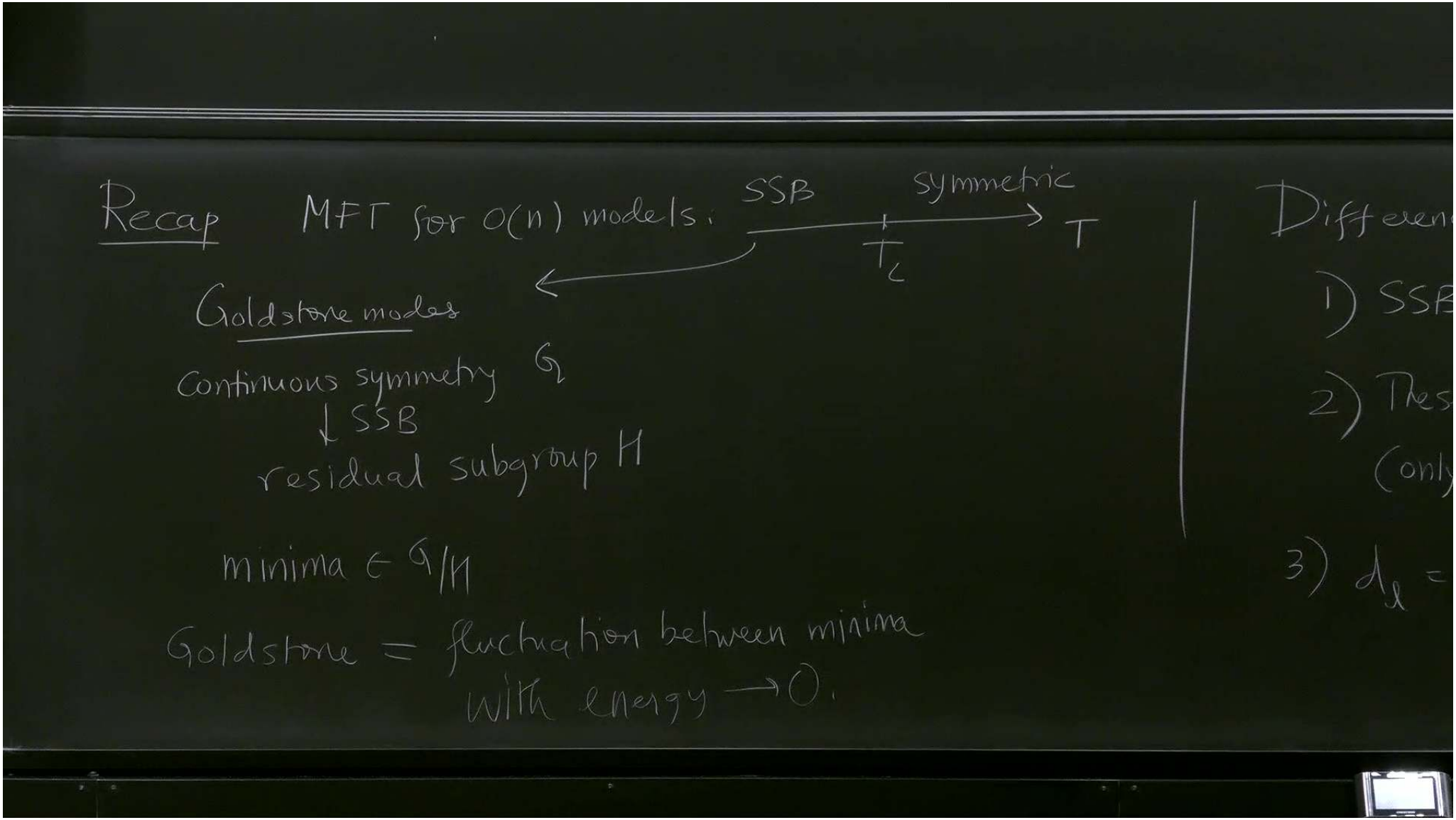

Recap MFT for O(n) models:
SSB
symmetric
T
T_c
Goldstone modes
Continuous symmetry G
↓ SSB
residual subgroup H
minima ∈ G/H
Goldstone = fluctuation between minima
with energy → 0.
Differen
1) SSB
2) Thes
(only
3) d_l =

Differences b/w continuous & discrete:

1) SSB phase has gapless excitations (Goldstone mode)

2) These gapless modes are present for a range of T
(only for $T \to T_c$ in discrete)

3) $d_l = 2$, continuous when $T > 0$ (Mermin - Wagner theorem: no ordering / SSB / Goldstone mode in $d \leq 2$ for $T > 0$)

1, discrete

Questions: I) Phase diagram of $O(N)$ models in different d.

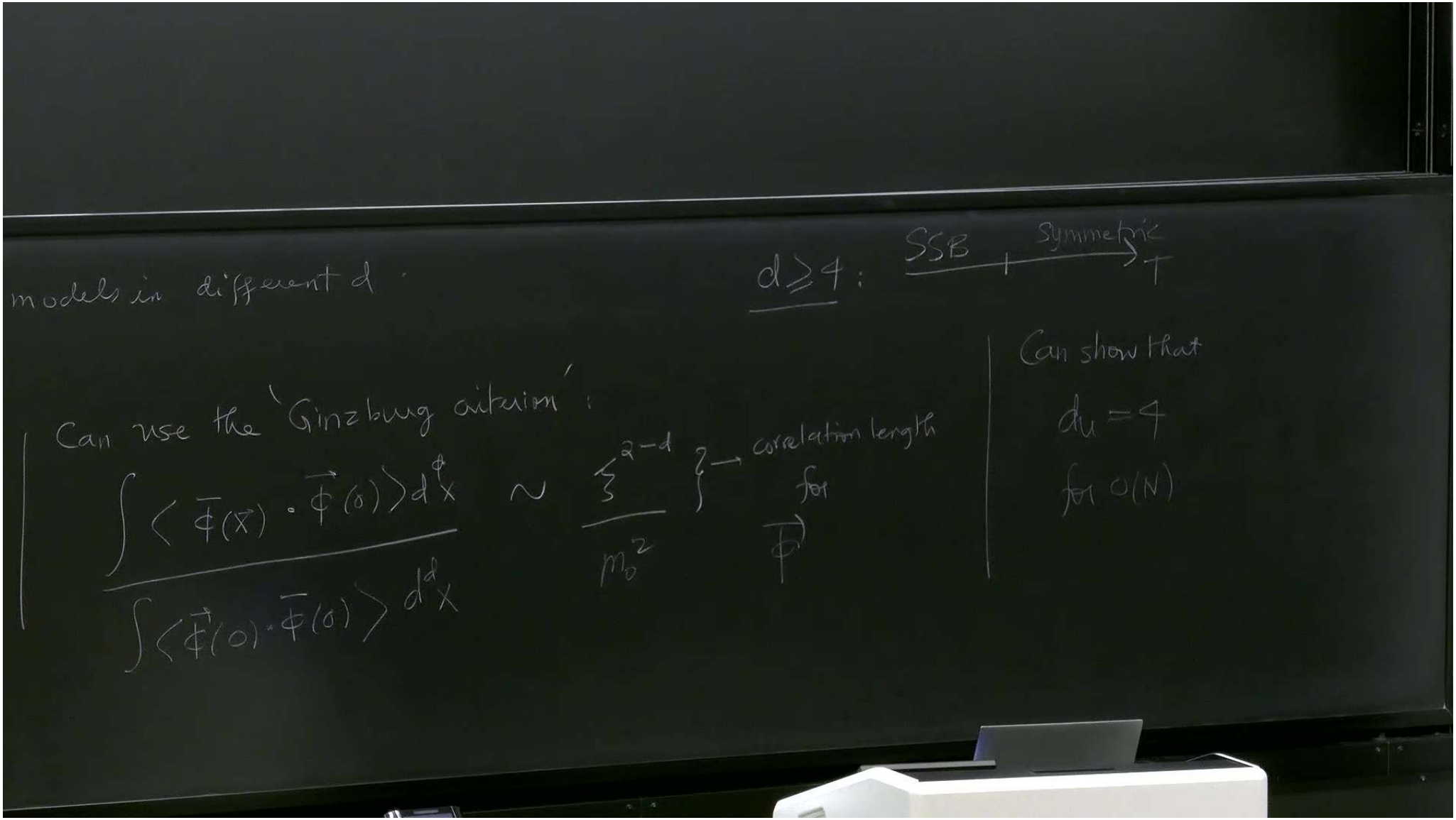
II) In particular, when $d=2$?

Recall, MFT is exact when $d \geq d_u$

Ising model: $d_u = 4$

Can use the 'Ginzburg criterion':

$$\frac{\int \langle \vec{\phi}(x) \cdot \vec{\phi}(0) \rangle d^d x}{\int \langle \vec{\phi}(0) \cdot \vec{\phi}(0) \rangle d^d x} \sim \frac{\xi^{2-}}{m_0^2}$$


models in different d
$d \geq 4$: SSB ⟷ Symmetric
Can use the 'Ginzburg criterion':
$\frac{\int \langle \vec{\phi}(x) \cdot \vec{\phi}(0) \rangle d^d x}{\int \langle \vec{\phi}(0) \cdot \vec{\phi}(0) \rangle d^d x} \sim \frac{\xi^{2-d}}{m_0^2}$
ξ → correlation length for $\vec{\phi}$
Can show that
$d_u = 4$
for $O(N)$

* In $d=3$, expect that MFT is qualitatively correct with modified critical exponents.

Following Ising model strategy, let $\mathcal{L} \sim |\nabla\phi|^2 + r\frac{|\phi|^2}{2} + \frac{u}{4!}|\phi|^4$

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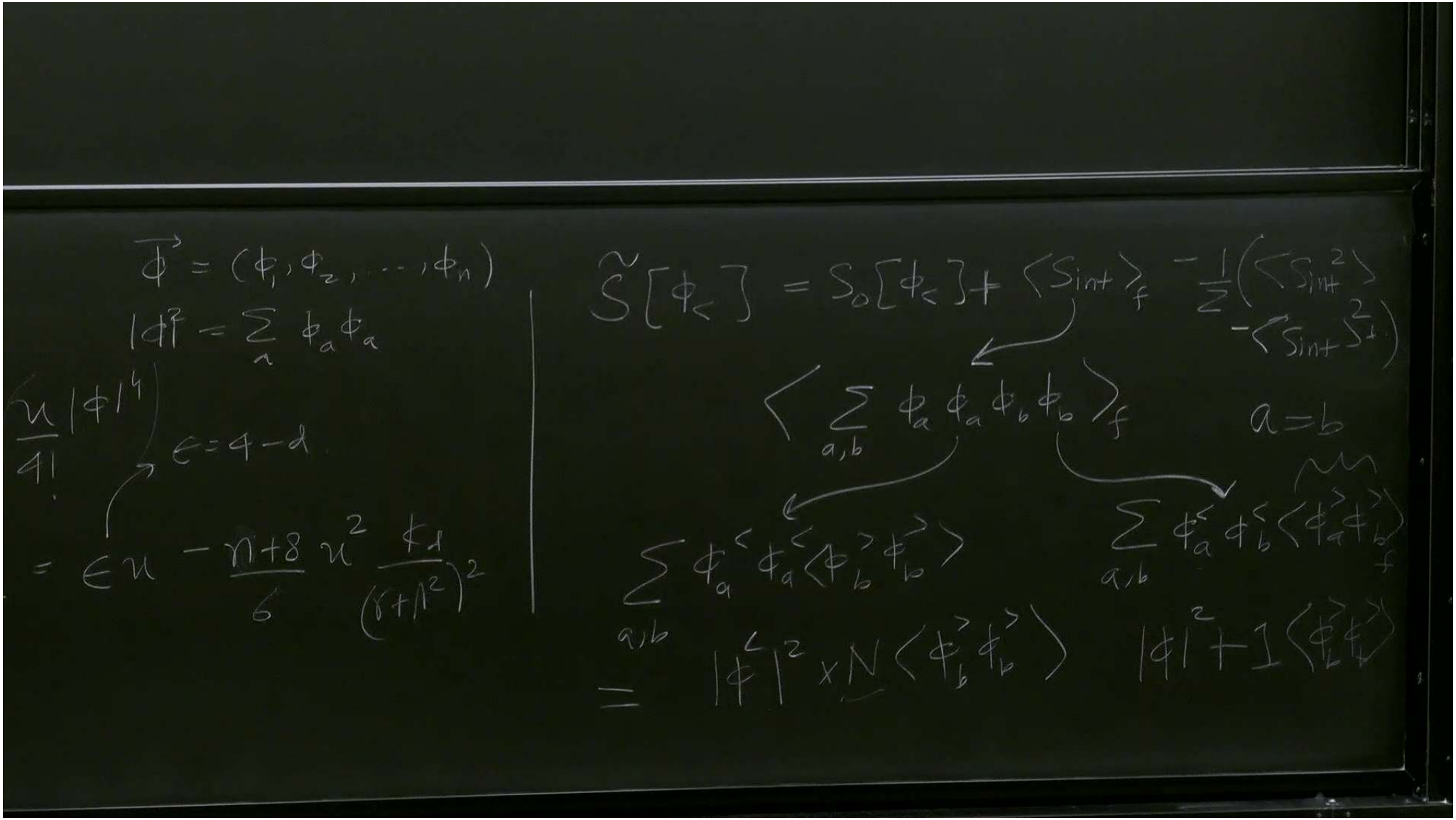
Following Ising model strategy, let $\mathcal{L} \sim |\nabla\phi|^2 + r\frac{|\phi|^2}{2} + \frac{u}{4!}|\phi|^4$

do perturbative RG in u;

Result $\quad \frac{dr}{dl} = 2r + \frac{N+2}{6} u \frac{K_d}{r+\Lambda^2}$;


$\vec{\phi} = (\phi_1, \phi_2, \cdots, \phi_n)$
$|\phi|^2 = \sum_a \phi_a \phi_a$
$\frac{u}{4!}|\phi|^4$
$\epsilon = 4 - d$
$= \epsilon u - \frac{n+8}{6} u^2 \frac{k_d}{(r+1)^2}$
$\tilde{S}[\phi_<] = S_0[\phi_<] + \langle S_{int} \rangle_f - \frac{1}{2}(\langle S_{int}^2 \rangle - \langle S_{int} \rangle^2)$
$\langle \sum_{a,b} \phi_a \phi_a \phi_b \phi_b \rangle_f$
$a = b$
$\sum_{a,b} \phi_a^< \phi_a^< \langle \phi_b^> \phi_b^> \rangle$
$= |\phi^<|^2 \times N \langle \phi_b^> \phi_b^> \rangle$
$\sum_{a,b} \phi_a^< \phi_b^< \langle \phi_a^> \phi_b^> \rangle_f$
$|\phi|^2 + 1 \langle \phi_b^> \phi_b^> \rangle$

* In $d=3$, expect that MFT is qualitatively correct with modified critical exponents.

$\vec{\phi} = (\phi_1, \phi_2, \cdots, \phi_n)$

$|\vec{\phi}|^2 = \sum_a \phi_a \phi_a$

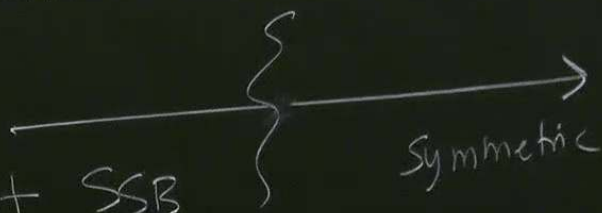
Following Ising model strategy, let $\mathcal{L} \sim |\nabla\vec{\phi}|^2 + r\frac{|\vec{\phi}|^2}{2} + \frac{u}{4!}|\vec{\phi}|^4$

do perturbative RG in u (ϵ-expansion) $\epsilon = 4-d$

Result $\frac{dr}{dl} = 2r + \frac{n+2}{6} u \frac{K_d}{r+\Lambda^2}$; $\frac{du}{dl} = \epsilon u - \frac{n+8}{6} u^2 \frac{K_d}{(r+\Lambda^2)^2}$

Finally: in $d=3$ we again have SSB $\xrightarrow{T}$ symmetric for all n.

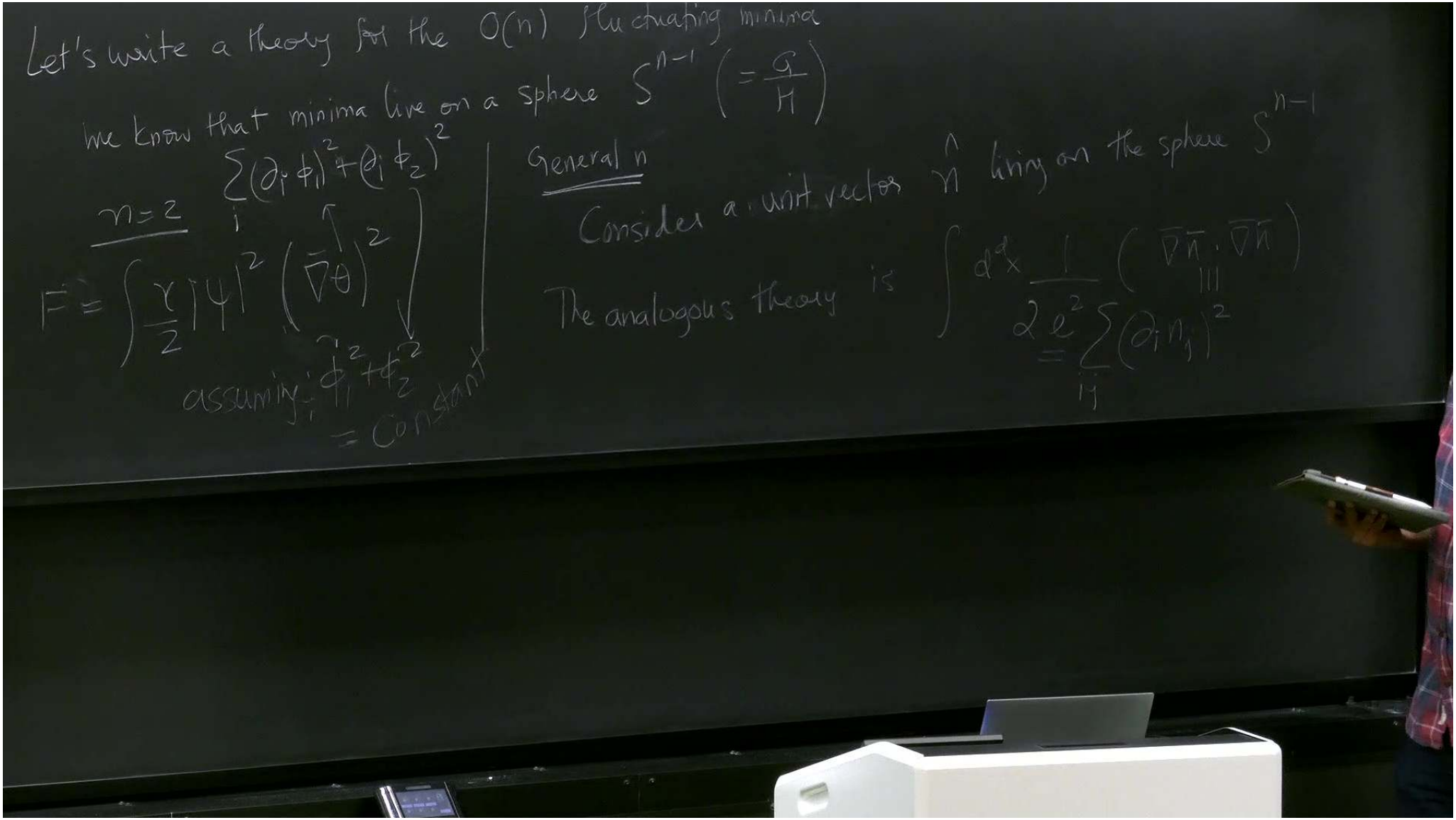
But we really want to understand $d=2$



a) symmetric

b) something else

Let's write a theory for the O(n


Let's write a theory for the O(n) fluctuating minima
We know that minima live on a sphere S^{n-1} $\left(=\frac{G}{H}\right)$
$n=2$
$\sum_i (\partial_i \phi_1)^2 + (\partial_i \phi_2)^2$
$F = \int \frac{\gamma}{2} |\psi|^2 \left(\vec{\nabla}\theta\right)^2$
assuming $\phi_1^2 + \phi_2^2 = \text{constant}$
General n
Consider a unit vector $\hat{n}$ living on the sphere S^{n-1}
The analogous theory is
$\int d^d x \frac{1}{2e^2} \left(\vec{\nabla} n_i \cdot \vec{\nabla} n_j\right)$
$= \sum_{ij} (\partial_i n_j)^2$

$d\lambda$

* $\vec{n} \in S^{n-1} = G/H$ (nonlinear σ-models (NLσM))

models of the form $F[\vec{n}]$ are called 'sigma models'

- the constraint $\hat{n} \cdot \hat{n} = 1$ is equivalent to a nonlinearity in F

* $F \sim \frac{1}{2e^2} \int |\partial \vec{n}|^2 d^d x$; "e^2" behaves like T. $\left(Z \sim e^{-\frac{1}{T}F}\right)$

we will use the notation $F = \frac{1}{2T} \int |\partial \vec{n}|^2 d^d x$

$$= |\phi^*|^2 \times N \langle \phi_b^* \phi_b \rangle \quad |\phi|^2 \chi^{-1} \langle \phi_b \phi \rangle$$

$L \sigma M)$

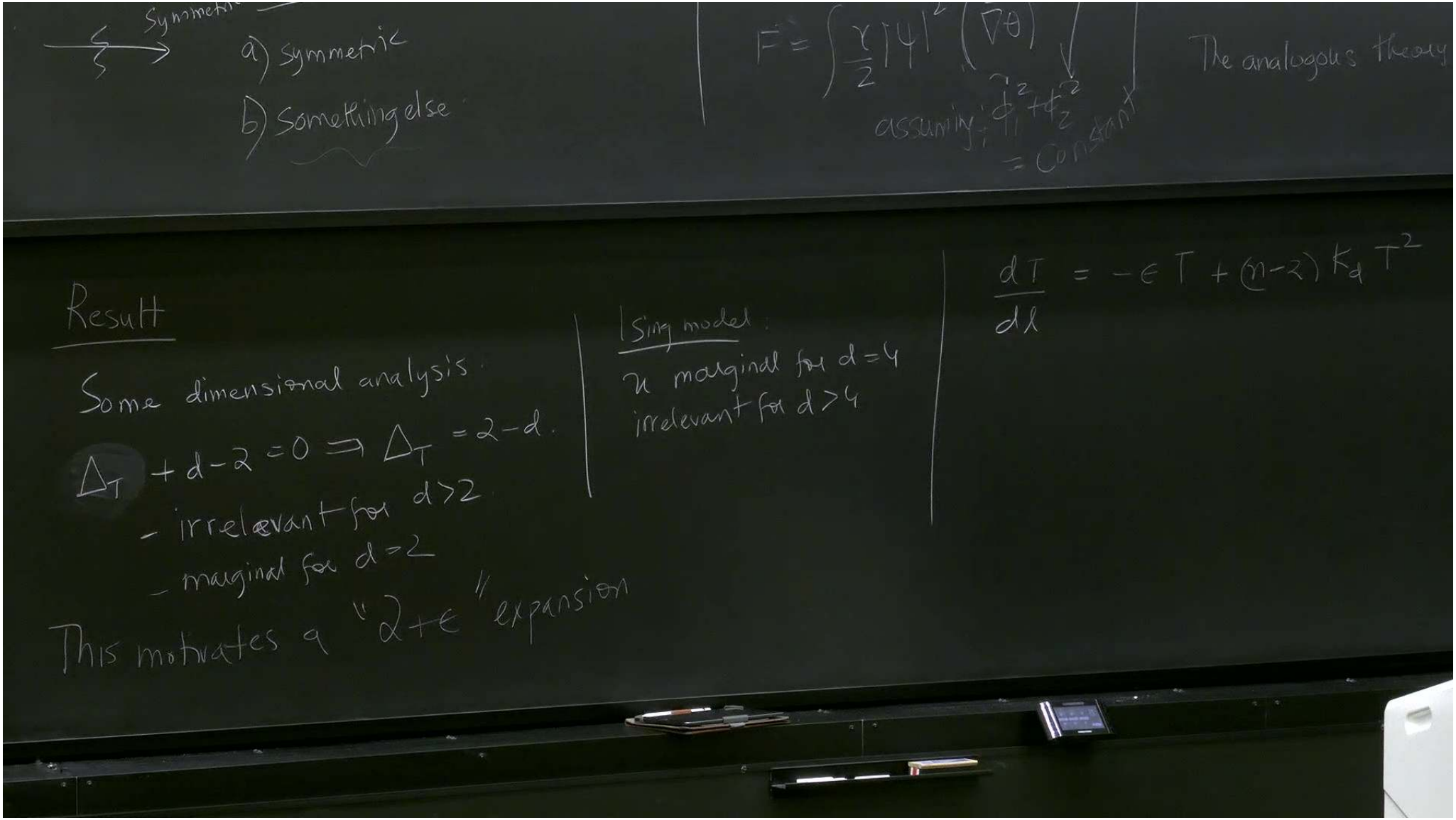
F

$\sim e^{-\frac{1}{T}(F}$

$\frac{1}{2T} \int |\vec{\nabla} \vec{n}|^2 d^d x$

Suppose we did RG for this model;
we are interested in how T flows under RG.

* at $T \to \infty$, small and large fluctuations come with equal weight $\Rightarrow$ expect that ordered state will be destabilized.
* at small T, large fluctuations are suppressed.


Symmetric
a) symmetric
b) Something else
$F = \int \frac{r}{2} |\psi|^2 (\nabla\theta)$
assuming $\frac{\dot{q}_1^2 + \dot{q}_2^2}{2}$ = constant
The analogous theory
Result
Some dimensional analysis:
$\Delta_T + d - 2 = 0 \Rightarrow \Delta_T = 2 - d$
- irrelevant for $d > 2$
- marginal for $d = 2$
This motivates a "$2+\epsilon$" expansion
Ising model:
n marginal for $d = 4$
irrelevant for $d > 4$
$\frac{dT}{dl} = -\epsilon T + (n-2) K_d T^2$

Symmetric not SSB

a) symmetric

b) Something else

$F = \int \frac{\gamma}{2} |\psi|^2 (\nabla\theta)^2$

assuming: $\frac{\phi_1^2 + \phi_2^2}{2}$ = constant

Consider

The analogous theory is:

Result

Some dimensional analysis:

$\Delta_T + d - 2 = 0 \Rightarrow \Delta_T = 2 - d$.

- irrelevant for $d > 2$
- marginal for $d = 2$

This motivates a "$2+\epsilon$" expansion

Ising model

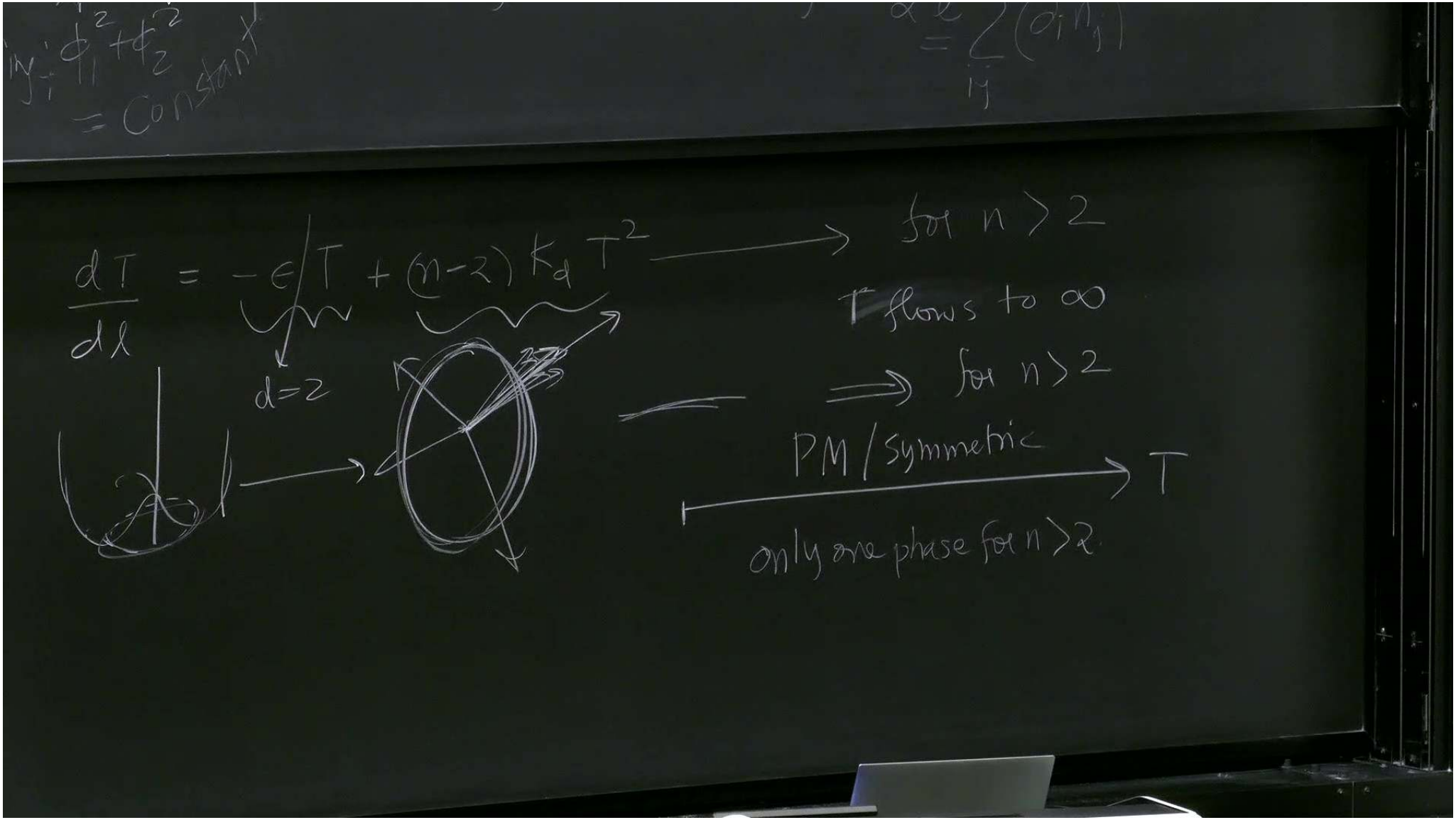
n marginal for $d = 4$

irrelevant for $d > 4$

$J \to \infty$

$\frac{dT}{dl} = -\epsilon T + (n-2) K_d T^2$

$d = 2$


$\frac{dT}{dl} = -\epsilon T + (n-2) K_d T^2$
d=2
for $n > 2$
T flows to ∞
for $n > 2$
PM / symmetric
T
only one phase for $n > 2$

Result

Some dimensional analysis:

$\Delta_T + d - 2 = 0 \Rightarrow \Delta_T = 2 - d$.

- irrelevant for $d > 2$
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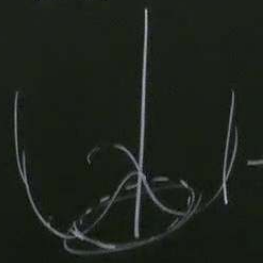
Ising model:

n marginal for $d = 4$

irrelevant for $d > 4$

$J \sum \sigma\sigma$

$\frac{dT}{dl} =$



$d=2$ MW: not-SSB $\xrightarrow{\text{symmetric}}$ T

This suggests

RG with NLσM: $\xrightarrow{\text{symmetric}}$ $n>2$

but for $n=2$ (XY model): $\boxed{\frac{dT}{d\ell}=0}$

to all orders in perturbation theory (ϵ)!

$= |\phi|^{\times N} \langle \psi_b \psi_b \rangle$

This suggests Something else → Symmetric T

also Symmetric, just a different phase

Let's look at 2-point correlations

In high-T phase, $\mathcal{L} \sim |\nabla\psi|^2 + \mu^2\psi^2$; find $\langle \psi^\dagger(r)\,\psi(0)\rangle \sim \frac{e^{-r/\xi}}{\sqrt{r}}$

This motivates a "$2+\epsilon$" expansion

low T Let's look at

$\langle \theta(x)\,\theta(0) \rangle$ where $\theta \to$ minimum of MFT

$= -\frac{T}{2\pi} \log(\Lambda x)$

(last class)

To compare with high-T, $\langle e^{-i\theta(x)} e^{i\theta(0)} \rangle$ "Wick's identity"

$= e^{-\frac{1}{2}\langle (\theta(x)-\theta(0))^2 \rangle}$

$\approx e^{-\frac{T}{2\pi}\log \Lambda x} \approx \frac{1}{(\Lambda x$

we want

identity "

$$-\frac{1}{2}\left\langle(\theta(x)-\theta(0))^2\right\rangle$$

$$e^{-\frac{T}{2\pi}\log\Lambda x} \approx \frac{1}{(\Lambda x)^{T/2\pi}}$$

We get power-law correlations over a range of T where T is a marginal coupling & doesn't flow under RG.

$\left(Z \sim e^{-\beta F}\right)$ we will use the notation $\beta = \frac{1}{2T}$

* There is a phase transition

for $d=2$ XY model $\langle \psi^\dagger \psi \rangle \sim \frac{1}{r^{T/2\pi}}$ $\Big\}$ $\langle \psi^\dagger \psi \rangle \sim e^{-r/\xi}$ $\longrightarrow T$

- The two phases have same symmetry
- Unlike SSB phases, here there is no local order parameter.
- The "Landau paradigm" (all phases are given by $\langle O \rangle \neq 0$)

$\left(Z \sim e^{-\beta F}\right)$ we will use the notation $\beta = \frac{1}{2T}$

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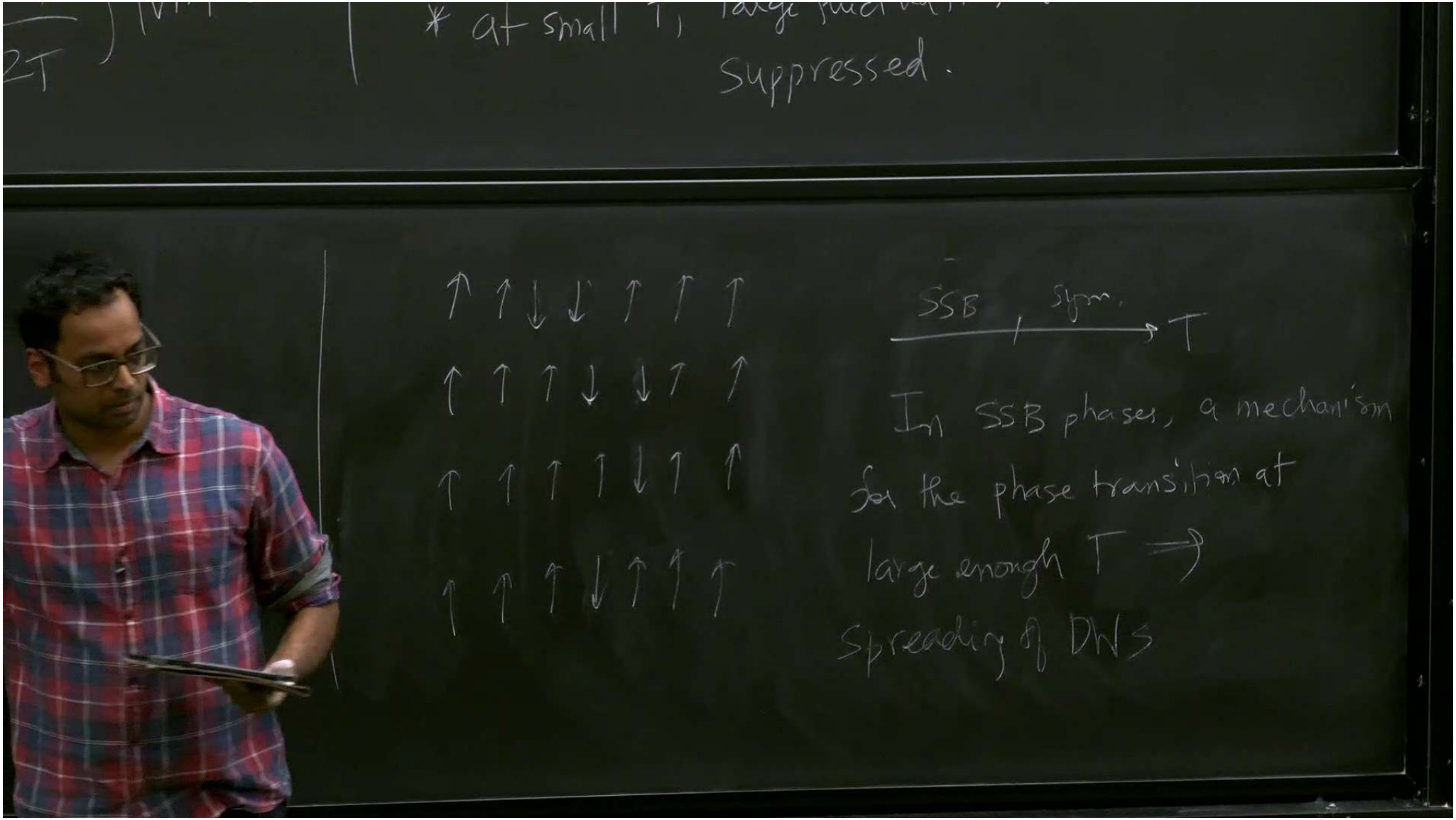
- The two phases have same symmetry
- Unlike SSB phases, here there is no local order parameter.
- The "Landau paradigm" (all phases are given by $\langle G, \pm 1 \rangle$) is incomplete.


* at small T,
suppressed.
SSB, Sym.
T
In SSB phases, a mechanism for the phase transition at large enough T ⇒ spreading of DWs

* at small T, large fluctuations suppressed.

SSB, Sym. ⟶ T

In SSB phases, a mechanism for the phase transition at large enough T ⟹ Spreading of DWs

spreading of DNS

$\frac{1}{r^2}(y, -x)$ ($\times n$ for winding number n)

$\langle \vec{r}^2 \rangle = \frac{1}{r^2}$

$r = L$

$\int d^2x \, \frac{1}{r^2} + \Delta F_{core} = \frac{\pi}{T} \log \frac{L}{a} + \Delta F_{core}$

$\int r \, dr \, \frac{1}{r^2}$

prob (vortex)

$= \frac{1}{Z} \left(\frac{L}{a}\right)^2 e^{-\frac{\pi}{T} \log \frac{L}{a} - \Delta F_{core}}$

$= \left(\frac{L}{a}\right)^{2 - \frac{\pi}{T}} \times \frac{e^{-\Delta F_{core}}}{Z}$.