Title: Lecture - Quantum Theory (Core), PHYS 605

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Subject: Quantum Foundations

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Feynman rules for $i\mathcal{M}$

$$\langle f|S|i\rangle \text{ or } i\mathcal{M} \xleftrightarrow{\text{LSZ}} \langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle \xleftarrow{\text{Feynman diagrams}} \Delta_F$$

diagrams

$$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle \text{ in } \varphi^4 \text{ theory}$$

$\updownarrow$ LSZ

$\left(\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4\right)$

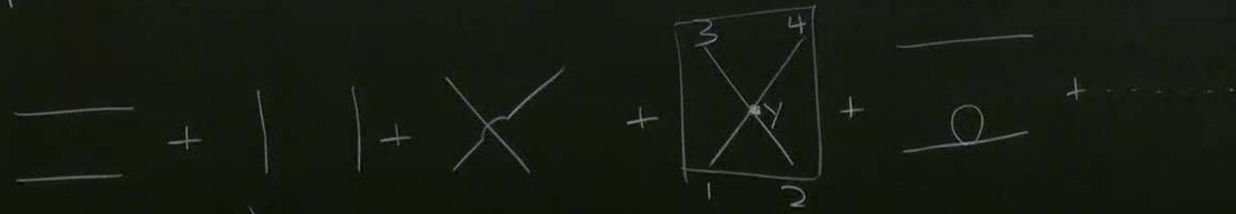
$2\to 2$ scattering

Last

Last time: $\delta^4(\Sigma p)$ per connected component

first interesting

$$\langle\Omega|T\varphi_1\varphi_2\varphi_3\varphi_4|\Omega\rangle =$$



$$\langle f|S|i\rangle_\times = i^4\int\prod_i d^4x_i\, e^{i\lambda_i x_i\cdot p_i}(\partial_i^2+m^2)(-i\lambda)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{3y}\Delta_{4y}$$

$$= (-i\lambda)\int d^4y\, e^{iy(p_1+p_2-p_3-p_4)}(-i)^4$$

$$= (-i\lambda)(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$(\partial_i^2+m^2)\Delta_{iy} = -i\delta^{(4)}(x_i-y)$$

$$\boxed{iM_X = -i\lambda}$$ ← no propagators for external lines!

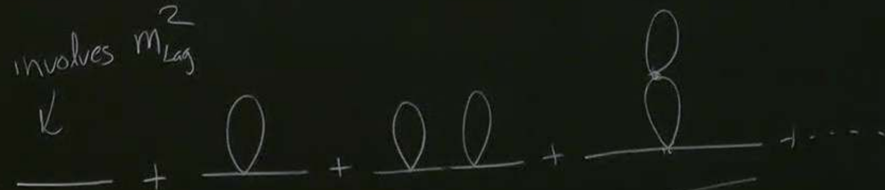
→ Amputate

Amputation

m_{ph}^2

$$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle =$$

involves m_{Lag}^2



corrections to free propagator

expect to depend on physical mass

related to amplitude for particle to propagate from x_2 to x_1

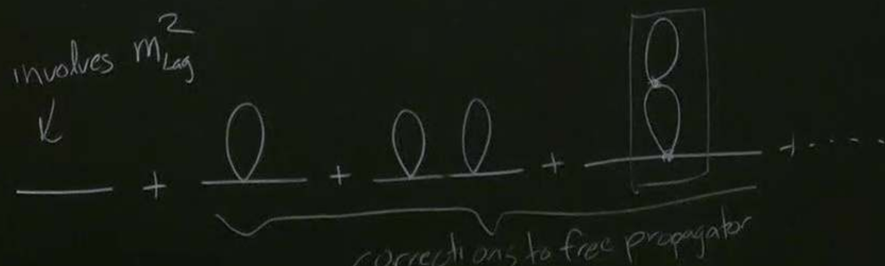
$\partial^2 + m_{ph}^2$ kills Δ_F + corrections

$i\mathcal{M}_\times = -i\lambda$ ← no propagators for external lines!

Amputation

m^2_{ph}

$G_2(x_1-x_2) = \langle\Omega|T\varphi_1\varphi_2|\Omega\rangle =$

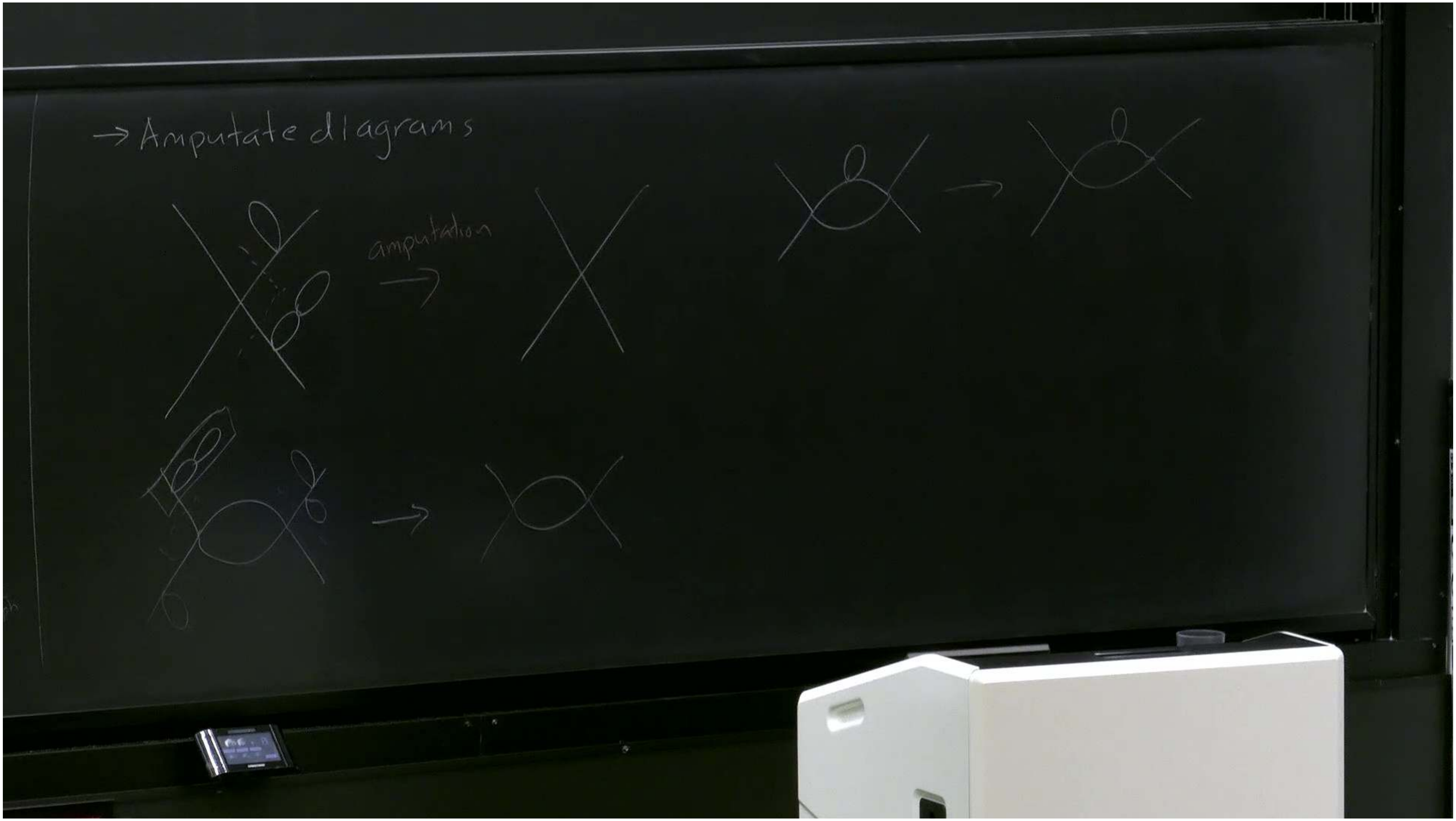


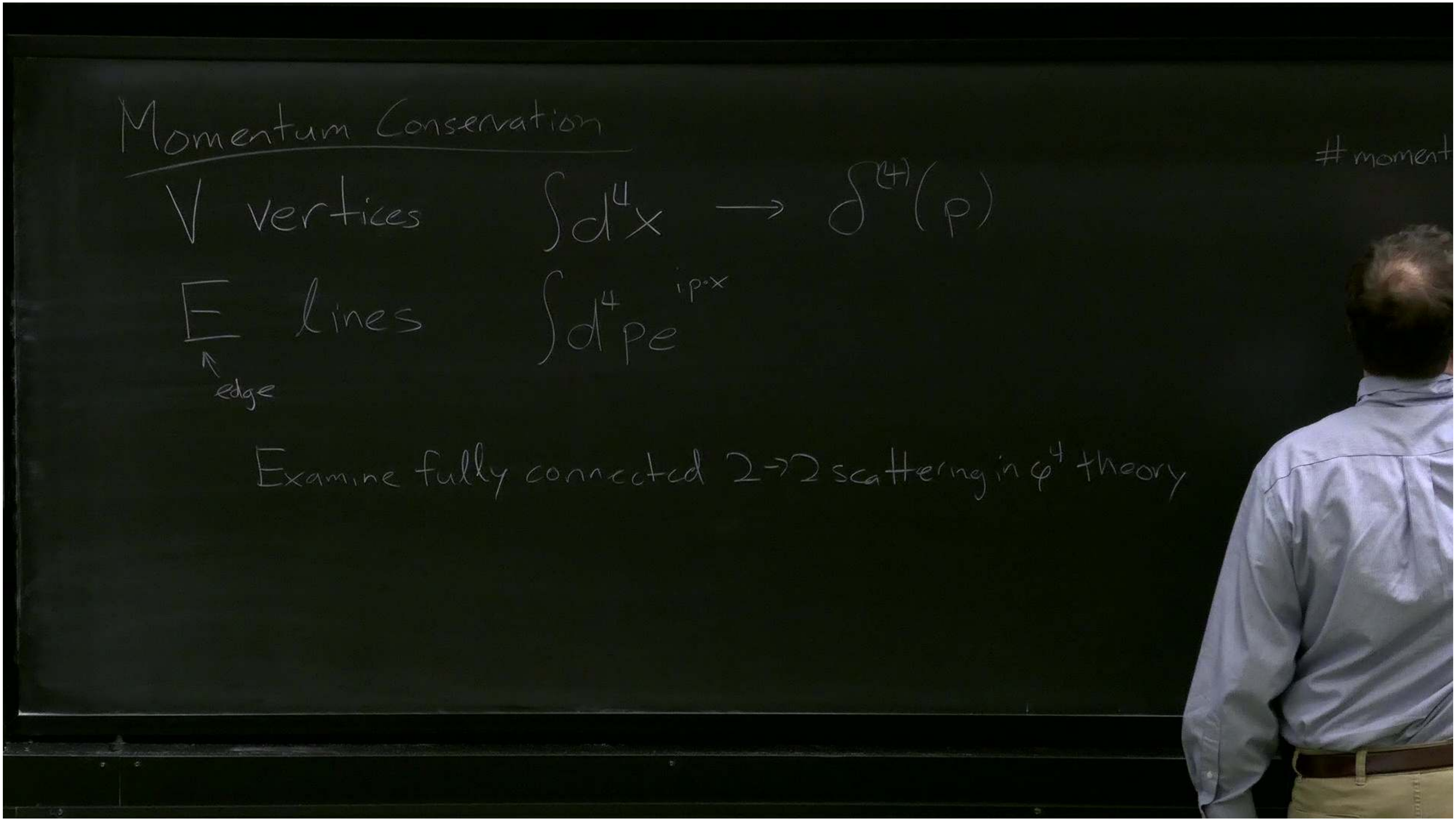
expect to depend on physical mass

related to amplitude for particle to propagate from x_2 to x_1

$\partial^2 + m^2_{ph}$ kills Δ_F + corrections $\quad \tilde{G}_2(p) = \frac{i}{p^2 - m^2_{ph} + i\epsilon}$ + finite $p^2 \to m^2_{ph}$

→ Amputate


→ Amputate diagrams
amputation


Momentum Conservation
#moment
V vertices $\int d^4x \longrightarrow \delta^{(4)}(p)$
E lines $\int d^4p\, e^{ip\cdot x}$
edge
Examine fully connected $2 \to 2$ scattering in φ^4 theory

$\boxed{\begin{array}{c}\#\text{ momentum integrals}\\ \text{in } i\mathcal{M}\end{array}} = E - \underset{\substack{\uparrow\\ \text{no } \int d^4p\\ \text{for external}}}{4} - (V-1) = E - V - 3 = E - V + \chi = \boxed{\#\text{loops}}$

Euler
Characteristic

$$\chi = V - E + \underset{\substack{\nwarrow\\ \#\text{ loops}}}{L}$$

$$\chi = 1 - 4 + 0 = -3$$

Feynman rules for $i\mathcal{M}$

$i\mathcal{M}$ = sum of connected, amputated diagrams

interesting contribution

asymptotic to

1. internal $\xrightarrow{p}$ $= \dfrac{i}{p^2 - m_{lag}^2 + i\varepsilon}$

2. external $\xrightarrow{p}$ $= 1$

$$= (-i\lambda)(2\pi)^4 \delta^{(4)}(p_1+p_2-p_3-p_4)$$

$$\boxed{\text{...um integrals ... } \mathcal{M}} = E - \underset{\substack{\uparrow \\ \text{no } \int d^4p \\ \text{for external}}}{4} - (V-1) = E-V-3 = E-V+\chi = \boxed{\#\text{loops}}$$

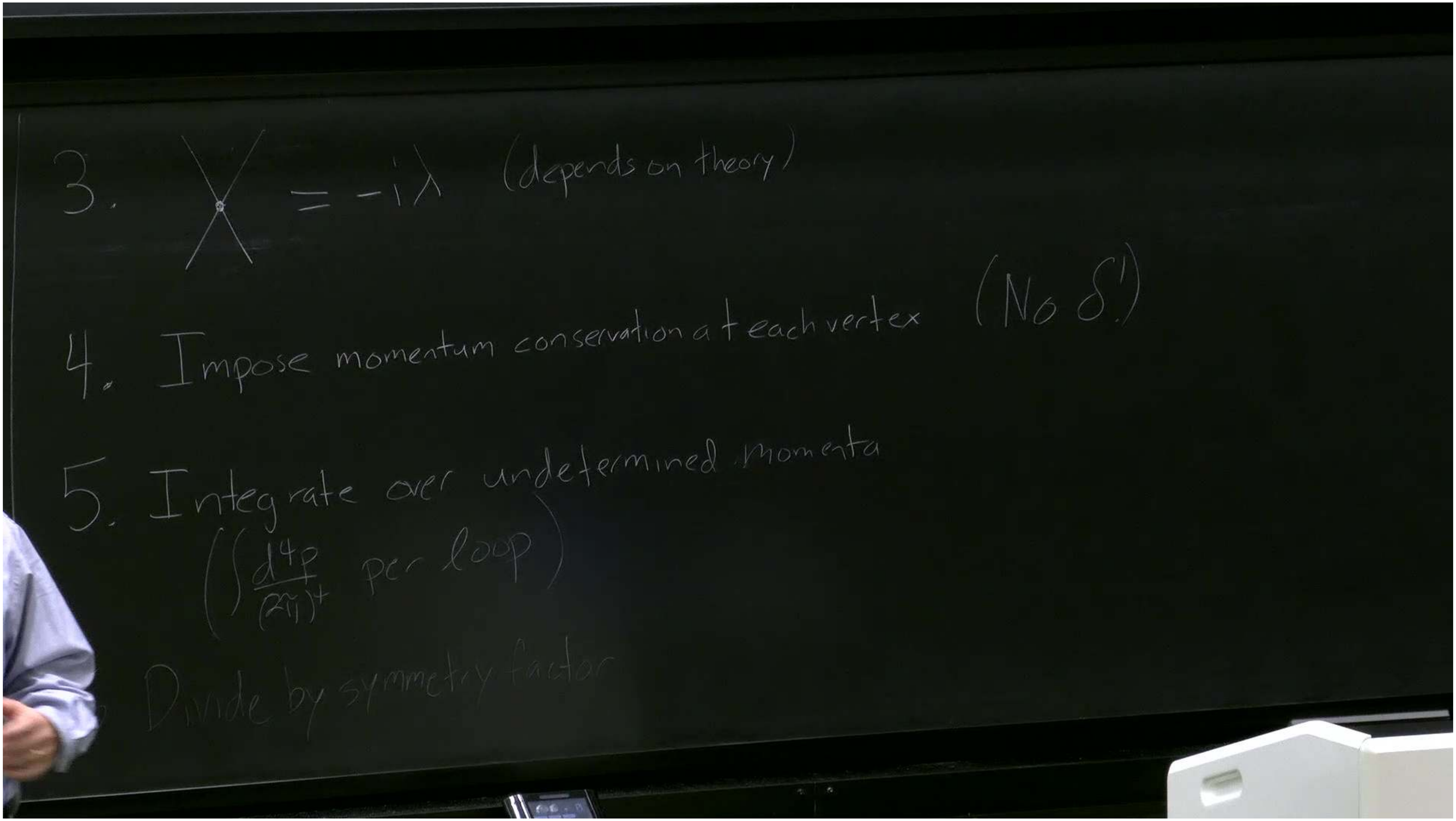
...r

...istic

$$\chi = V - E + \underset{\nwarrow \#\text{ loops}}{L}$$

$$\chi = 1 - 4 + 0 = -3$$

$$\Delta_{12} = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{ip(x_1-x_2)}}{p^2 - m^2_{Lag} + i\varepsilon}$$


3. = −iλ (depends on theory)
4. Impose momentum conservation at each vertex (No δ!)
5. Integrate over undetermined momenta
(∫ d^4p/(2π)^4 per loop)
Divide by symmetry factor

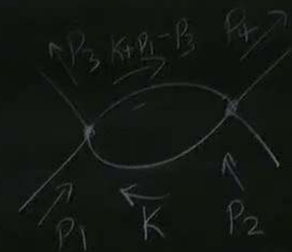
$-i\lambda$ (depends on theory)

momentum conservation at each vertex $\left(\text{No } \delta!\right)$

e over undetermined momenta

per loop)

symmetry factor



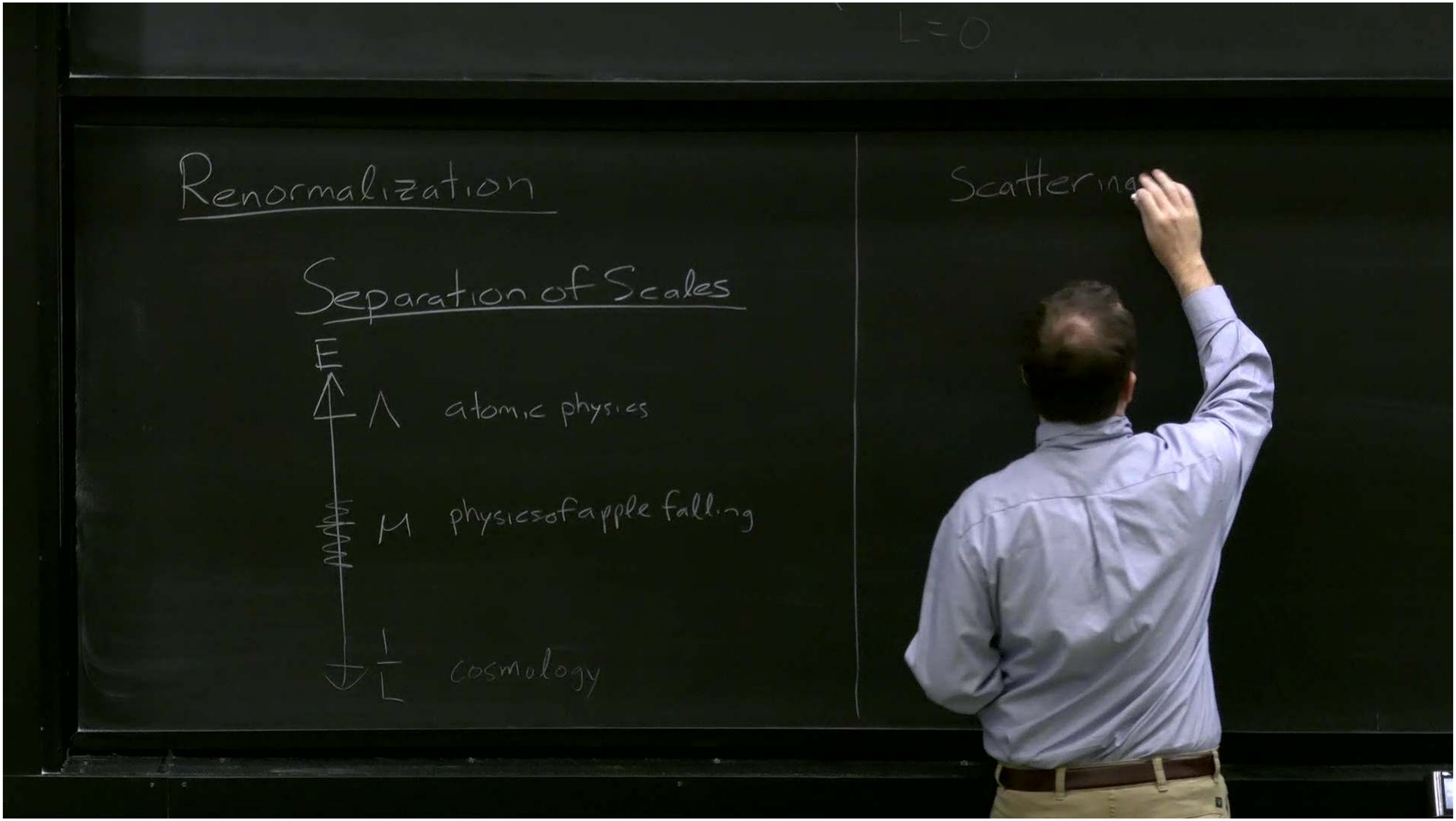
$$= \frac{(-i\lambda)^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-m^2+i\epsilon} \cdot \frac{i}{(k+p_1-p_3)^2-m^2+i\epsilon}$$

left

$$k+p_1 = k+p_1-p_3+p_3$$

$$p_2+\cancel{k}+p_1-p_3 = p_4+\cancel{k}$$

$$= i\mathcal{M}$$


L=0
Renormalization
Separation of Scales
E
Λ atomic physics
μ physics of apple falling
1/L cosmology
Scattering

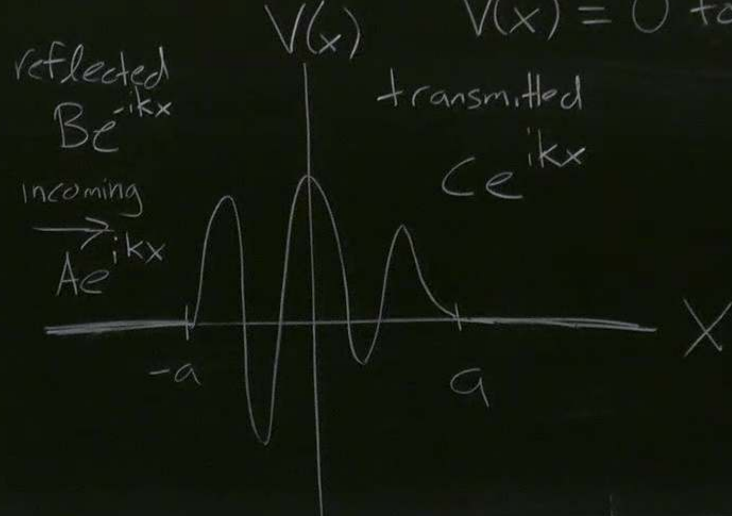
L = 0

Scattering in a local potential

$$H = \frac{p^2}{2m} + V$$

local potential

$$V(x) = 0 \text{ for } |x| > a$$

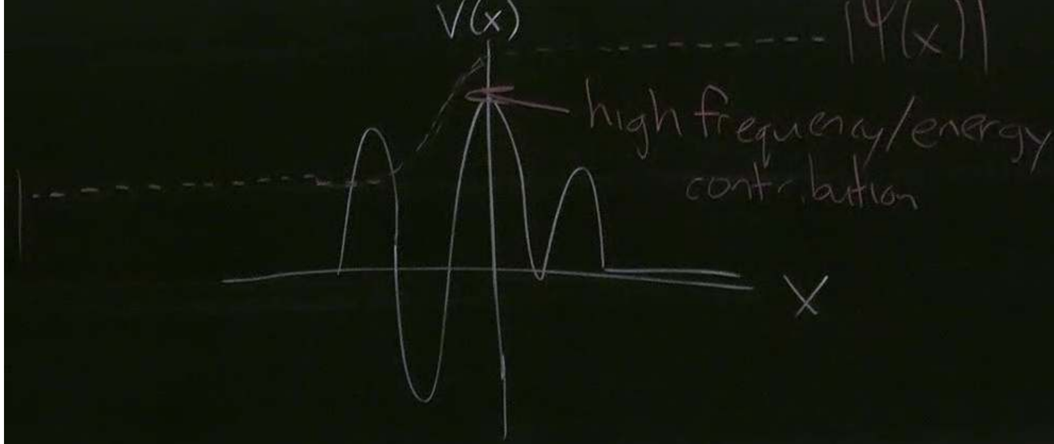


Expect: low energy probes cann

Expect: low energy probes cannot learn about detailed shape of V

$p << \frac{1}{a}$ local potential aproximation

$$V(x) = c_0\,\delta(x) + c_1\,\delta'(x) + \cdots$$



$$T(p) = f(c_0, c_1)$$

but calculation involves high energies

$\rightarrow c_i(\mu)$ constants depend on energy at which measured

Renormalization of φ^3

$$\mathcal{L}_{naive} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{g}{3!}\varphi^3$$

 $= -ig\int d^4x$

$$\text{LSZ: } \left.\begin{aligned} \langle\Omega|\varphi(x)|\Omega\rangle &= 0 \\ \langle k|\varphi(x)|\Omega\rangle &= e^{ikx} \end{aligned}\right\} \text{ shift + rescale}$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} + \underset{\text{counterterm}}{\mathcal{L}_{ct}}$$

$$\mathcal{L}_0 = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\mathcal{L}_{int} = \frac{1}{3!} Z_g g \varphi^3$$

$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 + \frac{1}{2}(Z_m - 1)m^2\varphi^2 + Y\varphi$$

4 independent parameters (degeneracy between Z_g & g and Z_m & m)

LSZ eliminate 2 $\to$ 2 physical parameters

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} + \mathcal{L}_{ct}$$

counterterm

$$Z_1 = 1 + \mathcal{O}(g)$$

$$Y = 0 + \mathcal{O}(g)$$