Title: Lecture - Energy Operators in Particle Physics, QFT, and Gravity - June 2025

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Collection/Series: Energy Operators in Particle Physics, QFT, and Gravity - June 6-13, 2025

Subject: Particle Physics, Quantum Fields and Strings

Date: June 11, 2025 - 11:00 AM

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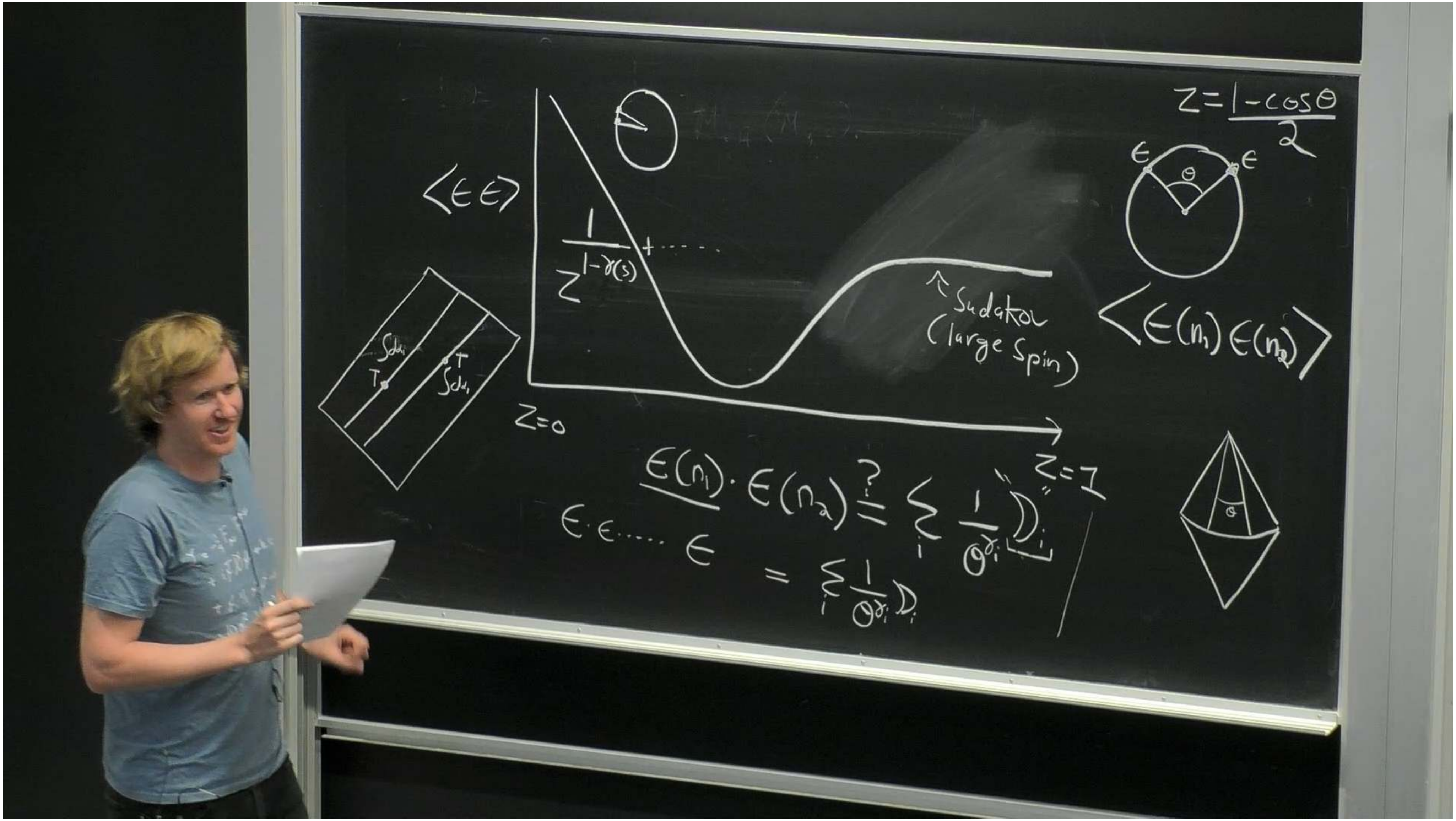
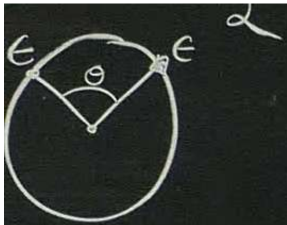
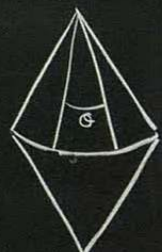


$Z = \frac{1-\cos\theta}{2}$
$\langle \mathcal{E}\mathcal{E} \rangle$
$\frac{1}{Z^{1-\gamma(s)}} + \cdots$
Sudakov (large Spin)
$Z=0$
$Z=1$
$\langle \mathcal{E}(n_1)\mathcal{E}(n_2) \rangle$
$\mathcal{E}(n_1)\cdot\mathcal{E}(n_2) \stackrel{?}{=} \sum_i \frac{1}{\theta^{\gamma_i}} \mathcal{D}_i$
$\mathcal{E}\cdot\mathcal{E}\cdots\cdots\mathcal{E} = \sum_i \frac{1}{\theta^{\gamma_i}} \mathcal{D}_i$



$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle$



$\mathcal{O} \to (\Delta, J)$

$\mathcal{O}^{\mu_1,\ldots,\mu_J}(x)$

index free: $\mathcal{O}(x,z) = \mathcal{O}^{\mu_1\ldots\mu_J} z_{\mu_1} z_{\mu_2}\cdots z_{\mu_J}$

$z^2=0$

$\mathcal{O}(x,\lambda z) = \lambda^J \mathcal{O}(x,z)$

More General Detector:

$$\mathcal{D}(n) = \lim_{x_+ \to \infty} \frac{(x_+)^{\Delta(J)-J}}{(n\cdot\bar{n})^{\Delta(J)}} \int_{-\infty}^{\infty} dx^+ \, \mathcal{O}^{\Delta(J)}_{\underbrace{+\cdots\cdots+}_{J}}(x)$$

$$\Delta_L = 1 - J$$

$n \to \ell n$ $\quad J_L = 1 - \Delta(J) \to$ celestial dimension

- collinear spin

Light transform: $(\Delta, J) \longleftrightarrow (J_L = 1-\Delta,\ \Delta_L = 1-J)$

Recall for Local OPE:

$$\mathcal{O}_{\Delta}(x)\,\mathcal{O}_{\Delta_2}(0) = \sum \frac{1}{x^{\Delta_1+\Delta_2-\Delta_3}}\,\mathcal{O}_{\Delta_3}$$

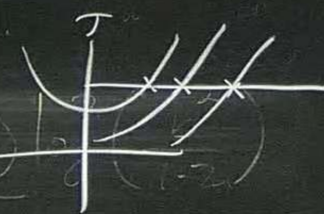
Two light ray operators:

$$\mathbb{D}^{\Delta_1, J_1}(n_1)\,\mathbb{D}^{\Delta_2, J_2}(n_2) = \sum \frac{1}{\Delta\theta^{\chi}}\,\mathbb{D}^{\Delta_3, J_3}(\theta_1)$$

$z^{\chi/2}$

Dimension (Δ_L):

$$(1-J_1)+(1-J_2) = 1-J_3$$



$$J_3 = J_1 + J_2 - 1 \Rightarrow \text{Selection rule}$$

$$\epsilon\cdot\epsilon \quad J_3 = 2+2-1 = 3 \quad \gamma(3)$$

Collinear Spin: $\Theta \to J_L = 1$

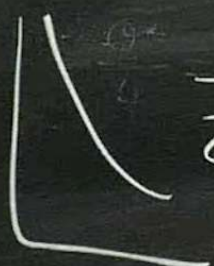
$$(1-\Delta_1)+(1-\Delta_2) = -\kappa + (1-\Delta_3)$$

Rewrite in terms of twist $\tau_i = \Delta_i - J_i$

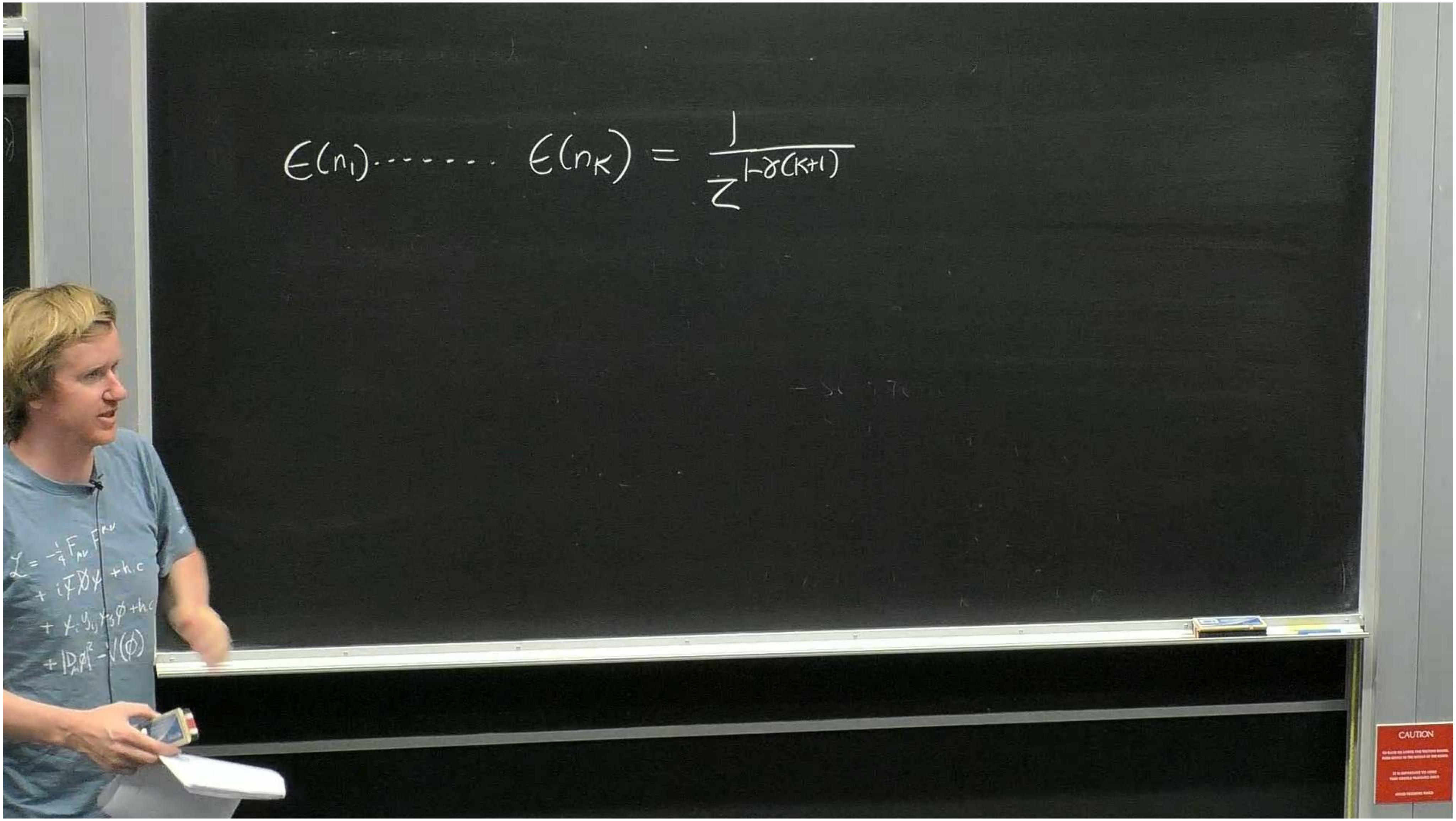
$\gamma = \tau_1 + \tau_2 - \tau_3$ $\rightarrow$ dominated by lowest twist operators appearing in OPE

$$\mathcal{E} \cdot \mathcal{E} = \sum_i \frac{1}{\theta^{4-\tau_i}} \mathbb{D}_{\tau_i}^{J=3}$$

$\rightarrow$ Need to understand leading twist.



$$\frac{1}{z^{1-\gamma(3)}} + \text{const} + \cdots$$



$\epsilon(n_1)\cdots\cdots\epsilon(n_K) = \frac{1}{z^{1-\gamma(K+1)}}$
CAUTION

TWIST-TWO operators: $\quad d=4$

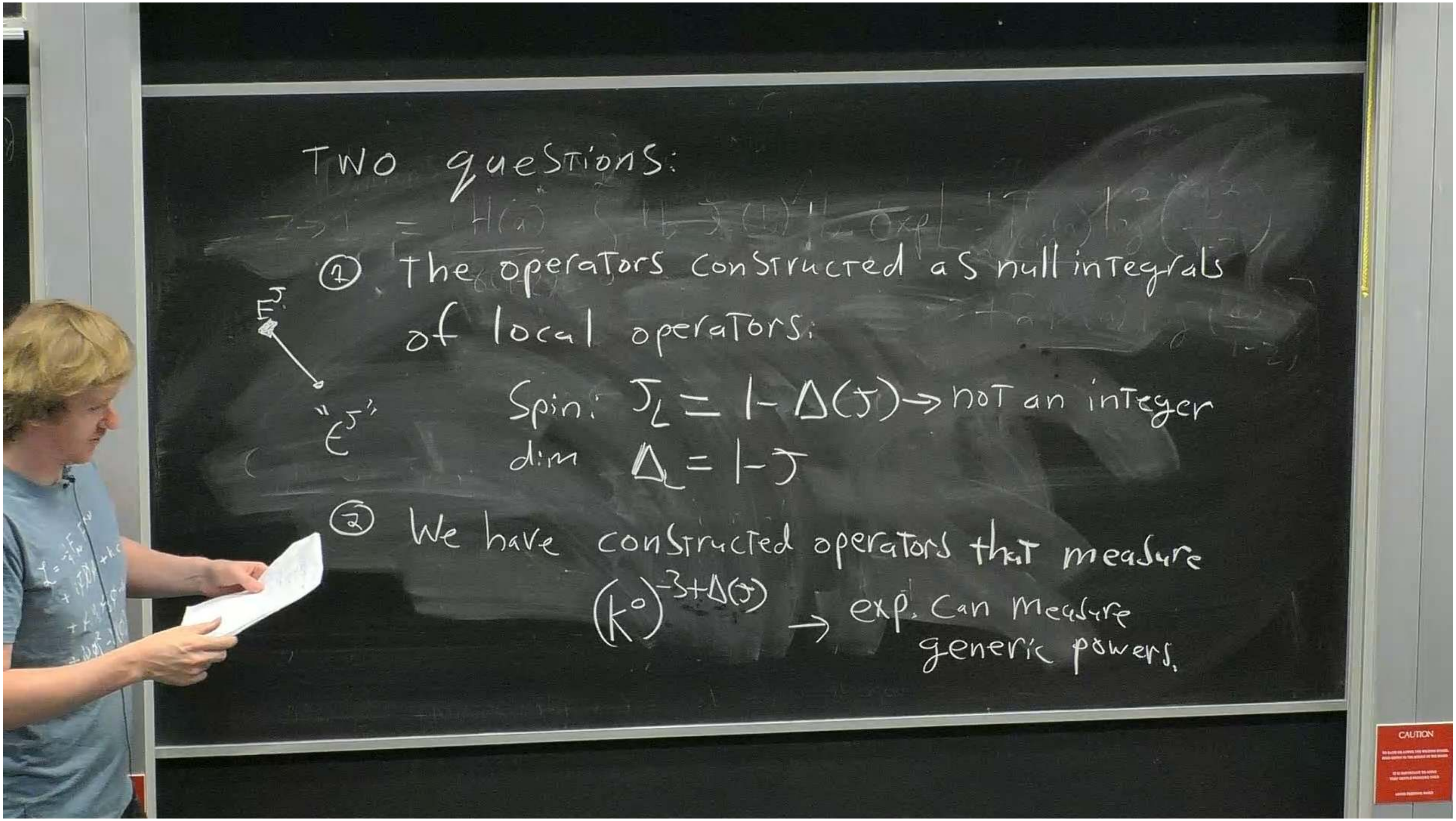
$$\mathcal{O}_\phi^{[J]} = \frac{1}{2^J}\,\phi\,(iD^+)^J\,\phi \qquad \Delta = 2+J, \quad \Delta - J = 2$$

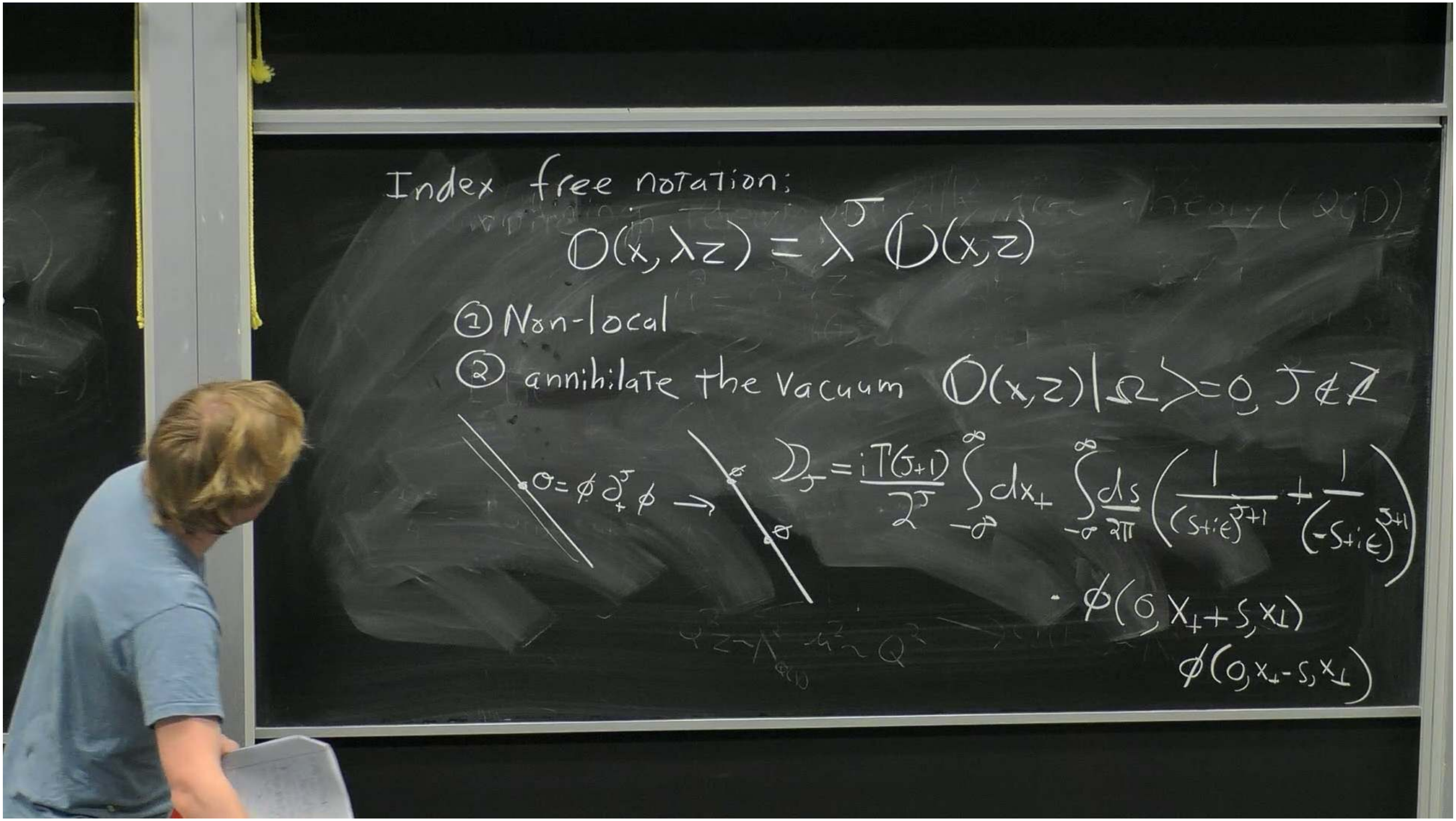
$$\mathcal{O}_\xi^{[J]} = \frac{1}{2^J}\,\bar{\psi}\,\gamma^+ (iD^+)^{J-1}\,\psi$$

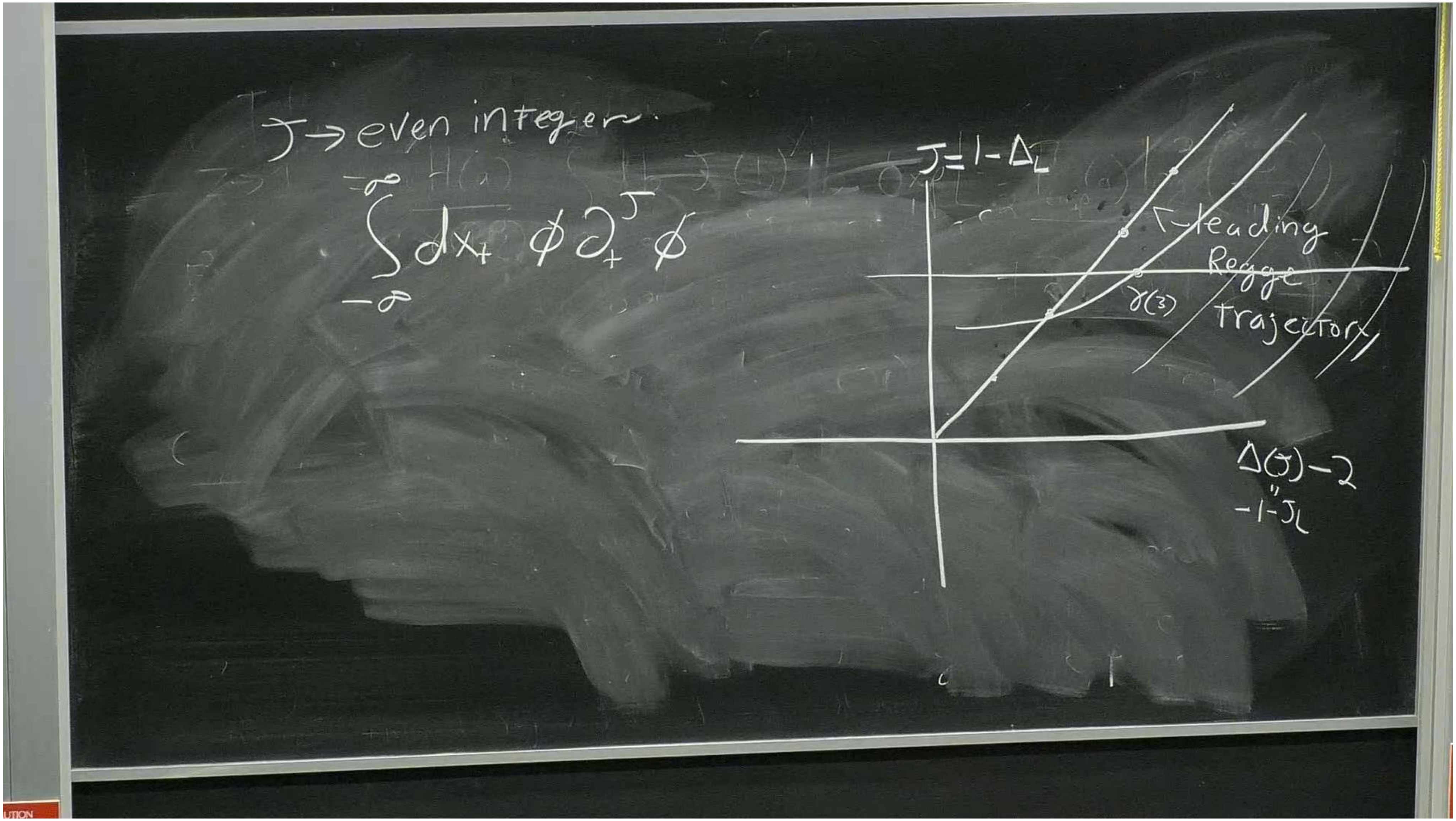
$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}\,F_c^{i+}\,(iD^+)^{J-2}\,F_c^{i+}$$

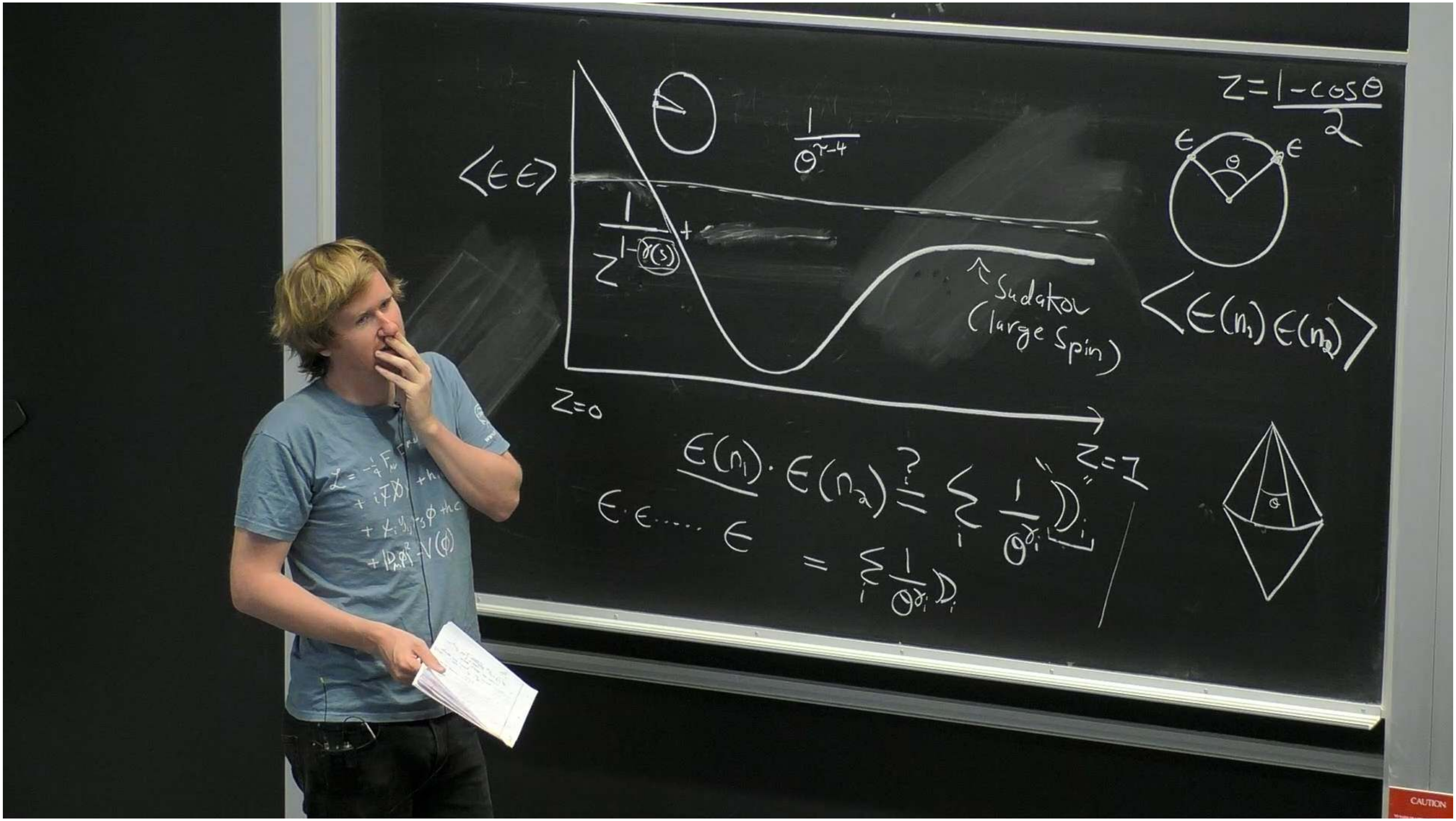
$$\mathbb{D}^J_{\tau=2}\,|X\rangle =$$

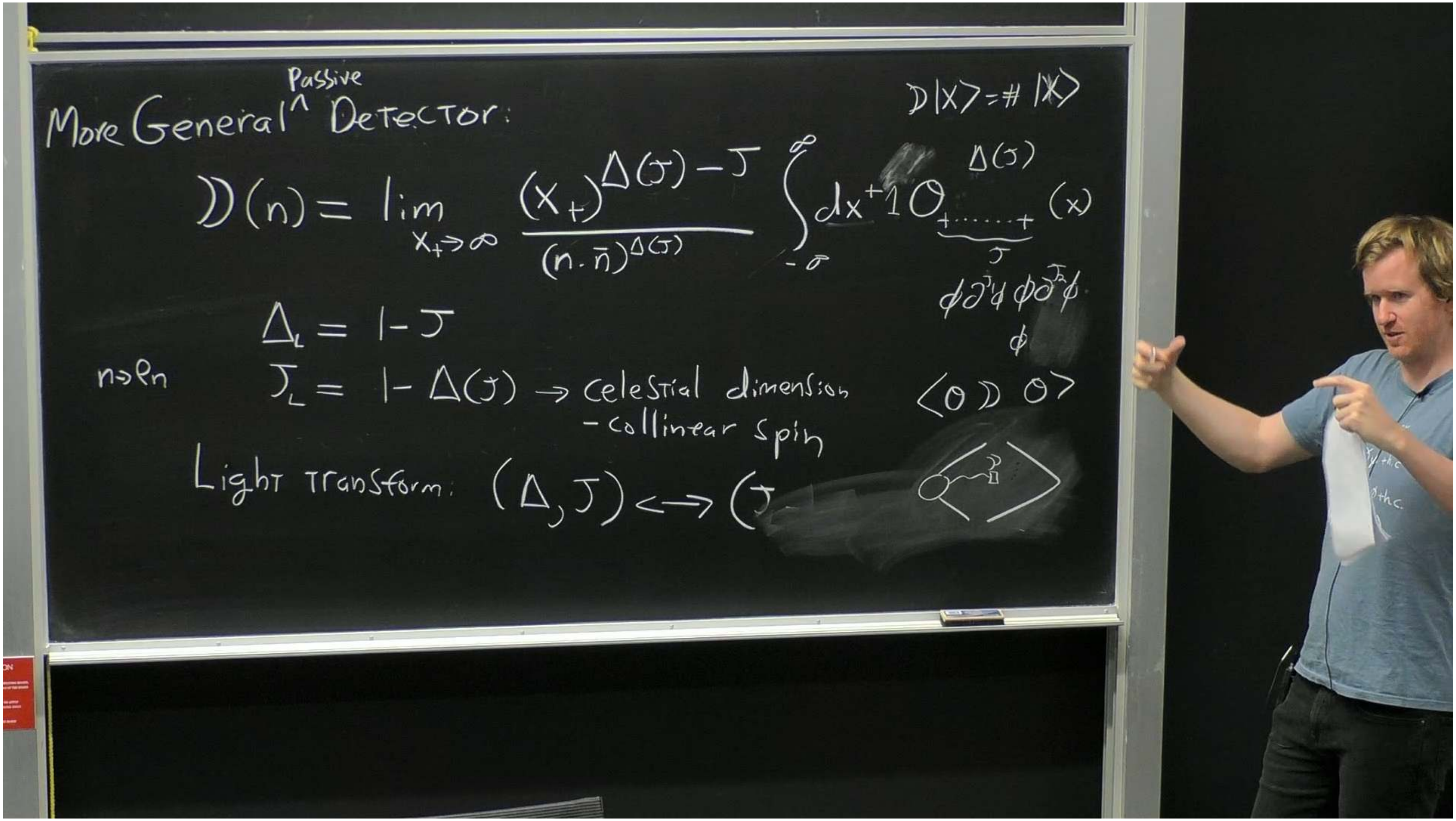
$$= \sum_{\substack{k\in X \\ k = \phi,\xi,g}} (k^0)^{-1+J}\,\delta^{(2)}(\Omega_n - \Omega_k)\,|X\rangle$$


Two questions:
① The operators constructed as null integrals
of local operators:
E^J
"$\mathcal{E}^J$"
Spin: $J_L = 1 - \Delta(J) \to$ not an integer
dim $\Delta_L = 1 - J$
② We have constructed operators that measure
$(k^0)^{-3+\Delta(J)} \to$ exp. can measure
generic powers.


Index free notation:
$\mathbb{O}(x, \lambda z) = \lambda^J \mathbb{O}(x, z)$
① Non-local
② annihilate the vacuum $\mathbb{O}(x,z)|\Omega\rangle = 0,\ J \notin \mathbb{Z}$
$\mathcal{O} = \phi\, \partial_+^J \phi \rightarrow$
$\mathcal{D}_J = \frac{i\Gamma(J+1)}{2^J} \int_{-\infty}^{\infty} dx_+ \int_{-\infty}^{\infty} \frac{ds}{2\pi} \left(\frac{1}{(s+i\epsilon)^{J+1}} + \frac{1}{(-s+i\epsilon)^{J+1}} \right)$
$\cdot\, \phi(0, x_+ + s, x_\perp)$
$\phi(0, x_+ - s, x_\perp)$


J → even integer
$\int_{-\infty}^{\infty} dx_+ \, \phi \, \partial_+^J \phi$
J = 1 - Δ_L
leading Regge trajectory
γ(3)
Δ(J) − 2
−1 − J_L


$z = \frac{1-\cos\theta}{2}$
$\langle \epsilon\epsilon \rangle$
$\frac{1}{\theta^{\gamma-4}}$
$\frac{1}{z^{1-\gamma(3)}} +$
Sudakov (large Spin)
$\langle \epsilon(n_1)\epsilon(n_2) \rangle$
$z=0$
$z=1$
$\epsilon(n_1)\cdot\epsilon(n_2) \overset{?}{=} \sum_i \frac{1}{\theta^{\gamma_i}} D_i$
$\epsilon\cdot\epsilon\cdots\epsilon = \sum_i \frac{1}{\theta^{\gamma_i}} D_i$


More General Passive Detector:
D|X> = # |X>
$\mathcal{D}(n) = \lim_{x_+ \to \infty} \frac{(x_+)^{\Delta(J)-J}}{(n\cdot\bar{n})^{\Delta(J)}} \int_{-\infty}^{\infty} dx^+ \, \mathcal{O}^{\Delta(J)}_{\underbrace{+\cdots+}_{J}}(x)$
$\phi\partial^J\phi$ $\phi\partial^{J_2}\phi$
ϕ
$\Delta_L = 1 - J$
$n \to \rho n$
$J_L = 1 - \Delta(J) \to$ celestial dimension
- collinear spin
$\langle \mathcal{O} \, \mathcal{D} \, \mathcal{O} \rangle$
Light transform: $(\Delta, J) \longleftrightarrow (J$