Title: Lecture - Quantum Theory (Core), PHYS 605

Speakers: David Schmid

Collection/Series: Quantum Theory (Core), PHYS 605, September 2 - October 7, 2025

Subject: Quantum Foundations

Date: September 04, 2025 - 2:45 PM

URL: https://pirsa.org/25090023

Textbook Postulates for QT

- State space postulate: Associated to each system is a Hilb. space $\mathcal{H}$
  - the state of a system is described by a unit vector $|\psi\rangle \in \mathcal{H}$
  - states differing by a global phase are physically equivalent
  - in most settings, any unit vector corresponds to a state

(X, P)

- Evolution Postulate: Closed system evolutions are described by unitaries

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$$

for continuous time, $i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$ for some Hermitian

$$|\psi(t_2)\rangle = e^{\frac{-iH(t_2-t_1)}{\hbar}}|\psi(t_1)\rangle$$

space $\mathcal{H}$

$\in \mathcal{H}$

(X, P)

unitaries

for some Hermitian H

$\psi(t_1)\rangle$

Measurement Postulate: a measurement is associated to a collection $\{\Pi_k\}_k$ of projectors s.t. $\sum_k \Pi_k = \mathbb{I}$

$$P(k|\psi) = Tr\left[|\psi\rangle\langle\psi|\,\Pi_k\right] = \langle\psi|\Pi_k|\psi\rangle \quad \text{Born's Rule}$$

side note: any Hermitian operator is "equivalent" to a set of projectors together with some e-values

$$A = \sum_k a_k \Pi_k \qquad \langle A\rangle = \langle\psi|A|\psi\rangle = \sum_k a_k \langle\psi|\Pi_k|\psi\rangle = \sum_k a_k P(k|\psi)$$

$|\psi(t)\rangle = U(t)|\psi(0)\rangle$ for continuous time, $i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$ for some

$$|\psi(t_2)\rangle = e^{\frac{-iH(t_2-t_1)}{\hbar}}|\psi(t_1)\rangle$$

State update after measurement Postulate: after a measurement, the evolution is not unitary, but rather $|\psi\rangle \xrightarrow{k} \frac{\Pi_k|\psi\rangle}{\sqrt{P(k)}}$

e.g. if $\Pi_k = |k\rangle\langle k|$, $|\psi_{initial}\rangle \longrightarrow |k\rangle$ "collapse rule"

$|\Psi(t_1)\rangle$ $\qquad \sum_k \qquad \langle A\rangle = \langle\psi|A|\psi\rangle = \sum_k a_k \langle\psi|\Pi_k|\psi\rangle = \sum_k$

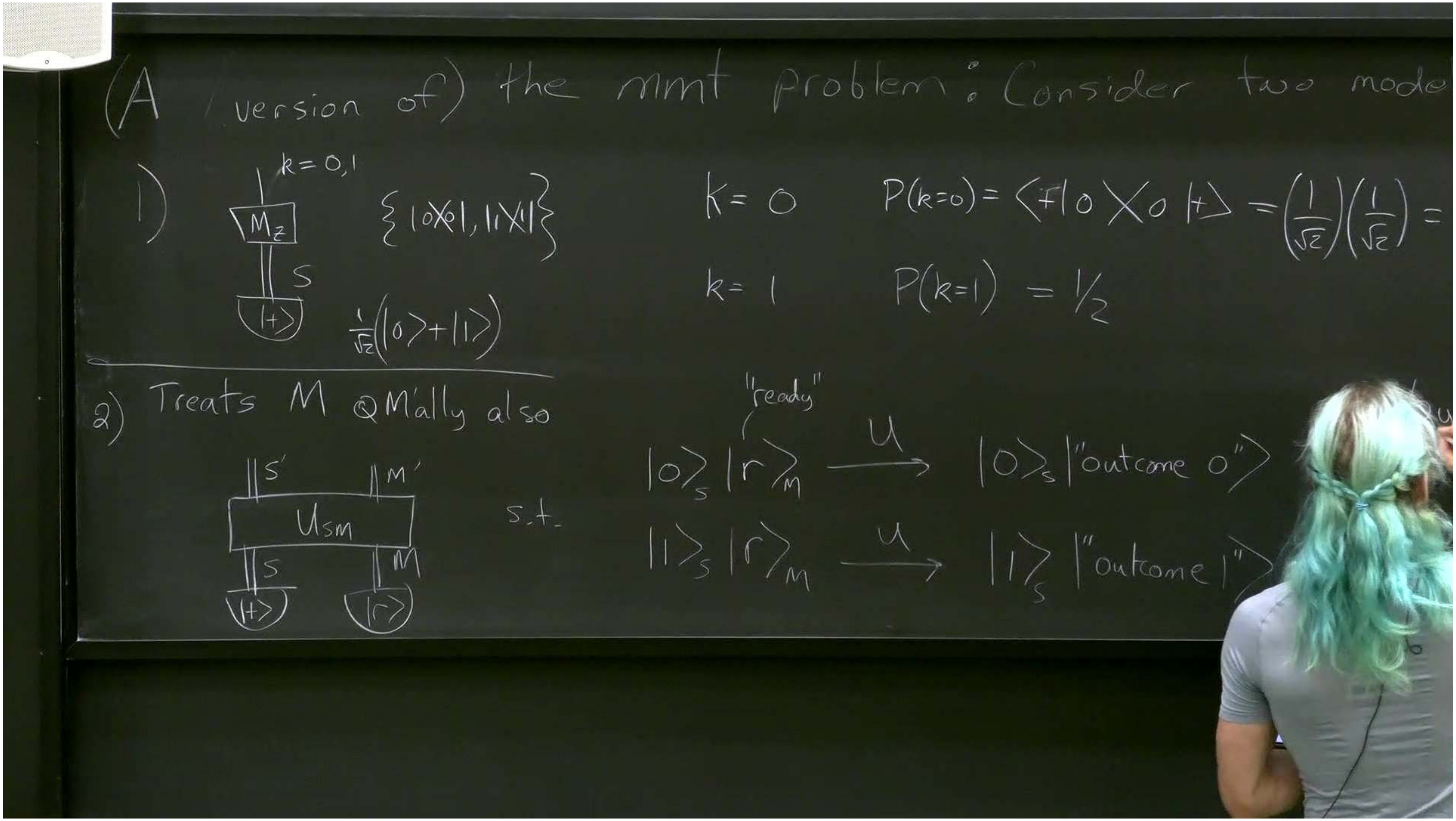
Orthodox/Textbook Interpretation (not Copenhagen)

1) Projective mmts $\longleftrightarrow$ Properties

   e.g. $\{|x\rangle\langle x|\}_x \longleftrightarrow$ Position

2) eigenstate-eigenvalue link: a state has a definite value for a property iff it is an e-state of the associated projector

   e.g. $|x\rangle$ has a definite position, namely x


(A version of) the mmt problem: Consider two mode
1)
k=0,1
M_z
S
|+⟩
{|0⟩⟨0|, |1⟩⟨1|}
1/√2 (|0⟩+|1⟩)
k = 0
P(k=0) = ⟨+|0⟩⟨0|+⟩ = (1/√2)(1/√2) =
k = 1
P(k=1) = 1/2
2) Treats M QM'ally also
S'
M'
U_SM
S
M
|+⟩
|r⟩
s.t.
"ready"
|0⟩_S |r⟩_M —U→ |0⟩_S |"outcome 0"⟩
|1⟩_S |r⟩_M —U→ |1⟩_S |"outcome 1"⟩

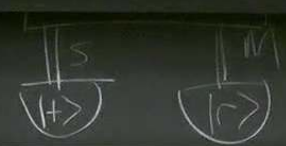
dels of a mmt process

$) = \frac{1}{2}$ state $\to |0\rangle$ $\frac{|0\rangle\langle 0|+\rangle}{\sqrt{P(0)}}$

$|1\rangle$

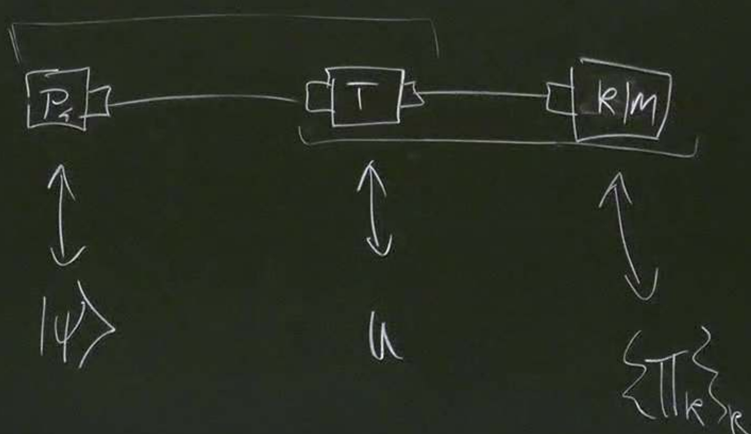
by linearity: $|+\rangle_S |r\rangle_M \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle_S |\text{outcome } 0\rangle_M + |1\rangle_S |\text{outcome } 1\rangle_M \right)$

$\underline{\mathbb{I}_S}$



$$|\uparrow\rangle_S |\uparrow\rangle_M \longrightarrow |\uparrow\rangle_S |\text{outcome } \uparrow\rangle$$

Operational QT : - mathematical symbols in the theory refer to la



$$Pr(k \mid P_s, T, M) = \langle \psi | U^\dagger \Pi_k U | \psi \rangle$$

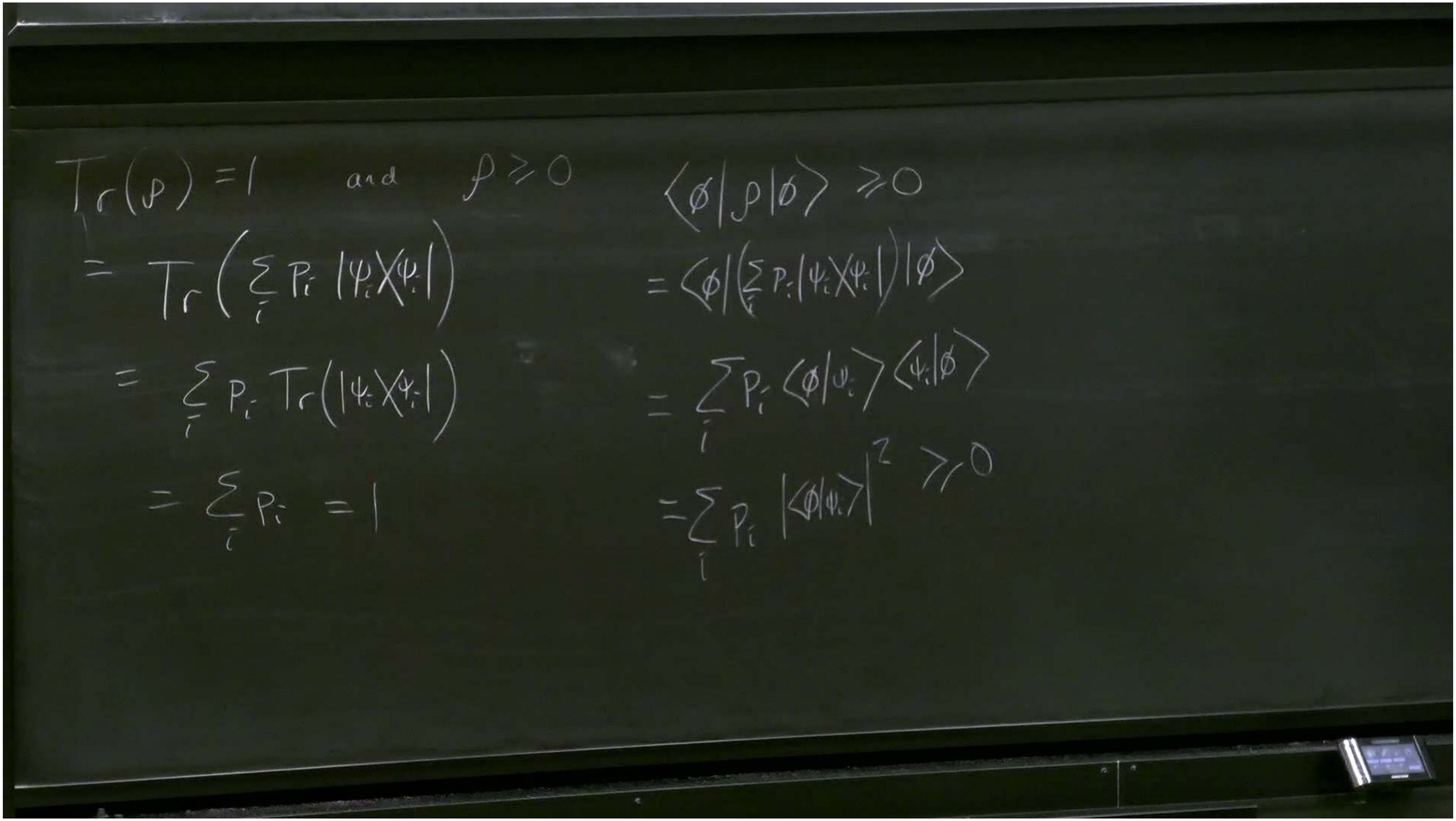
General Prepn Procedures:
- uncertainty
- access only to a subsystem
- noisy

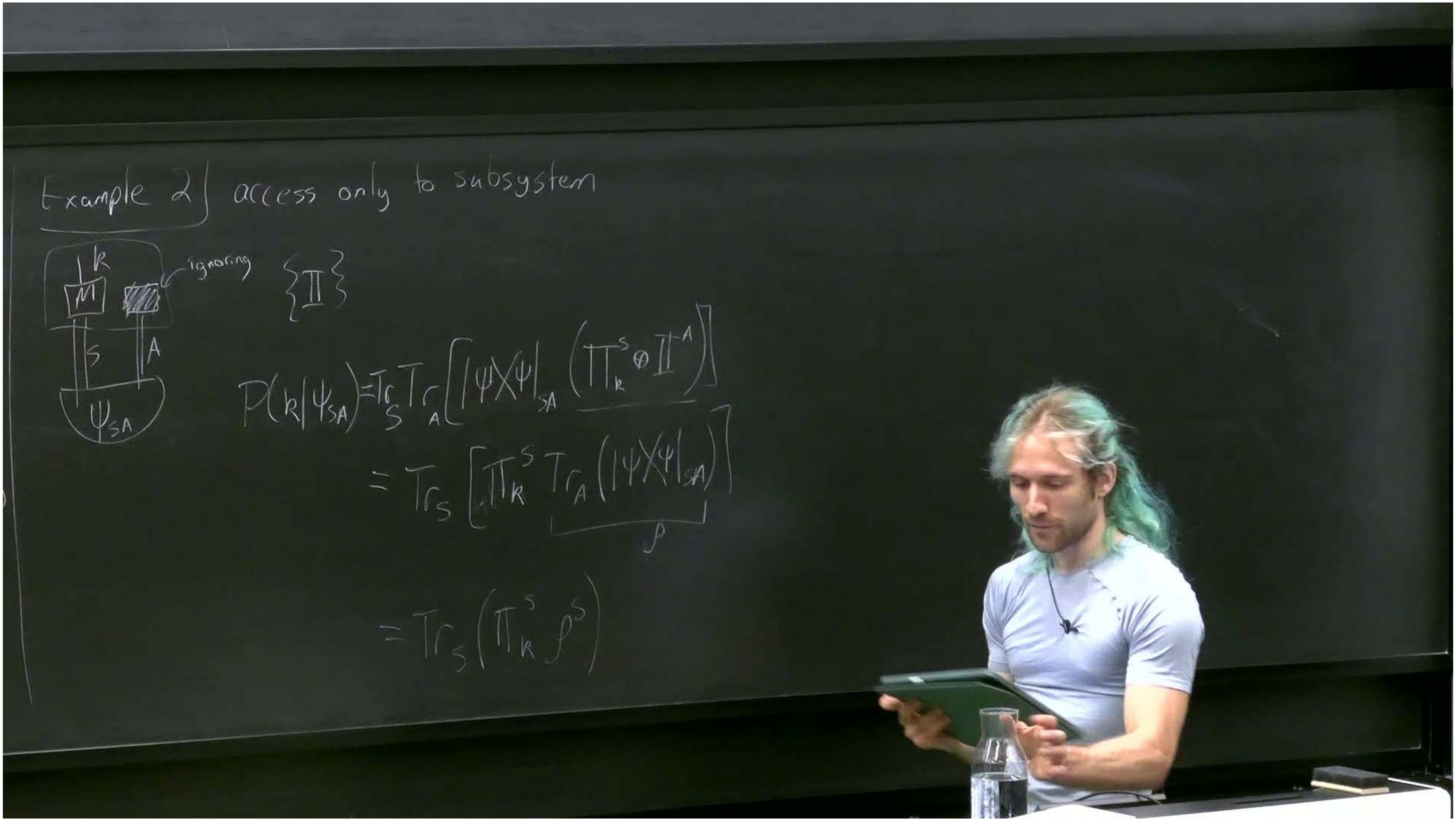
$Pr(k|\bar{P}$

Ex) Uncertainty about the procedure

Prepare $|\psi_i\rangle$ with prob p_i

"ensemble prepn procedure" $\bar{P} := \{p_i, |\psi_i\rangle\}_i$


$Tr(\rho) = 1$ and $\rho \geq 0$
$= Tr\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)$
$= \sum_i p_i Tr\left(|\psi_i\rangle\langle\psi_i|\right)$
$= \sum_i p_i = 1$
$\langle\phi|\rho|\phi\rangle \geq 0$
$= \langle\phi|\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)|\phi\rangle$
$= \sum_i p_i \langle\phi|\psi_i\rangle\langle\psi_i|\phi\rangle$
$= \sum_i p_i |\langle\phi|\psi_i\rangle|^2 \geq 0$


Example 2
access only to subsystem
ignoring
k
M
S
A
ψ_{SA}
$\{\mathbb{I}\}$
$P(k|\psi_{SA}) = Tr_S Tr_A [|\psi\rangle\langle\psi|_{SA} (\Pi_k^S \otimes \mathbb{I}^A)]$
$= Tr_S [\Pi_k^S Tr_A (|\psi\rangle\langle\psi|_{SA})]$
ρ
$= Tr_S (\Pi_k^S \rho^S)$

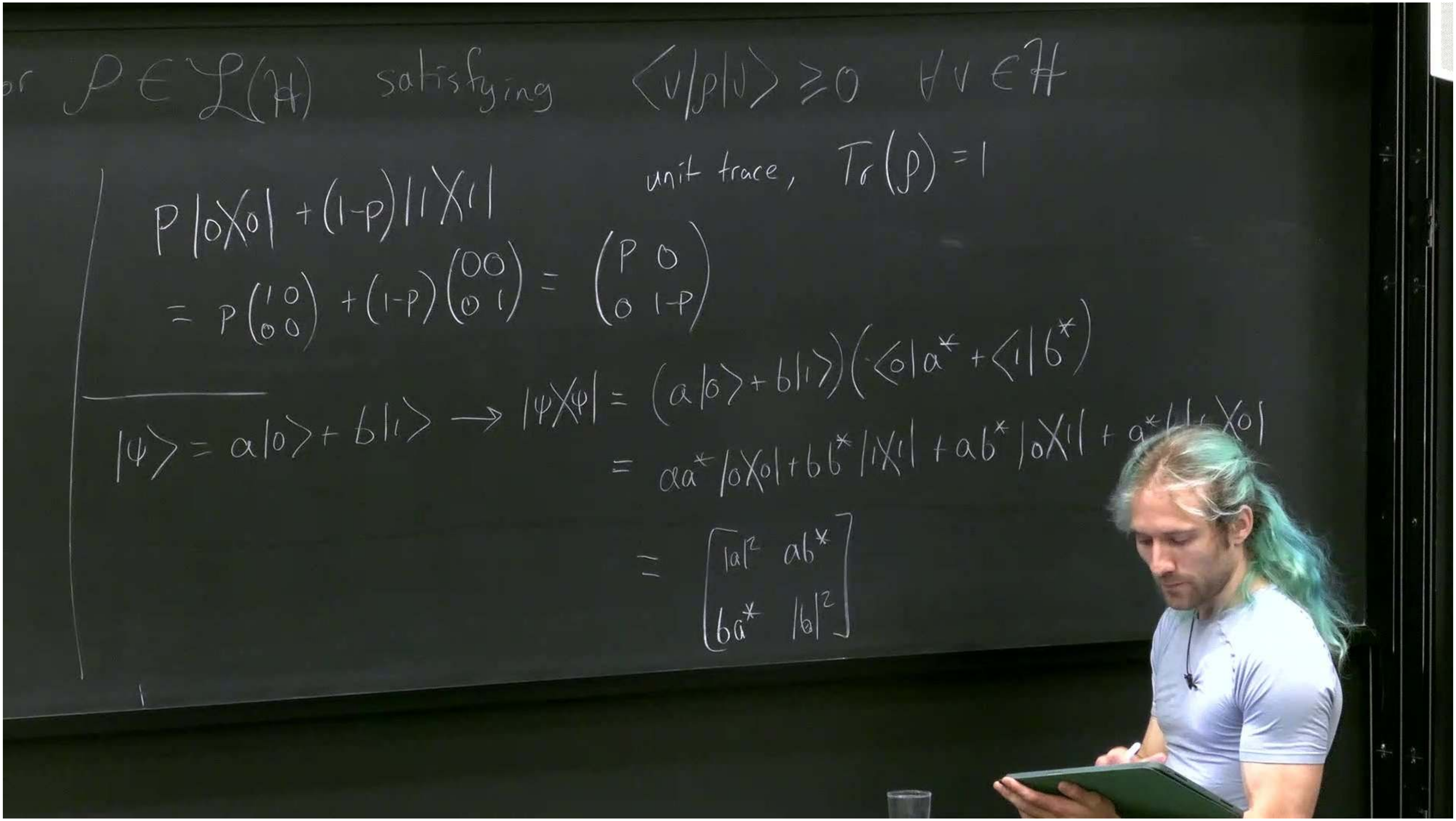
Postulate: Every Prepn Procedure corresponds to an operator $\rho \in \mathcal{L}$

$$|\psi\rangle \rightarrow \underline{|\psi\rangle\langle\psi|}$$

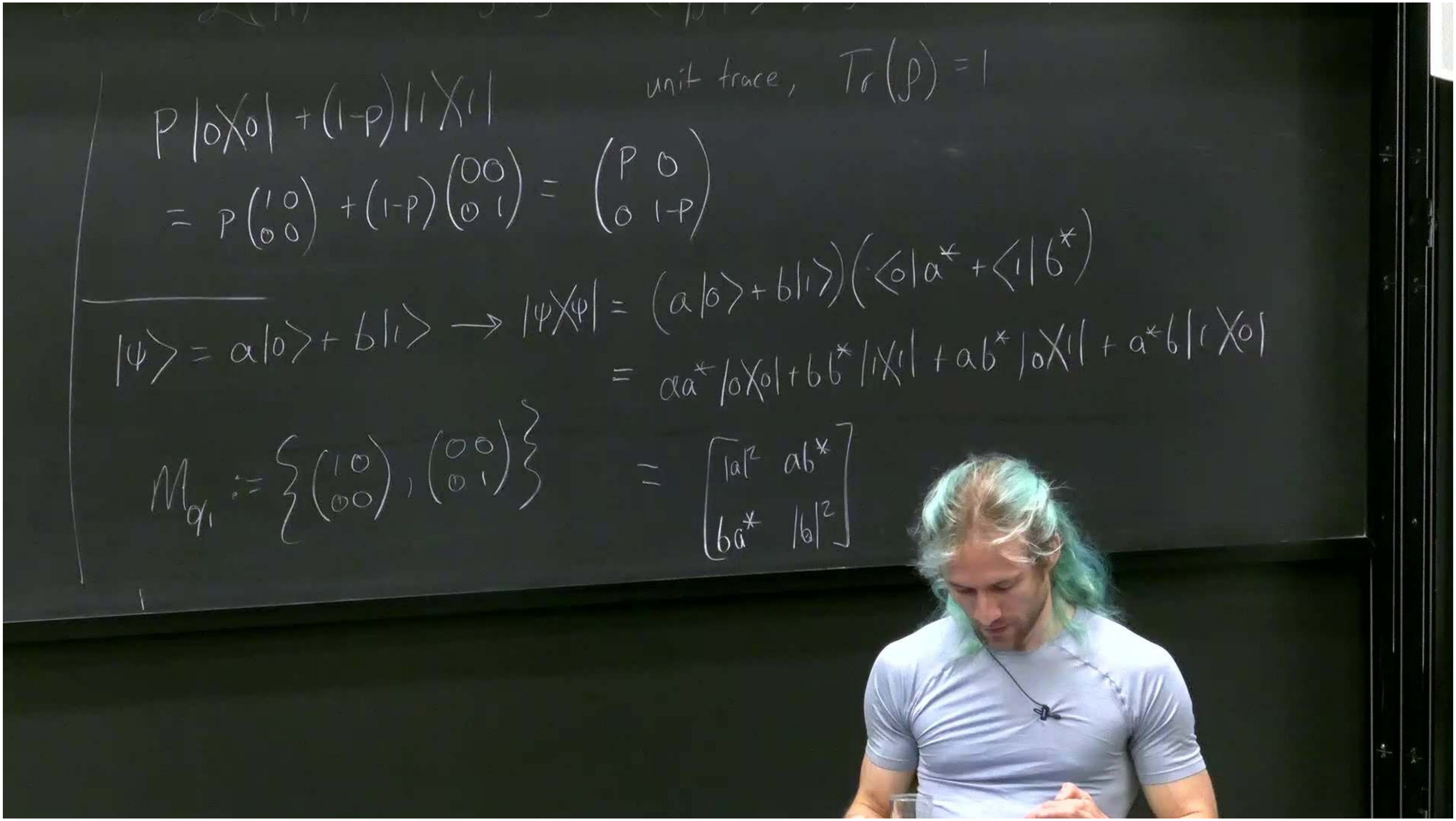
$$e^{i\theta}|\psi\rangle\langle\psi|e^{-i\theta} \qquad Tr\left(|\psi\rangle\langle\psi|\, \Pi_R\right)$$

e.g. $|\pm\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ \pm 1\end{bmatrix} \xrightarrow{\text{density matrix}} |\pm\rangle\langle\pm| = \frac{1}{2}\begin{bmatrix}1 & \pm 1\end{bmatrix}\begin{bmatrix}1\\ \pm 1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1 & \pm 1\\ \pm 1 & 1\end{bmatrix}$

mixed states: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$



or ρ ∈ 𝓛(𝓗) satisfying ⟨v|ρ|v⟩ ≥ 0 ∀v ∈ 𝓗
unit trace, Tr(ρ) = 1
p|0⟩⟨0| + (1-p)|1⟩⟨1|
= p(1 0; 0 0) + (1-p)(0 0; 0 1) = (p 0; 0 1-p)
|ψ⟩ = a|0⟩ + b|1⟩ → |ψ⟩⟨ψ| = (a|0⟩ + b|1⟩)(⟨0|a* + ⟨1|b*)
= aa*|0⟩⟨0| + bb*|1⟩⟨1| + ab*|0⟩⟨1| + a*b|1⟩⟨0|
= [|a|² ab*; ba* |b|²]


unit trace, Tr(ρ) = 1
p|0⟩⟨0| + (1−p)|1⟩⟨1|
= p(1 0; 0 0) + (1−p)(0 0; 0 1) = (p 0; 0 1−p)
|ψ⟩ = a|0⟩ + b|1⟩ → |ψ⟩⟨ψ| = (a|0⟩ + b|1⟩)(⟨0|a* + ⟨1|b*)
= aa*|0⟩⟨0| + bb*|1⟩⟨1| + ab*|0⟩⟨1| + a*b|1⟩⟨0|
= [|a|² ab*; ba* |b|²]
M_{q1} := {(1 0; 0 0), (0 0; 0 1)}