Title: Quantum cosmology, gravitational waves and fast radio bursts: common insights from Picard-Lefshetz theory and Resurgence

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Quantum cosmology, gravitational waves and fast radio bursts: common insights from Picard-Lefshetz theory and Resurgence

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Picard-Lefshetz/Resurgence

- real time Feynmann Path Integral: Quantum Cosmology++ (including yesterday's 2508.17578!)
- wave optics: diffraction vs refraction
- essential for GW and FRBs
- opens new precision probes of space-time, e.g. Hubble, DE
- complex saddles, thimbles

Step

II. THE SMOOTH STEP

The Feynman path integral of a non-relativistic particle of mass m in the potential V expresses the propagator in terms of a sum over histories

$$G(x_1, x_0; T) = \int_{x(0)=x_0}^{x(T)=x_1} e^{iS[x]/\hbar} \mathscr{D}x \,, \qquad (1)$$

interpolating between the initial x_0 and final position x_1, with the action

$$S[x] = \int_0^T \left(\frac{1}{2} m \dot{x}^2 - V(x) \right) \mathrm{d}t \,, \qquad (2)$$

$$V_{WS}(x) = \frac{V_0}{1 + e^{-2\alpha x}} \,, \qquad (8)$$

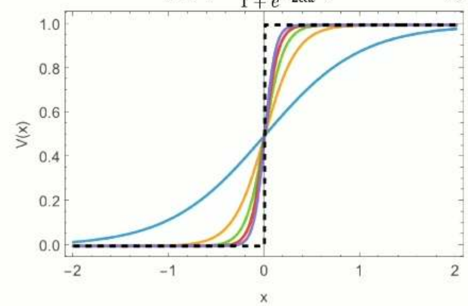

FIG. The smooth step potential with the strength $V_0 = 1$ and the steepness $\alpha = 1, 3, \ldots, 9$ (blue, yellow, green, red, and purple). The corresponding Heaviside step is plotted by the dashed black curve.

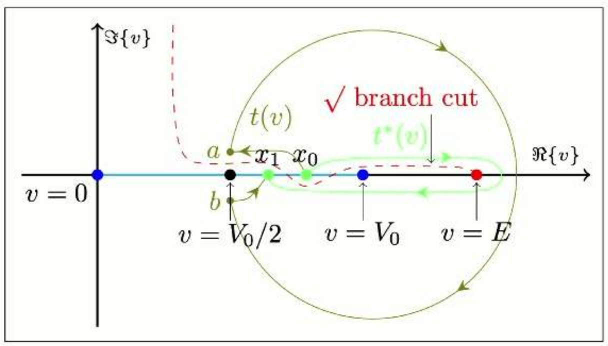

FIG. 13: Saddles for a reflection of a particle from the right. A direct contour choice of tracing the real line to the turning point and back corresponds to a counterintuitive notion of a particle running to infinity and back, instead of reflecting at the step. Instead, the red saddle illustrates a contour from the same equivalence class, *i.e.*, the same action, that moves consistently with intuition, first to the step, and then looping

Feldbrugge+ULP now!

Huygen's Principle: Path Integral

- Kirchoff integral $A(\mu) = \int e^{iS(\theta,\mu)} d\theta$
- $S = \nu[(\theta - \mu)^2/2 + \Psi(\theta)]$
- Highly oscillatory integral, even for $\Psi = 0$
- Stationary phase points: $\partial_\theta S = 0$ leads to (complex) Eikonal images θ_i where $\Psi'(\theta_j) = \mu - \theta_j$.
- flux/phase $1 + \Psi'' = 1/\mu$ through curvature expansion (known as *steepest descent*): exact as $\nu \to \infty$
- Geometric limit considers only *Real* solutions θ_i and gives up phase information (length of trajectory)
- Geometric optics applicable at short wavelengths for extended sources (e.g. optical gravitational lensing of finite size sources, stars)
- roughly 1/4 of the degrees of freedom of Eikonal

Resurgence

- at Saddle, second derivatives determine eikonal/geometric optics
- for finite wavelength, higher order derivatives contribute to wave form
- power series at saddle usually divergent: asymptotic series
- Borel Resummation, Hyperasymptotics: reveal non-perturbative effects, adjacent saddles!
- analogy: by examining deviations of LIGO template to measurement, predict if/when lensed image will arrive!

Diffractive/Evanescent Gravitational Wave Lensing

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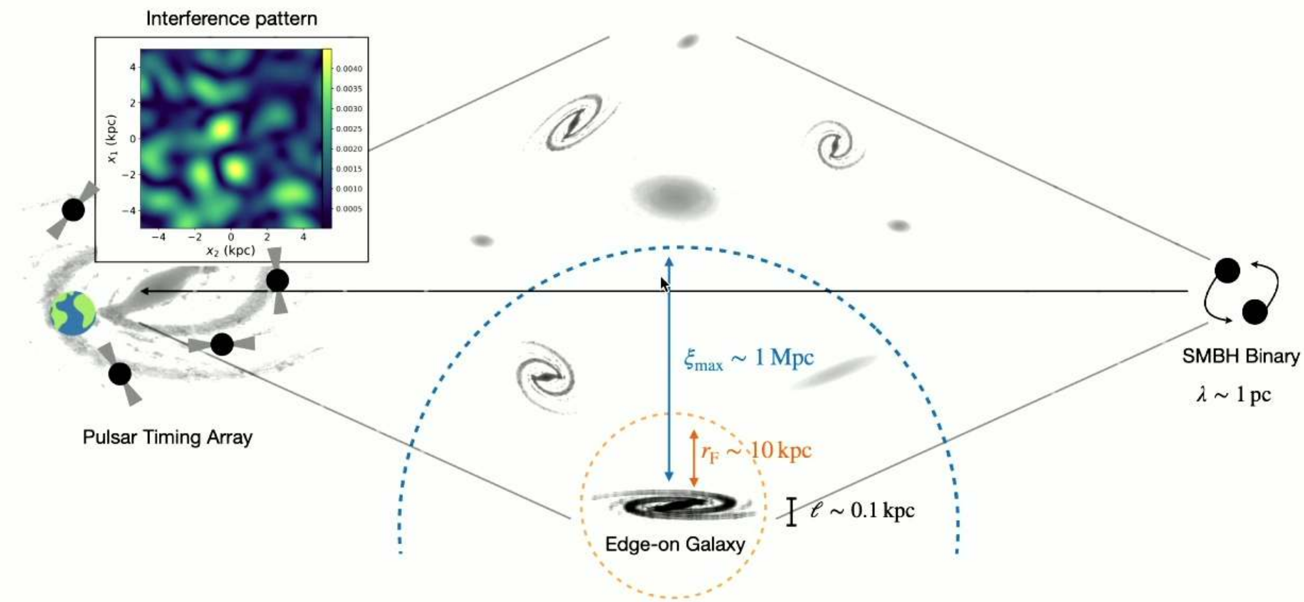

FIG. 1. Diffractive lensing of a nanohertz gravitational wave by galactic disks. In geometric optics, the Einstein radius sets the optical depth of a lens, which for a typical galactic disk is only $\sim 10\,\mathrm{kpc}$. In the diffractive limit, the maximum bending angle is $\alpha \sim \lambda/\ell$, leading to a maximum impact parameter $\xi_{\max} \sim r_F^2/\ell \sim 1\,\mathrm{Mpc}$ for edge-on galaxies. Hundreds of edge-on galaxies may be within 1 Mpc of any given line of sight. The colour-map superimposed over the Milky Way shows the interference pattern imprinted onto the strain of the gravitational wave.

Jow+ULP, PRL 134, (2025)

Microlensing: applications

- direct distance measurement by stellar microlensing: H_0.
- DM, stellar mass function, planets
- +plasma lensing: direct distance
- repeater macrolensing: direct distance
- contamination by plasma in lens galaxy?

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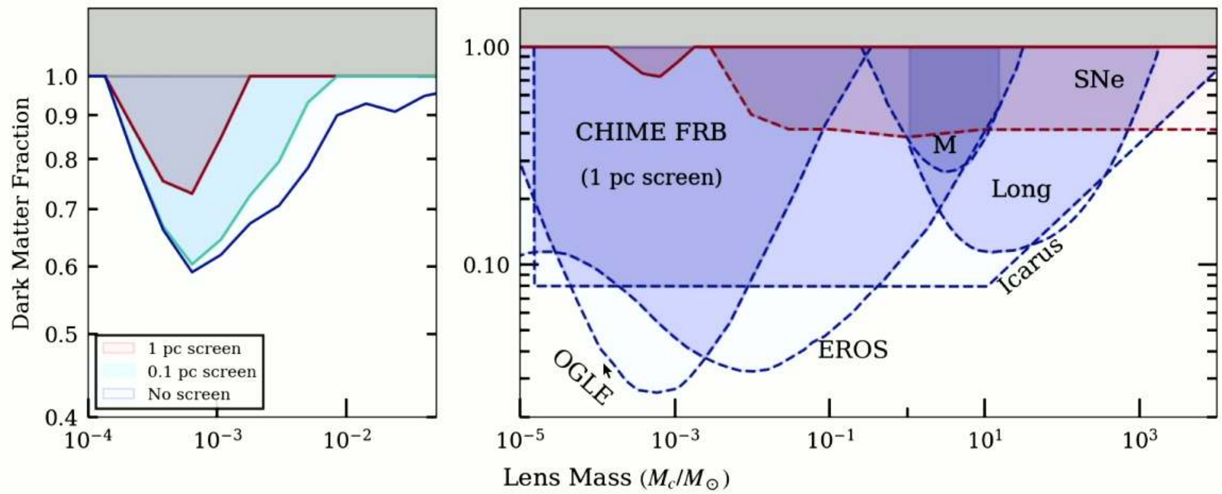

FIG. 5. Left: 95% constraints on PBHs as a function of scattering screen distance corresponding to the optical depth calculated in Fig. 3. We plot our fiducial (1 pc screen) model in red and suppress curves for screen distances of 10 and 100 pc because $\lambda < 3$ under those assumptions. Right: a collection of microlensing constraints on the fraction of dark matter composed of compact objects (such as PBHs), $f(M_c)$, assuming a monochromatic mass function peaked around M_c. We have shown Local Group PBH constraints in blue (M: MACHO [108], EROS [109], OGLE [30], Long: long-duration optical microlensing [110], Icarus [111]), and Local Universe constraints in red (SNe [11], CHIME/FRB, this work). CHIME/FRB lensing constraints depend on our two-screen scattering model, in which we have assumed that the average FRB is scattered by a screen at an effective distance of $1\,pc$, and our model for how DM correlates with distance. In these constraints, we have used Eq. (22) to define the exclusion limit as a function of M_c. Wave optics effects suppress our signal at $M \lesssim 1.5 \times 10^{-4}\ M_\odot$ and finite source size suppresses our signal at $M \gtrsim 3 \times 10^4\ M_\odot$. This shows that coherent FRB lensing has the potential to search new parameter space for exotic compact objects such as PBHs.

Leung++22 (CHIME)

Conclusions

- wave optics changes nature of astrophysical observables: Coherent FRB/pulsar/GW radiation one of the potentially most precise measurements in physics
- already makes pico-arcsecond images of pulsars
- Picard-Lefschetz theory provides alternative interpretation of optics, quantum mechanics: imaginary positions and trajectories
- FRB microlensing and PTA weak diffractive lensing may give new tool for Hubble Constant tension