Title: Phenomenological consequences of phase transitions occurred during inflation

Speakers: Haipeng An

Collection/Series: Charting the Future Symposium

Subject: Cosmology, Particle Physics, Strong Gravity

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Abstract:

In slow-roll inflationary models, the inflaton can undergo excursions on the order of the Planck scale, leading to significant changes in the properties of fields coupled to the inflaton, referred to as spectator fields. These changes may result in transitions between weakly and strongly interacting regimes, or even alterations in mass squared within the spectator field sector during inflation. Such dynamics can induce phase transitions, which have profound implications for the early Universe. In this talk, I will explore the phenomenological consequences of these phase transitions, focusing on the production of gravitational waves, curvature perturbations, non-Gaussianities, dark matter, and baryon number. I will also demonstrate how gravitational waves generated by scalar perturbations induced by phase transitions may potentially explain the alleged gravitational wave signals observed in recent pulsar timing array studies.

Consequences of phase transitions occurred during inflation

Haipeng An (Tsinghua University)

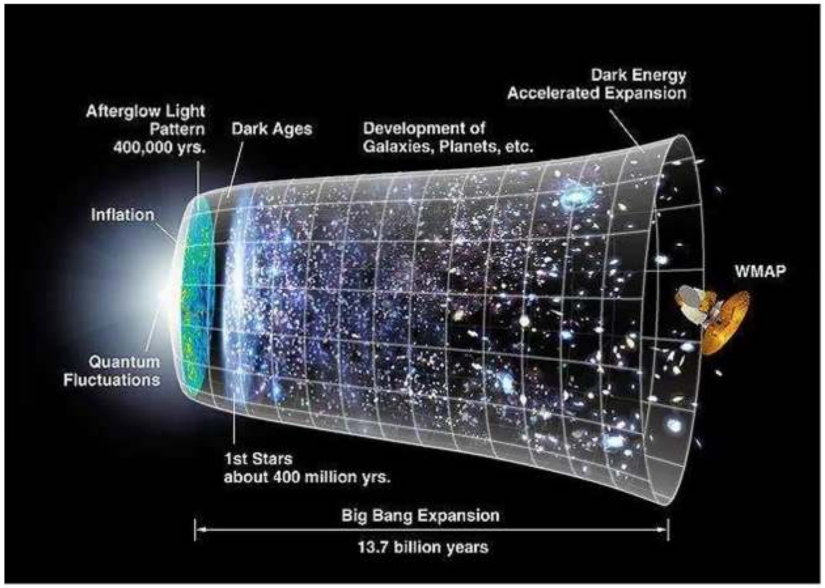
Charting the Future

August 25-29, 2025 @ Perimeter

2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou
2208.14857 w/ Xi Tong and Siyi Zhou
2304.02361 w/ Chen Yang
2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang
2409.05833 w/ Qi Chen, Yuan Yin
2411.12699 w/ Qi Chen, Yuhang Li, Yuan Yin

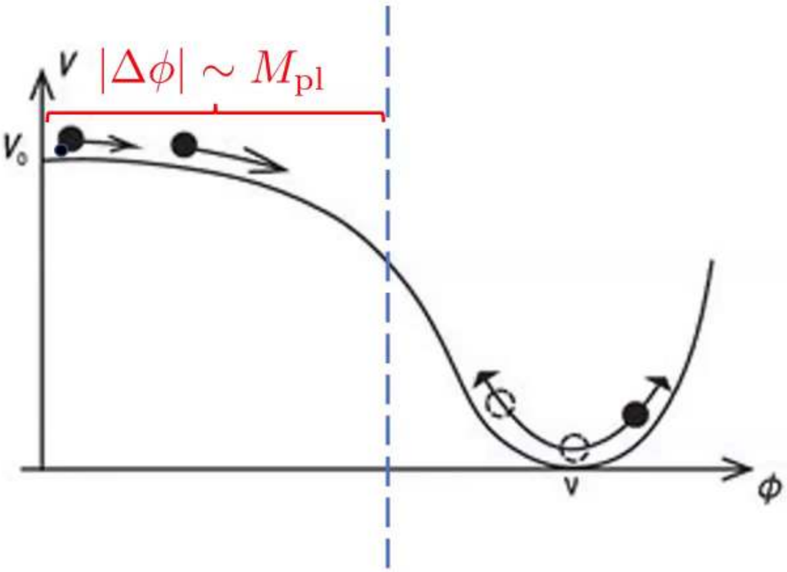
Very brief introduction of inflation

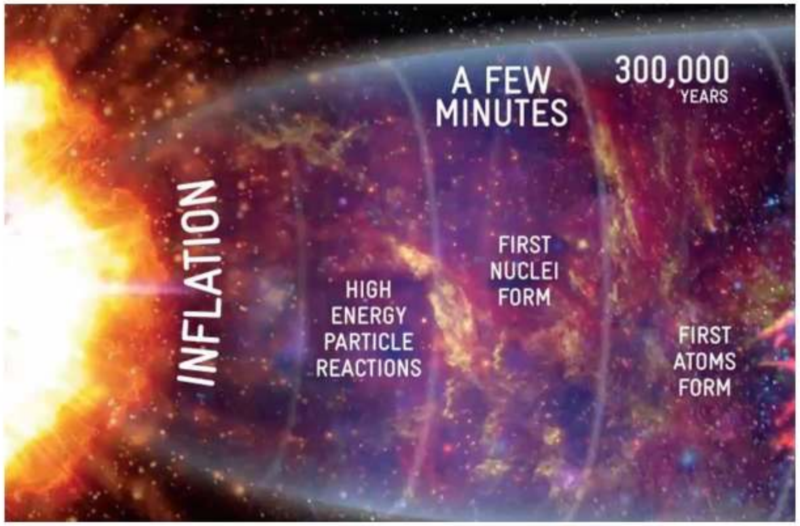
1. Solves the causality problem
2. Solves the flatness problem
3. Solves the magnetic monopole problem
4. Generates the seed of large scale structure



- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll inflation



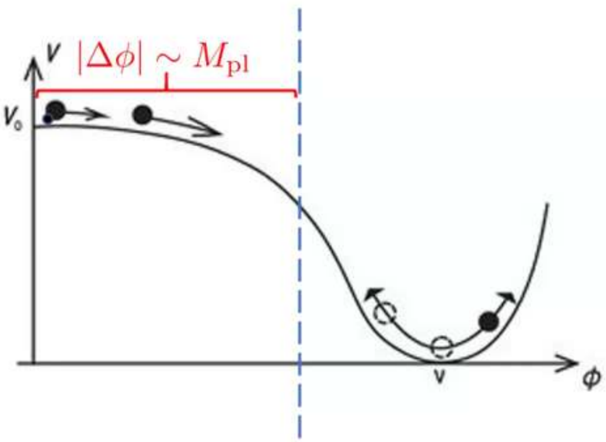


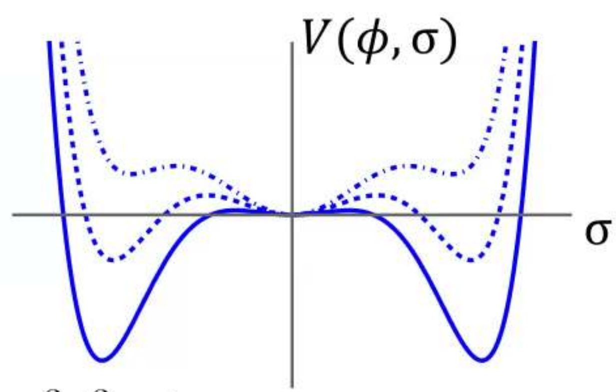
To generate enough e-folds, the excursion of the inflaton field must be very large, comparable to the M_{pl}.

Phase transitions in spectator sector triggered by the evolution of the inflaton field

- ϕ: inflaton field σ: spectator field

Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$





Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

Evolutions in the early universe

- Thermal expansion: temperature coupled to SM sector $T^2|H^2|$

T_{RH} → T_{EW} → T_{QCD} → T

- Inflation: ϕ coupled to spectator sectors $f(\phi)g(\sigma)$

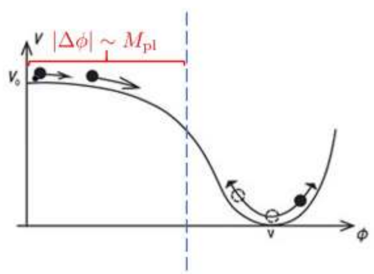
$f(\phi)$

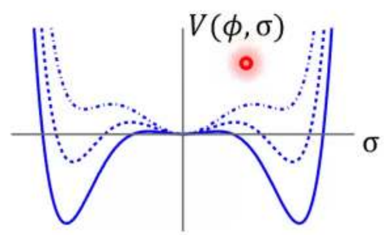
Grand Unification | Baryogenesis | SUSY Breaking | Dark sector Phase transitions | …

Consequences of the phase transitions

Evolution of inflaton → Phase transitions in spectator sector →

- Primary Gravitational waves
- Dark Matter
- Topological Defects
- Baryogenesis
- Curvature perturbation
  - → Primordial Non-Gaussianity
  - → Primordial Black Holes
  - → Large scale structure
  - → Secondary Gravitational waves





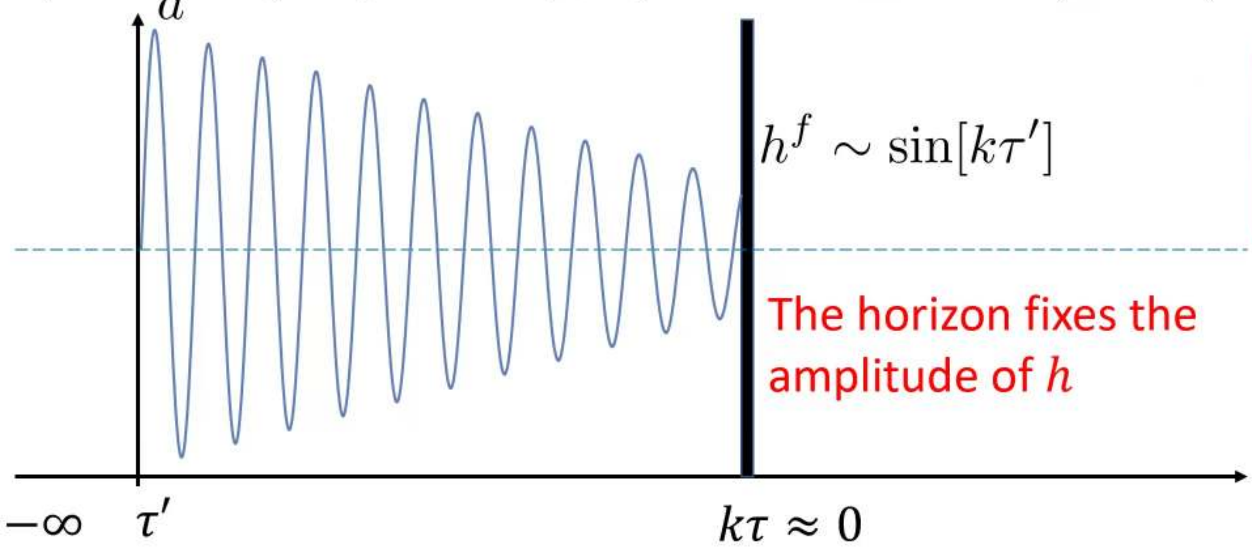
GWs from first-order phase transitions during inflation

- How to calculate GWs?
  - We linearize the Einstein equation: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$. GW is h^{TT}_{ij}.
  - We solve the Green's function first. (instantaneous and local source)
  - We convolute the Green's function with the source.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

GWs from first-order phase transitions during inflation

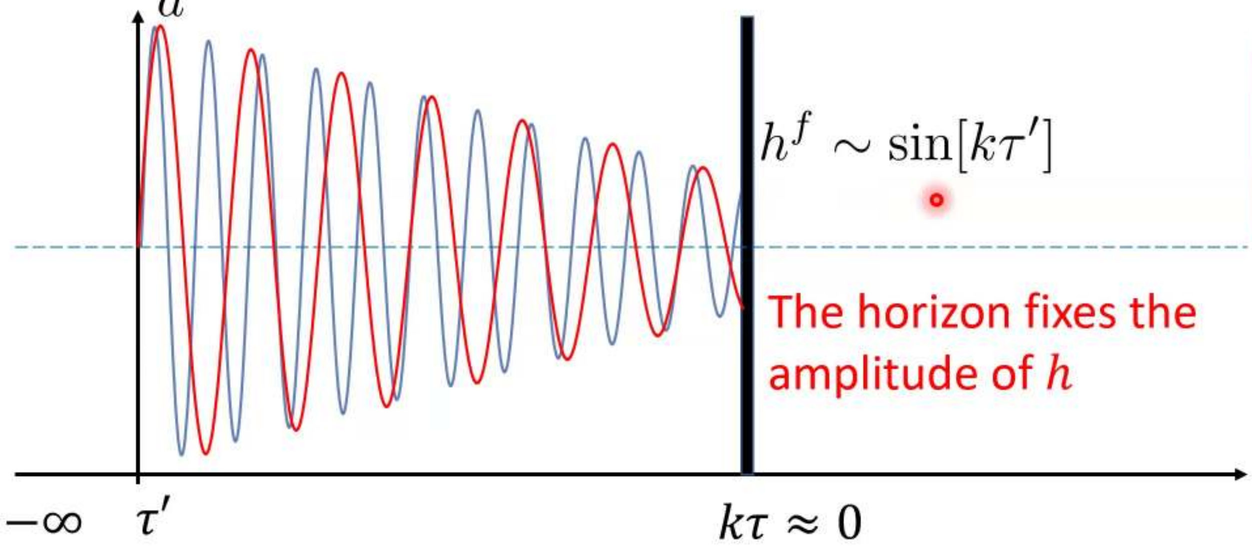
- $$h''(\tau, \mathbf{k}) + \frac{2a'}{a} h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$



$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$
GW is h^{TT}_{ij}.

GWs from first-order phase transitions during inflation

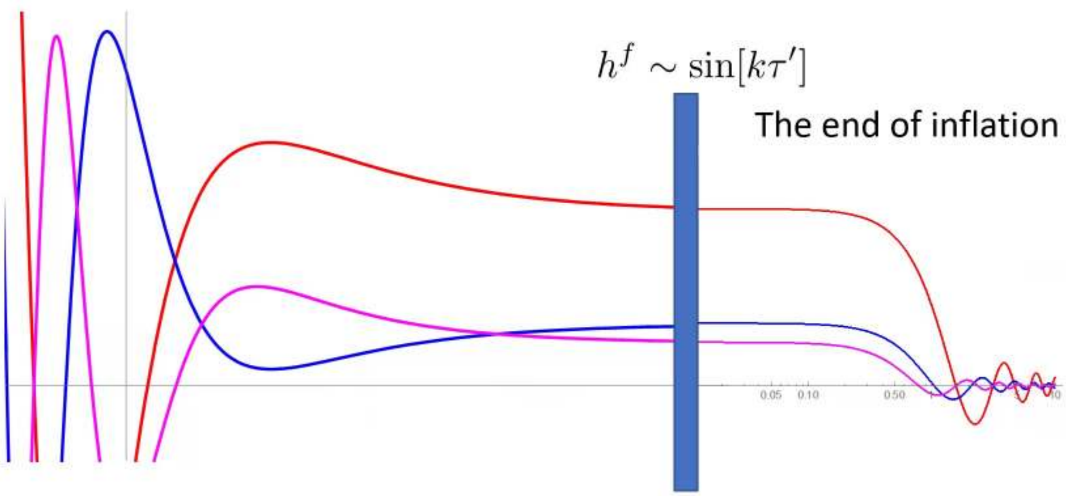
- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$

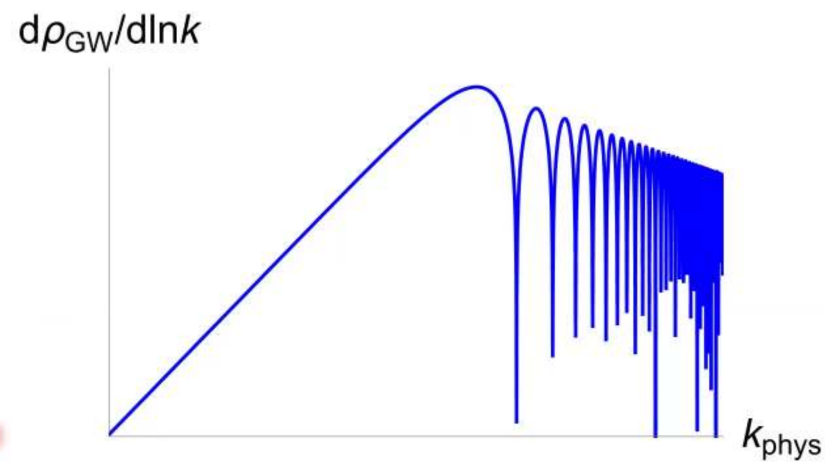


$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$
GW is h_{ij}^{TT}.

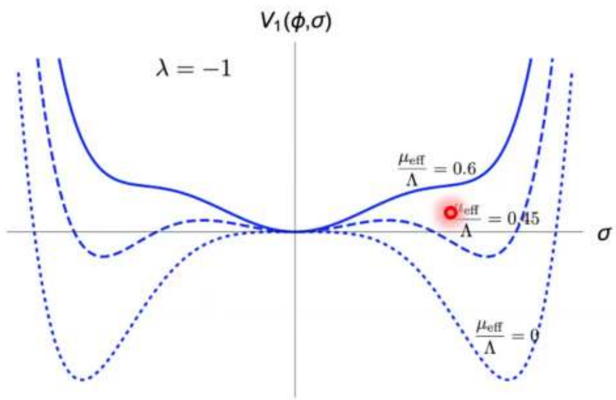
After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$.



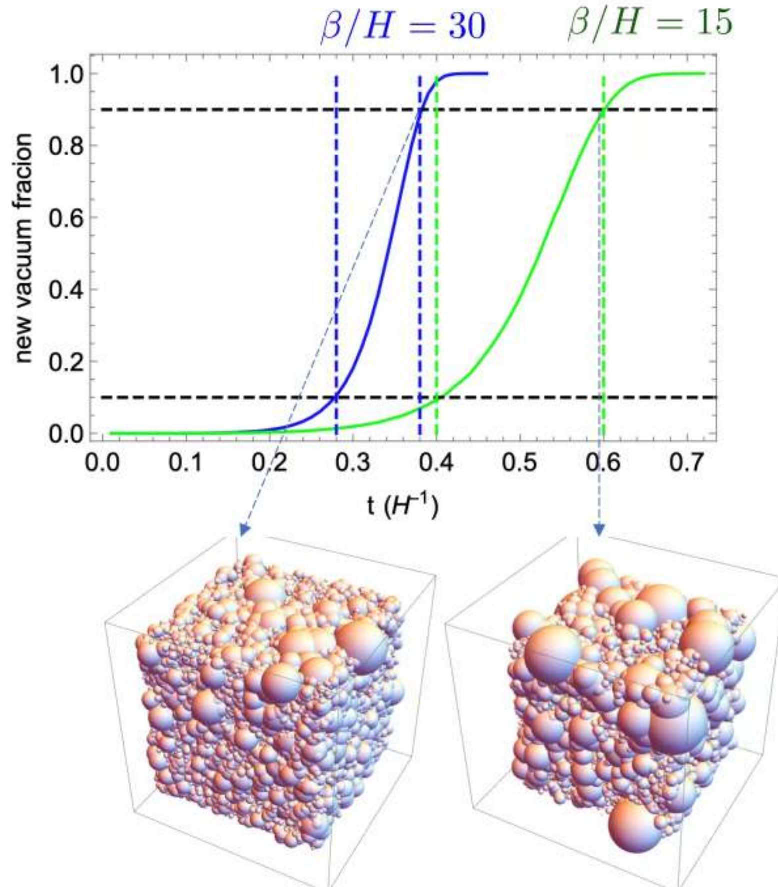


First-order phase transition during inflation

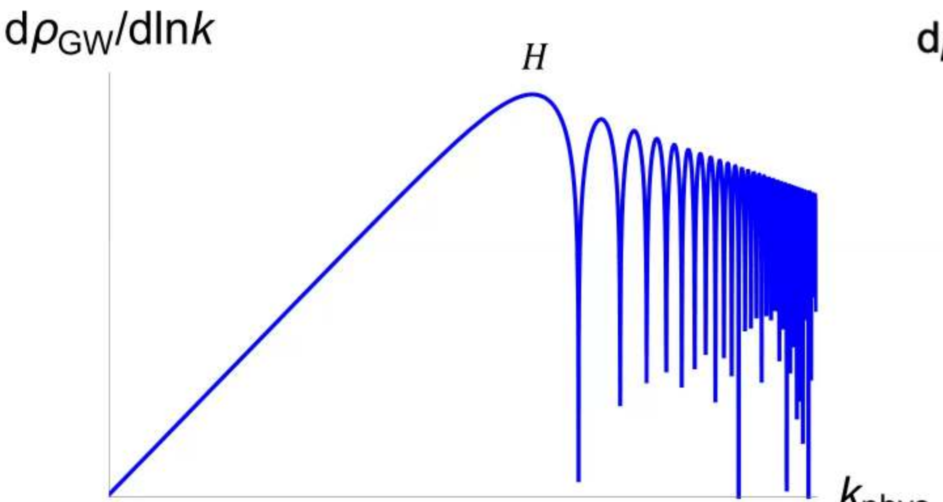


S_4 becomes smaller during

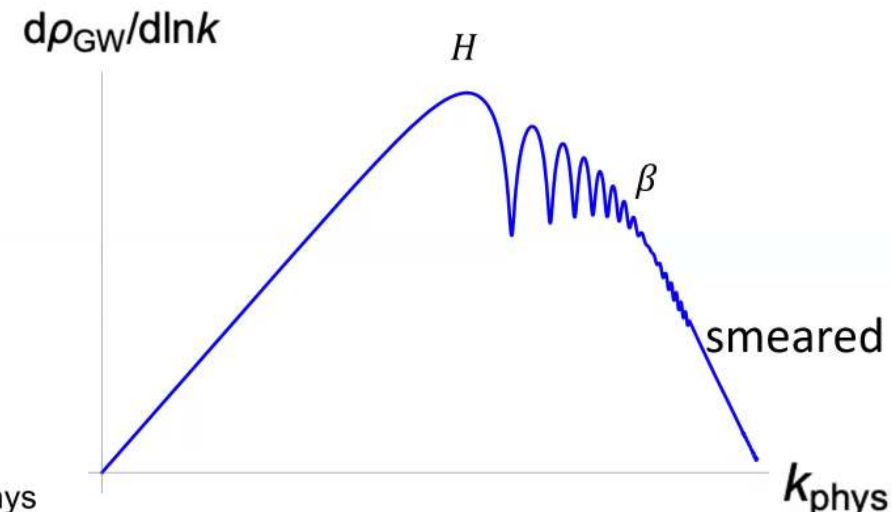
- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.



Spectrum of GW from a real source



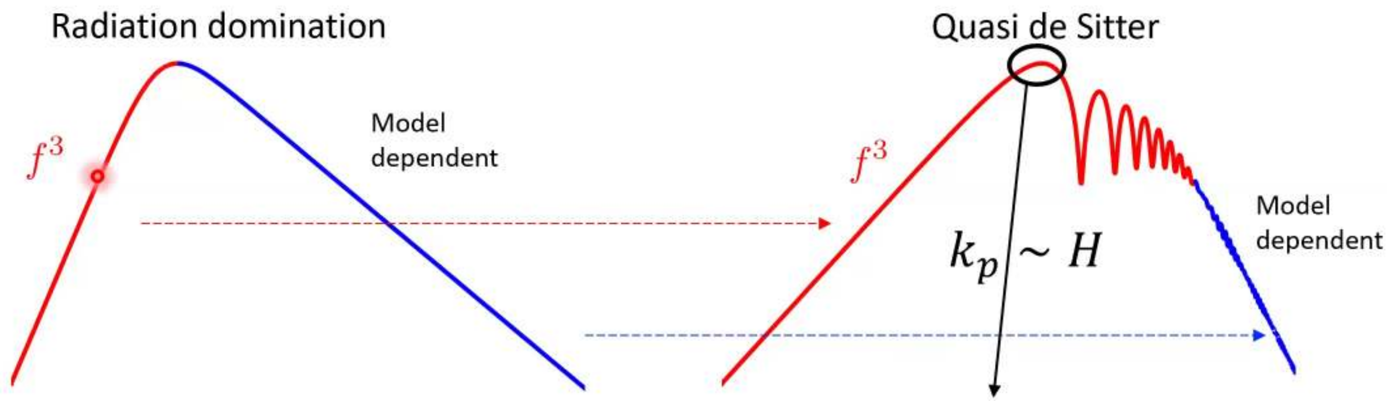
Instantaneous source



First-order phase transition

For phase transition to complete, $\beta = -\frac{dS_b}{dt} \gg H.$

Spectrum distortion by inflation



Cai, Pi, Sasaki 2018

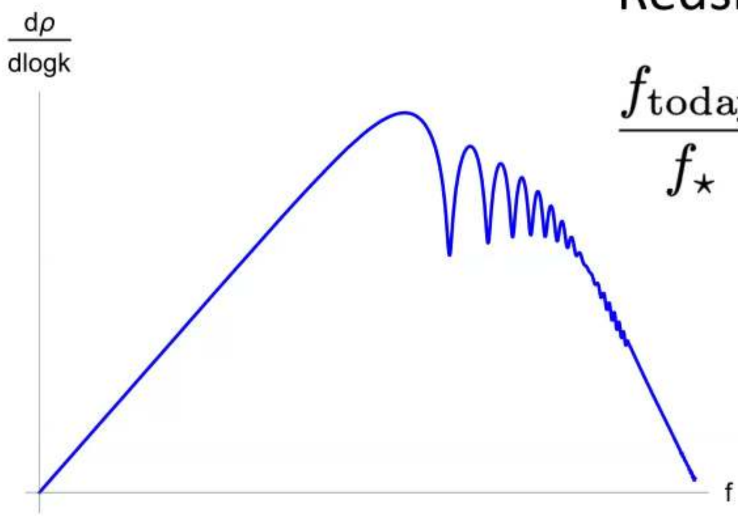
$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-12} \times \left(\frac{H_{\rm inf}}{0.1\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-17} \times \left(\frac{H_{\rm inf}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

First-order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering, and fast reheating



Redshift

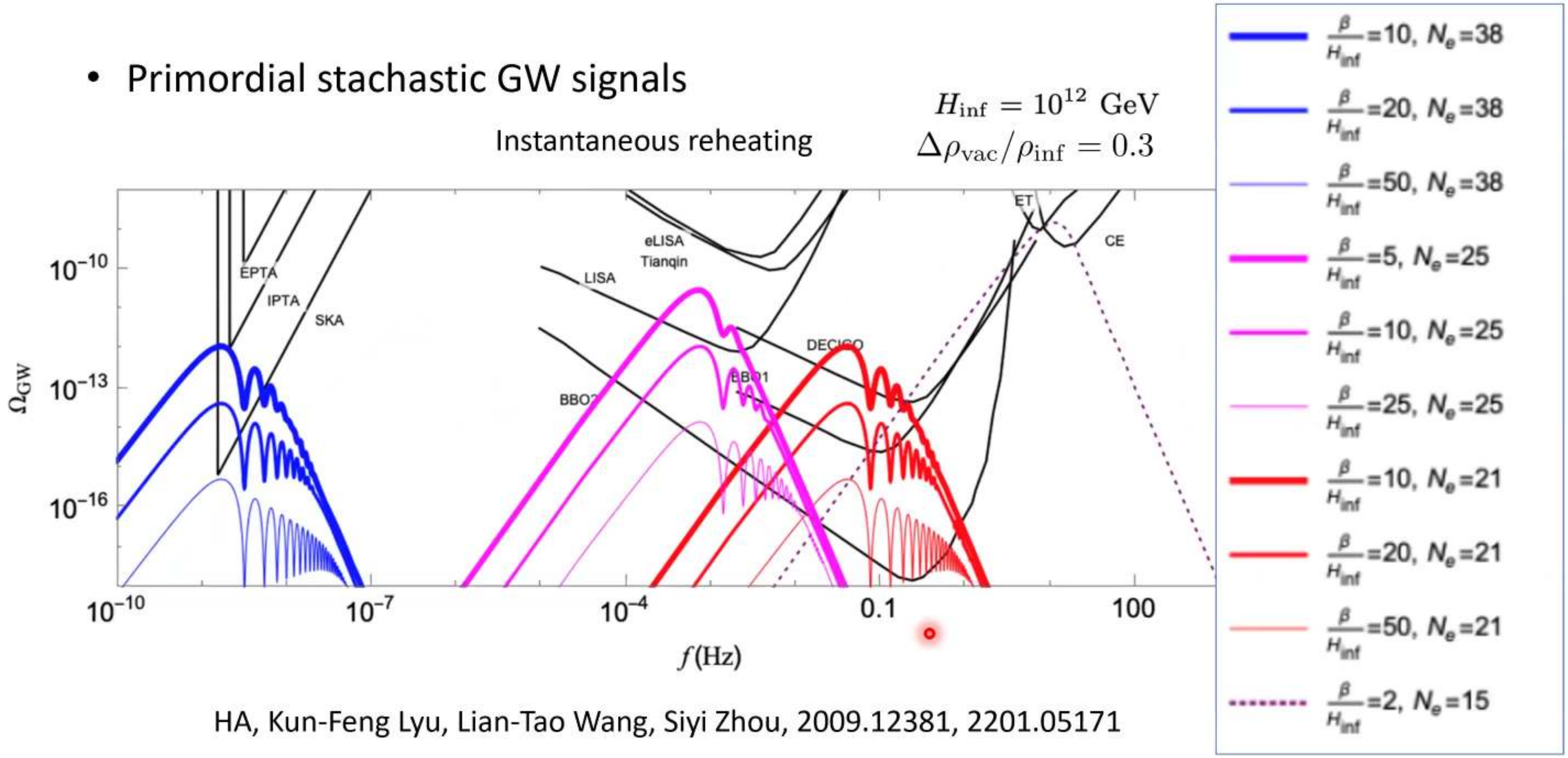
$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$\frac{a(\tau_\star)}{a_1} \downarrow e^{-N_e}$

N_e: e-folds before the end of inflation

First-order phase transition during inflation

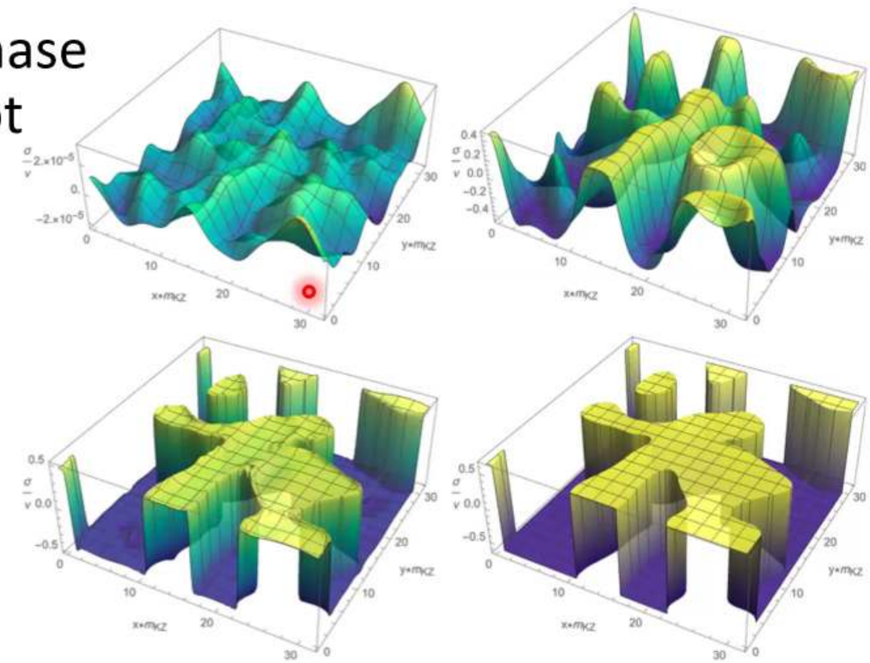
- Primordial stachastic GW signals



HA, Kun-Feng Lyu, Lian-Tao Wang, Siyi Zhou, 2009.12381, 2201.05171

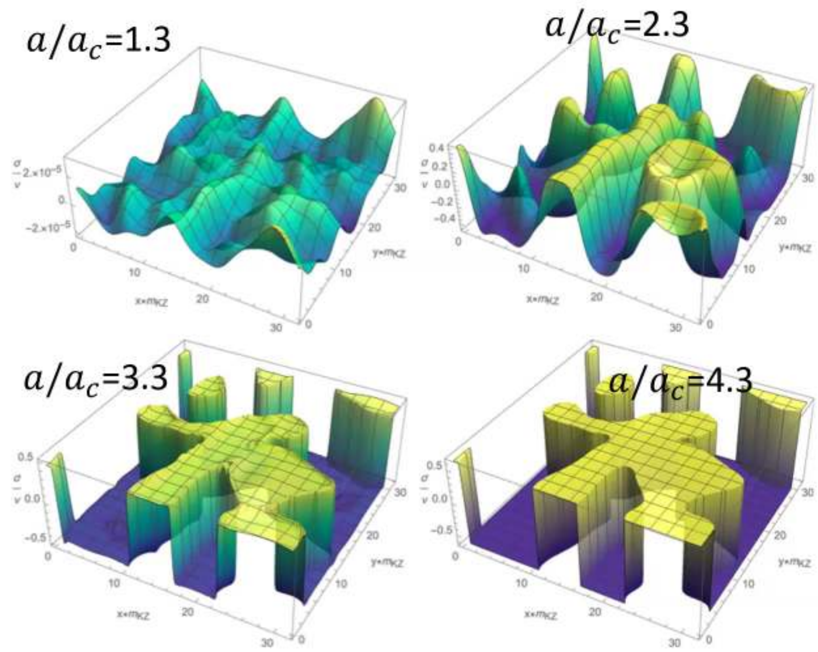
GWs from topological defects

- GWs directly from second-order phase transitions are small, usually cannot be detected.
- Phase transitions can produce topological defects:
  - Domain walls
  - Cosmic strings
  - Monopoles



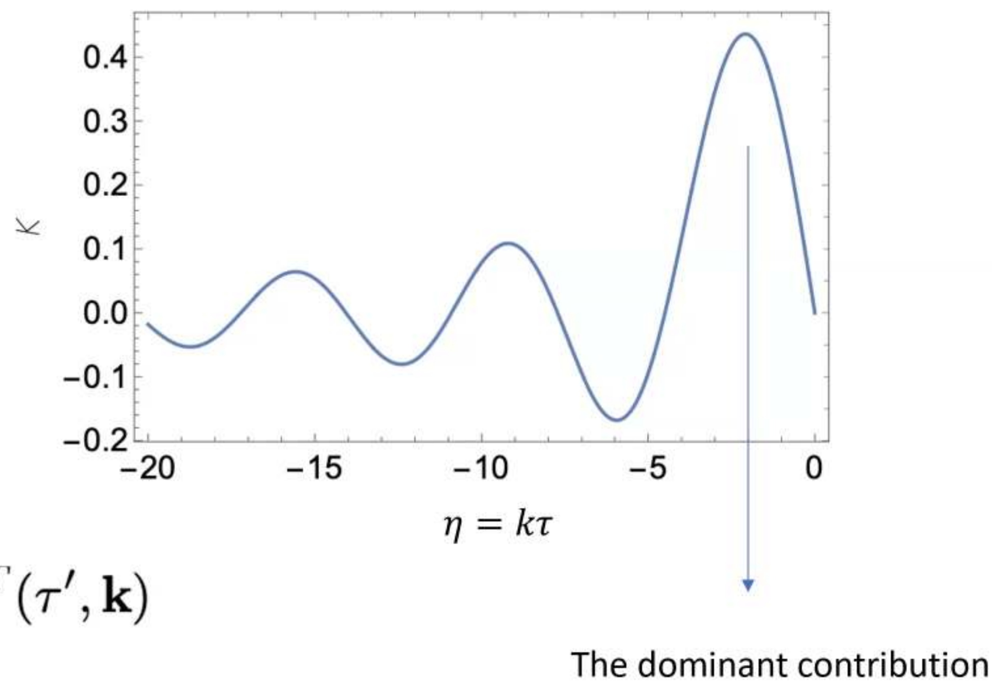
Formation of domain walls

- Symmetry breaking via a second order phase transition.
- We numerically solve the nonlinear evolution of σ field with $1000 \times 1000 \times 1000$ lattice.
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.



$$\tilde{h}^f_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k}\int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau')\tilde{T}^{TT}_{ij}(\tau',\mathbf{k})$$

Numerical results for GWs

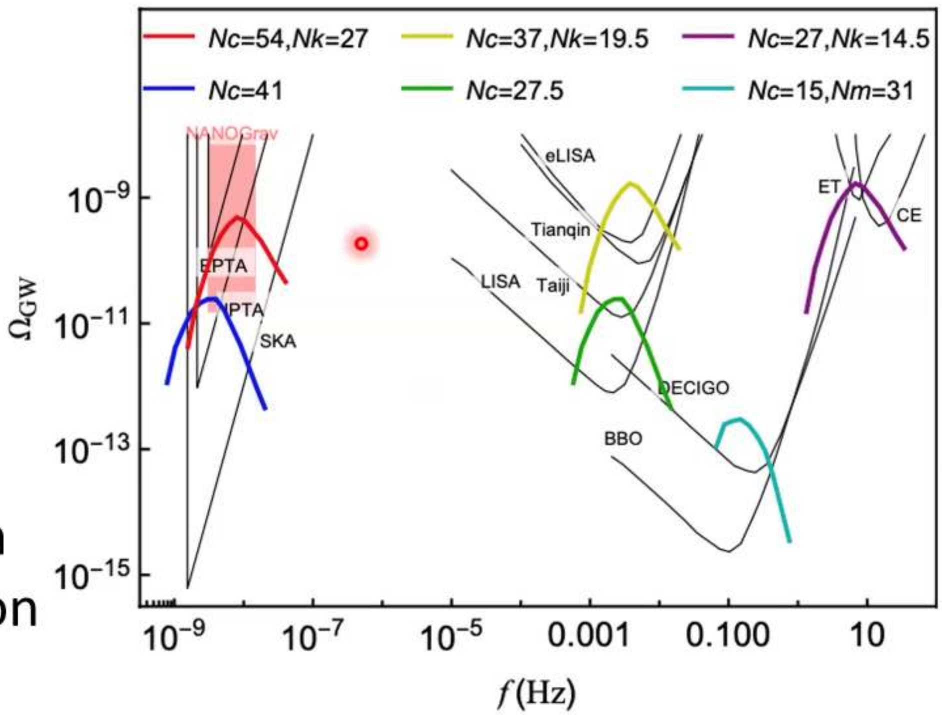
$$\Omega_{\rm GW}(f) = \Omega_R \times \rho_R^{-1}\,\frac{d\rho_{\rm GW}}{d\ln f}\bigg|_{\rm today}$$

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3}\frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

Intermediate stages matter:

- Instantaneous reheating
- Intermediate matter domination
- Intermediate kination domination

HA, Chen Yang, 2304.02361



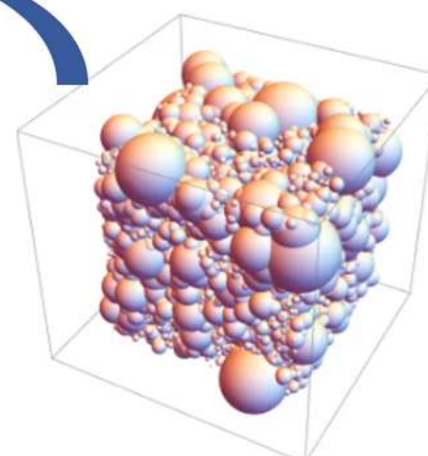
Induced curvature perturbation

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

$$V(\phi,\sigma) = V_0(\phi) + V_1(\phi,\sigma) \xrightarrow{\phi = \phi_0 + \delta\phi} \frac{\partial V_1}{\partial\phi_0}\delta\phi$$

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} \ ,$$



Induced curvature perturbation

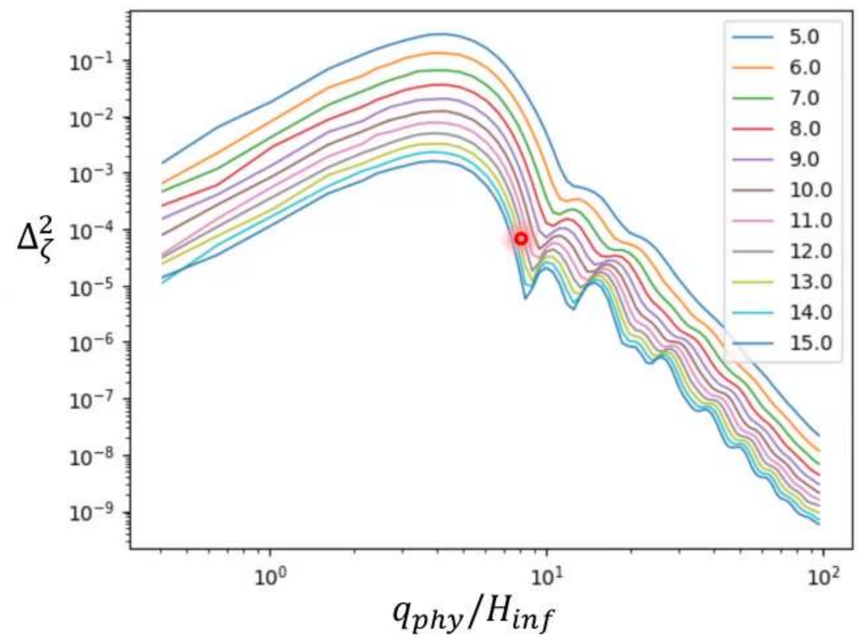
- After the collision of the bubbles, σ field oscillates and keeps producing ζ.
- The production of ζ lasts about H^{-1}, longer than β^{-1}.

$$\Delta_\zeta^{2\,(\mathrm{emp})}(q) = A_{\mathrm{ref}}\mathcal{F}\left(\frac{q_{\mathrm{phys}}}{H_{\mathrm{inf}}}\right)$$

$$\mathcal{F}(x) \ = \ \frac{x^3}{1+(\alpha_1 x)^4+(\alpha_2 x)^9}$$

$$A_{\mathrm{ref}} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\mathrm{pl}}}{\phi_0}\right)^2\left(\frac{H_{\mathrm{inf}}}{\beta}\right)^3\left(\frac{\Delta\rho}{\rho_{\mathrm{inf}}}\right)^2$$

$\mathcal{A} \approx 24$ $\qquad$ $\alpha_1 \approx 0.31, \alpha_2 \approx 0.17$



Scalar induced secondary GWs

- After inflation $\zeta \to \Phi, \Psi$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}^{\ \ lm}\mathcal{S}_{lm}\,,$$

$$\begin{aligned}\mathcal{S}_{ij} \;\equiv\; & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i\left(\Psi' + \mathcal{H}\Phi\right)\partial_j\left(\Psi' + \mathcal{H}\Phi\right) - \frac{2c_s^2}{3w\mathcal{H}^2}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)\,.\end{aligned}$$

Scalar induced GWs

Matarrese, Mollerach, and Bruni, astro-hp/9707278
Mollerach, Harari, and Matarrese, astro-hp/0310711
Ananda, Clarkson, and Wands, gr-qc/0612013
Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290
...

Scalar induced secondary GWs

$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

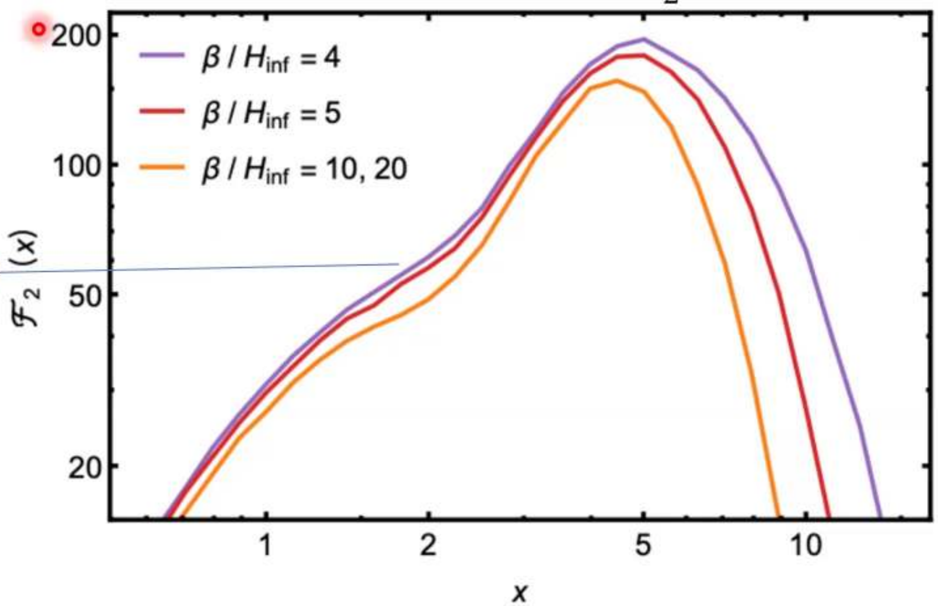
$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^2$$

$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

$$f_{\rm ref} = 10^{-9}\ {\rm Hz} \times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

$\mathcal{F}_2^{\rm max} \approx 200$

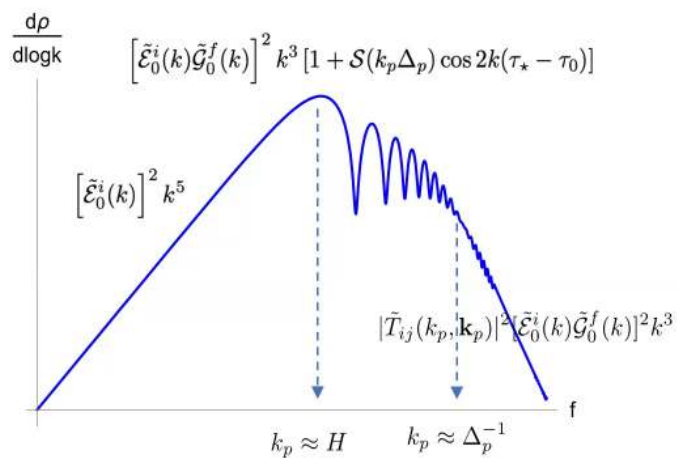


$\mathcal{F}_2$ collects information of the transfer functions.

Comparison between primary GW and secondary GW

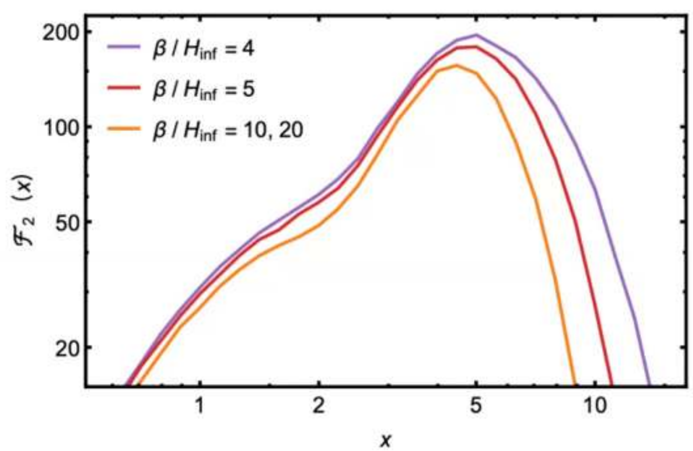
- Primary

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$



- Secondary

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^4$$

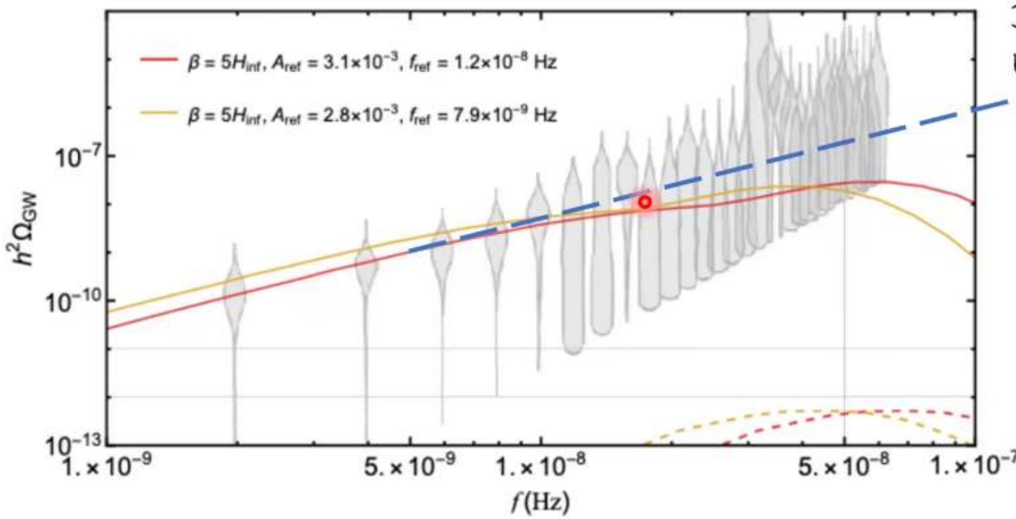


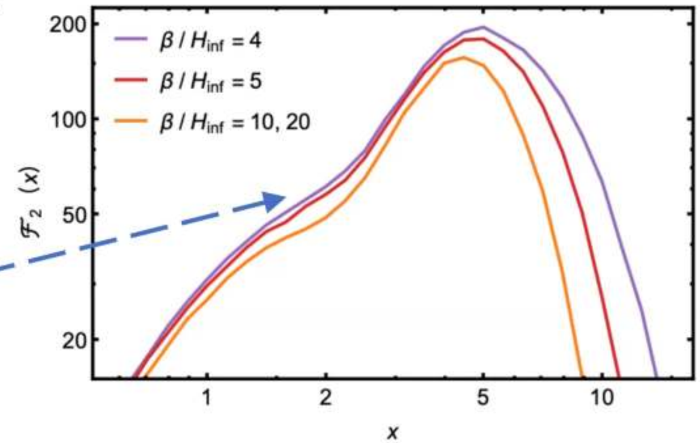
Observation from PTAs

HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

- The slope is around 2 in the IR region





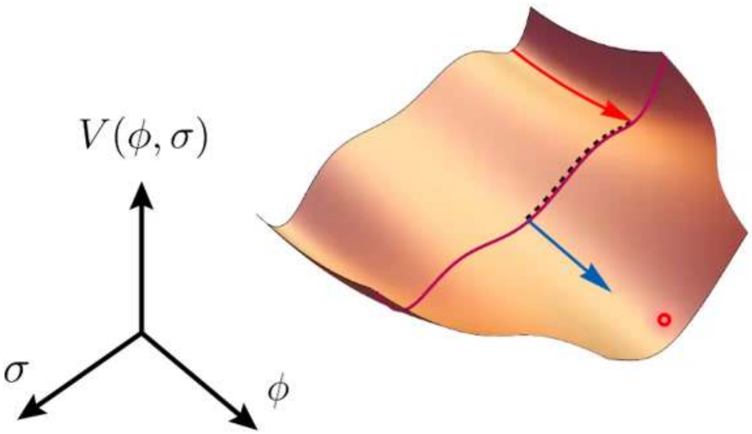
$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

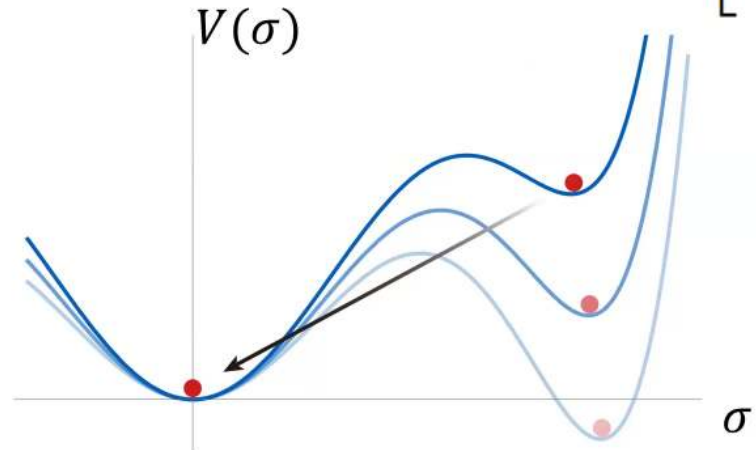
$$f = \frac{q}{2\pi a_0} = f_{\rm ref}\times\frac{q_{\rm phys}}{H_{\rm inf}}$$

$$f_{\rm ref} = 10^{-9}\ {\rm Hz}\times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

Primordial non-Gaussianity (quantum fluctuation)

- The evolution of ϕ_0 induces the evolution of σ_0. $\zeta = -H_{\rm inf}\left[\frac{\dot{\phi}_0\delta\phi + \dot{\sigma}_0\delta\sigma}{\dot{\phi}_0^2 + \dot{\sigma}_0^2}\right]$

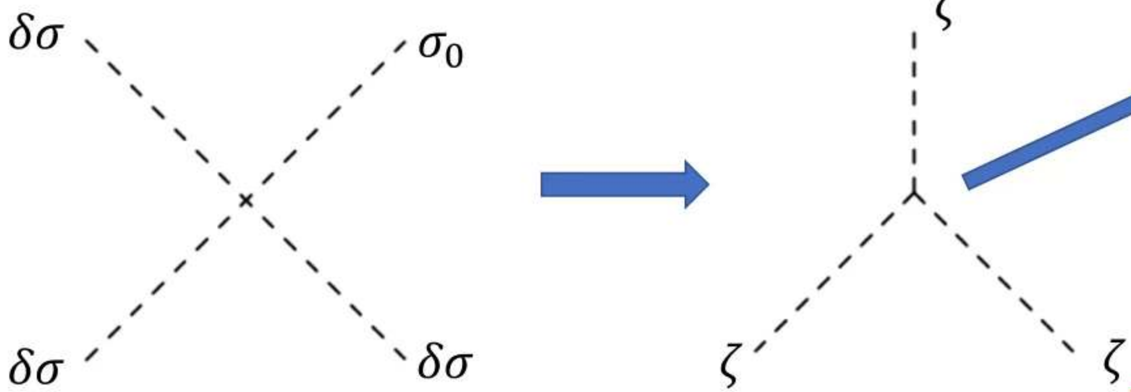




- $\delta\sigma$ also contributes to the curvature perturbation, and the interaction in the σ sector is strong.

Primordial non-Gaussianity (quantum fluctuation)

- 3pt function in the symmetry breaking phase



$$\mathcal{H}_I \sim \left[\frac{1}{6}\frac{\partial^3 V_\sigma}{\partial \sigma^3}\dot{\sigma}_0^3\right]\zeta^3$$

$$\dot{\sigma}_0 \propto \dot{\phi}_0$$

$$\mathcal{H}_I \sim \kappa^3 \Delta\rho\epsilon^{3/2}$$

$$\kappa = \frac{M_{\rm pl}}{\phi_0}$$

Enhancement: $\kappa^3 \frac{\Delta\rho}{\rho_{\rm inf}}\epsilon^{-1/2}$

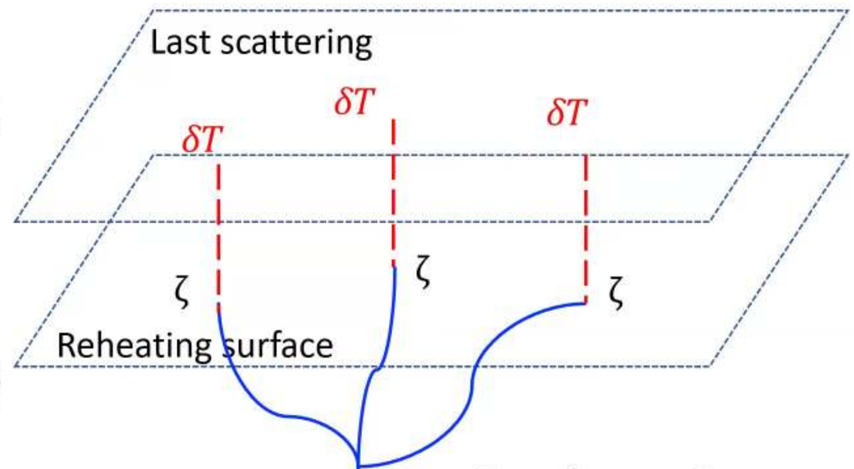
compared to single field inflation.

Primordial non-Gaussianity

- Calculate the three-point function using the in-in formalism.

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t)\right] \cdots \right]\right]\right\rangle$$

S. Weinberg, hep-th/0506236



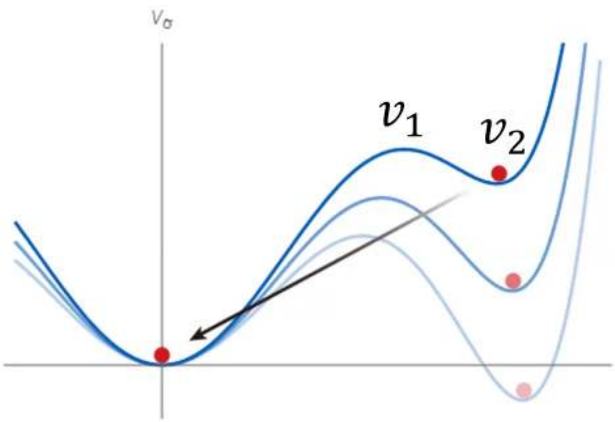
$$\mathcal{H}_I \sim \left[\frac{1}{6}\frac{\partial^3 V_\sigma}{\partial \sigma^3}\dot{\sigma}_0^3\right]\zeta^3$$

- Relevant operator, IR dominant. $\int d\tau \sim N_e \sim \epsilon^{-1/2}$
- $f_{NL} \sim O(1)$

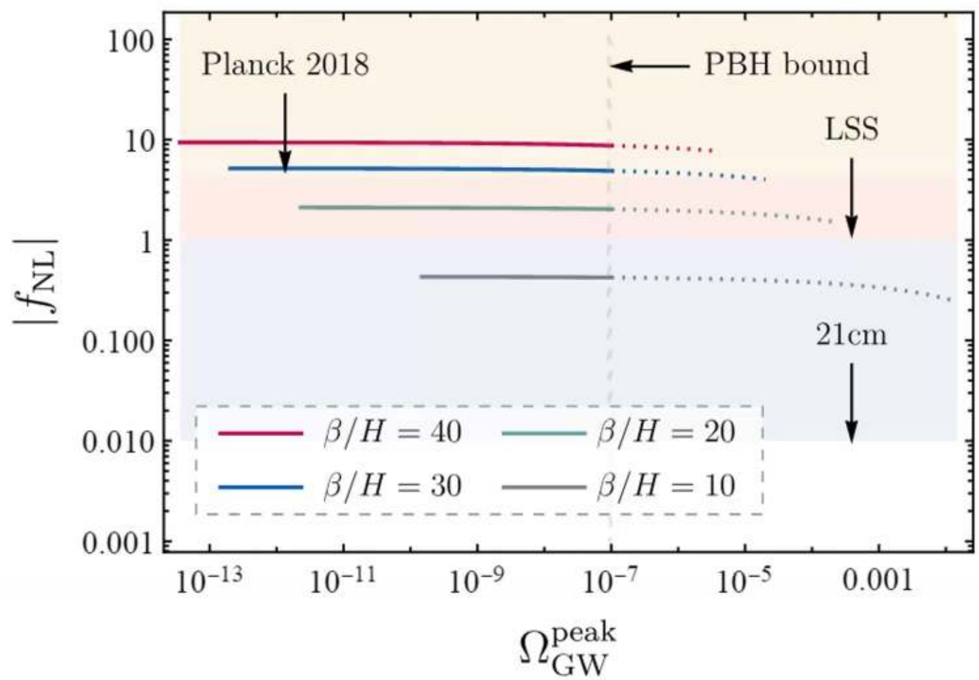
Primordial non-Gaussianity

$$f_{\rm NL} = \left(\frac{\beta}{H_{\rm inf}}\right)^3 \left(\frac{\Delta\rho}{\rho_{\rm inf}}\right) \left(\frac{\Delta_\star}{\mathcal{S}_E}\right)^3 \mathcal{F}(\Delta_\star)$$

$$\Omega_{\rm GW} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^4$$



$$\frac{v_2}{v_1} = 1 + \Delta$$



HA, Qi Chen, Yuhang Li, Yuan Yin, 2411.12699

Producing superheavy DM

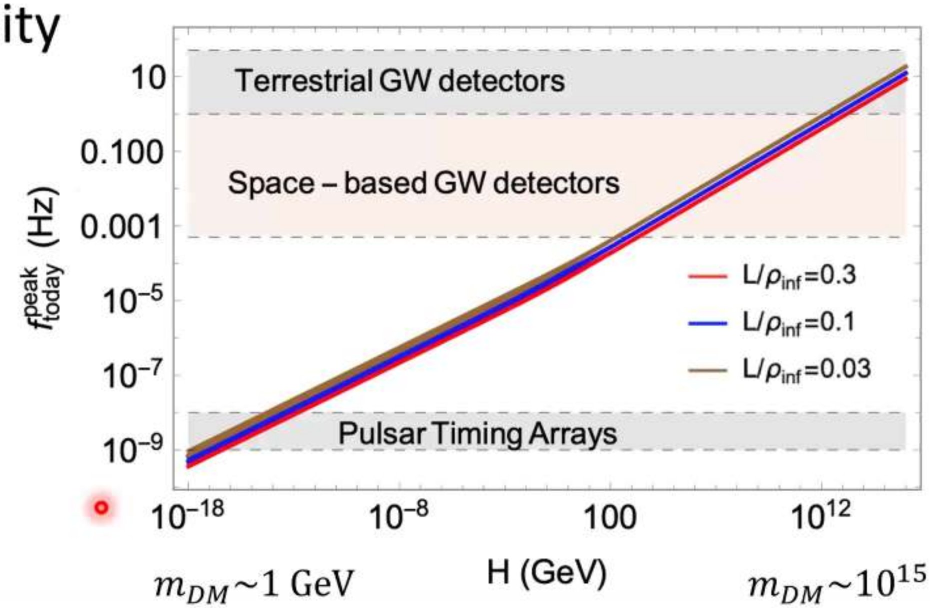
- Today's dark matter energy density

$$\rho_{DM}^{(0)} \approx \Delta\rho_{\rm vac} e^{-3(N_\star - N_{\rm aftar})}$$

$$\Omega_{\rm DM} \sim \frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}} \times \eta^{-1} \times e^{-3N_\star}$$

$$\eta = \frac{n_B^{(0)}}{n_\gamma} \approx 10^{-9}$$

$$f_{\rm today}^{\rm peak} \sim \frac{1}{2\pi}\left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^{-1/3} (H_{\rm inf} H_0^2)^{1/3}$$

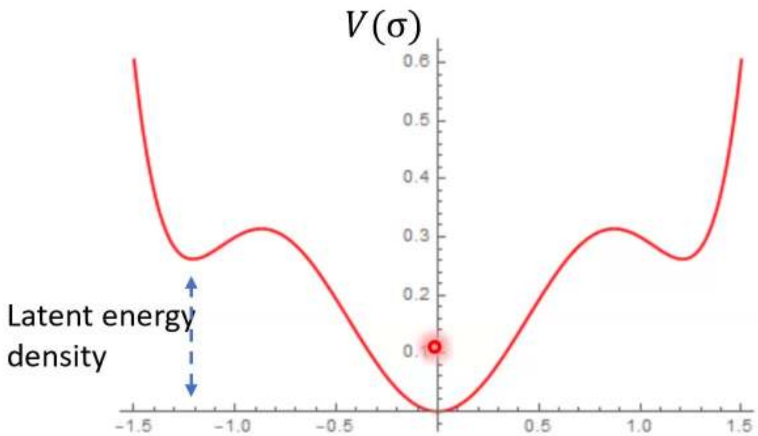


m_{DM}~1 GeV $\quad$ m_{DM}~10^{15} GeV

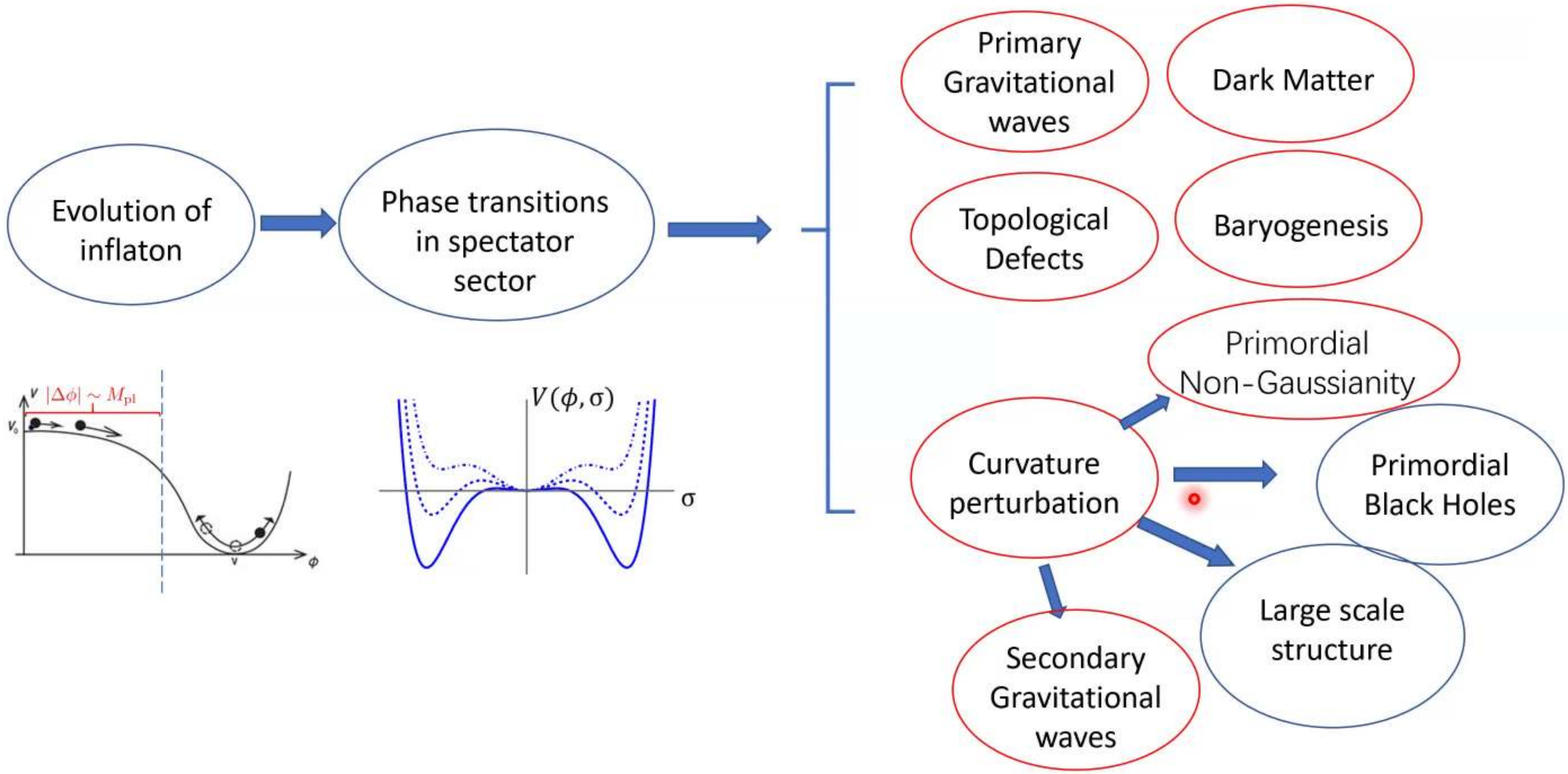
HA, Xi Tong, Siyi Zhou, 2208.14857

Producing superheavy DM

- Where does the latent energy go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is ***symmetry-restoration***, σ particles can be DM.



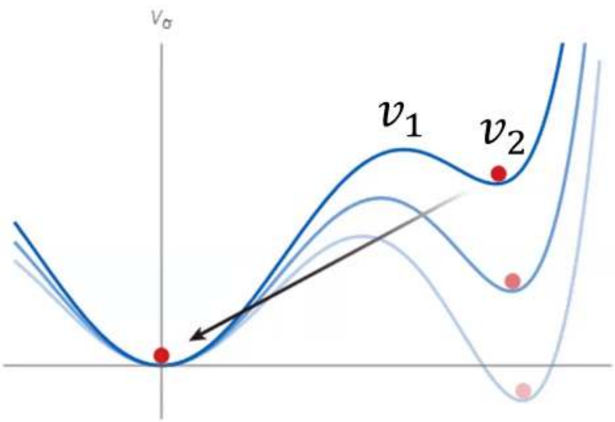
Summary


Evolution of inflaton
Phase transitions in spectator sector
$|\Delta\phi| \sim M_{\rm pl}$
$V(\phi,\sigma)$
σ
Primary Gravitational waves
Dark Matter
Topological Defects
Baryogenesis
Primordial Non-Gaussianity
Curvature perturbation
Primordial Black Holes
Large scale structure
Secondary Gravitational waves

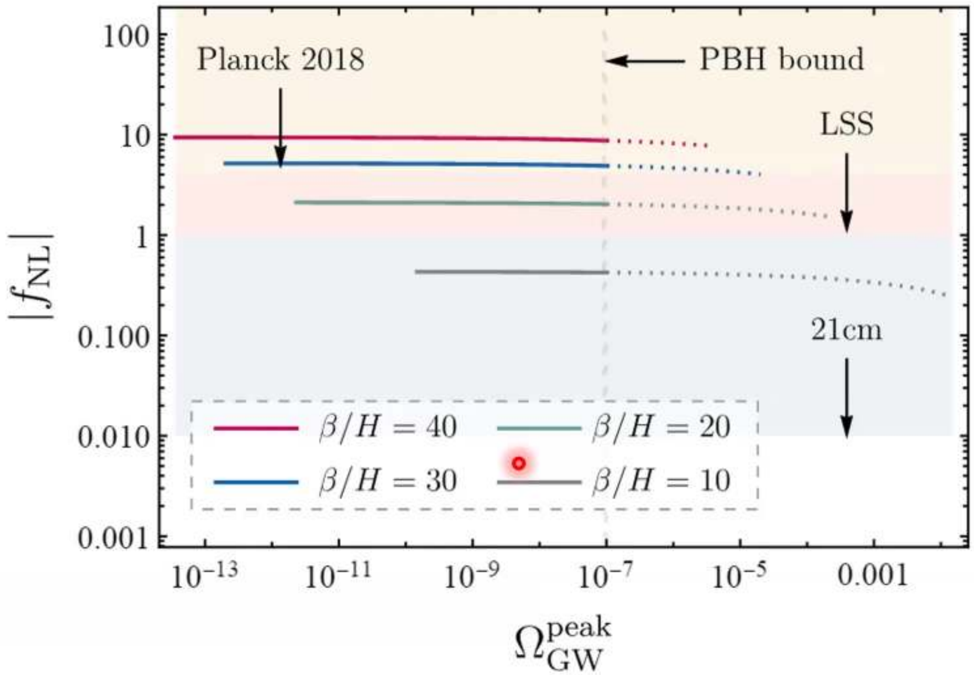
Primordial non-Gaussianity

$$f_{\rm NL} = \left(\frac{\beta}{H_{\rm inf}}\right)^3 \left(\frac{\Delta\rho}{\rho_{\rm inf}}\right) \left(\frac{\Delta_\star}{\mathcal{S}_E}\right)^3 \mathcal{F}(\Delta_\star)$$

$$\Omega_{\rm GW} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^4$$



$$\frac{v_2}{v_1} = 1 + \Delta$$



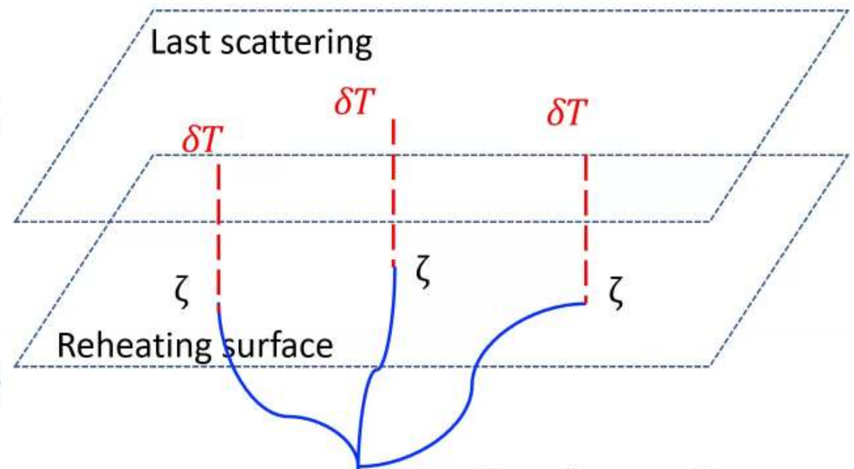
HA, Qi Chen, Yuhang Li, Yuan Yin, 2411.12699

Primordial non-Gaussianity

- Calculate the three-point function using the in-in formalism.

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} dt_N \int_{-\infty}^{t_N} dt_{N-1} \cdots \int_{-\infty}^{t_2} dt_1 \times \left\langle \left[H_I(t_1), \left[H_I(t_2), \cdots \left[H_I(t_N), Q^I(t)\right] \cdots \right]\right]\right\rangle$$

S. Weinberg, hep-th/0506236



$$\mathcal{H}_I \sim \left[\frac{1}{6}\frac{\partial^3 V_\sigma}{\partial \sigma^3}\dot{\sigma}_0^3\right]\zeta^3$$

- Relevant operator, IR dominant. $\int d\tau \sim N_e \sim \epsilon^{-1/2}$
- $f_{NL} \sim O(1)$