Title: Lecture

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Collection/Series: Energy Operators in Particle Physics, QFT, and Gravity - June 6, 9, 11, 2025

Subject: Particle Physics, Quantum Fields and Strings

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Energy Operators (cont'd)

$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} dt \lim_{r\to\infty} r^2\, n^i\, T_{0i}(t, r\hat{n})$$

(Sterman, Lee, ...)

1. Measures "weights" by k^0 of a free particle state.
2. $\mathcal{E}(\vec{n})|0\rangle = 0$
3. $[\mathcal{E}(\vec{n}_1), \mathcal{E}(\vec{n}_2)] = 0$

Lorentz Covariant detector op.

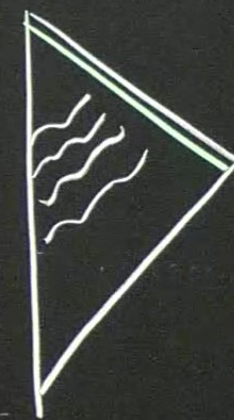
(massless excitations)

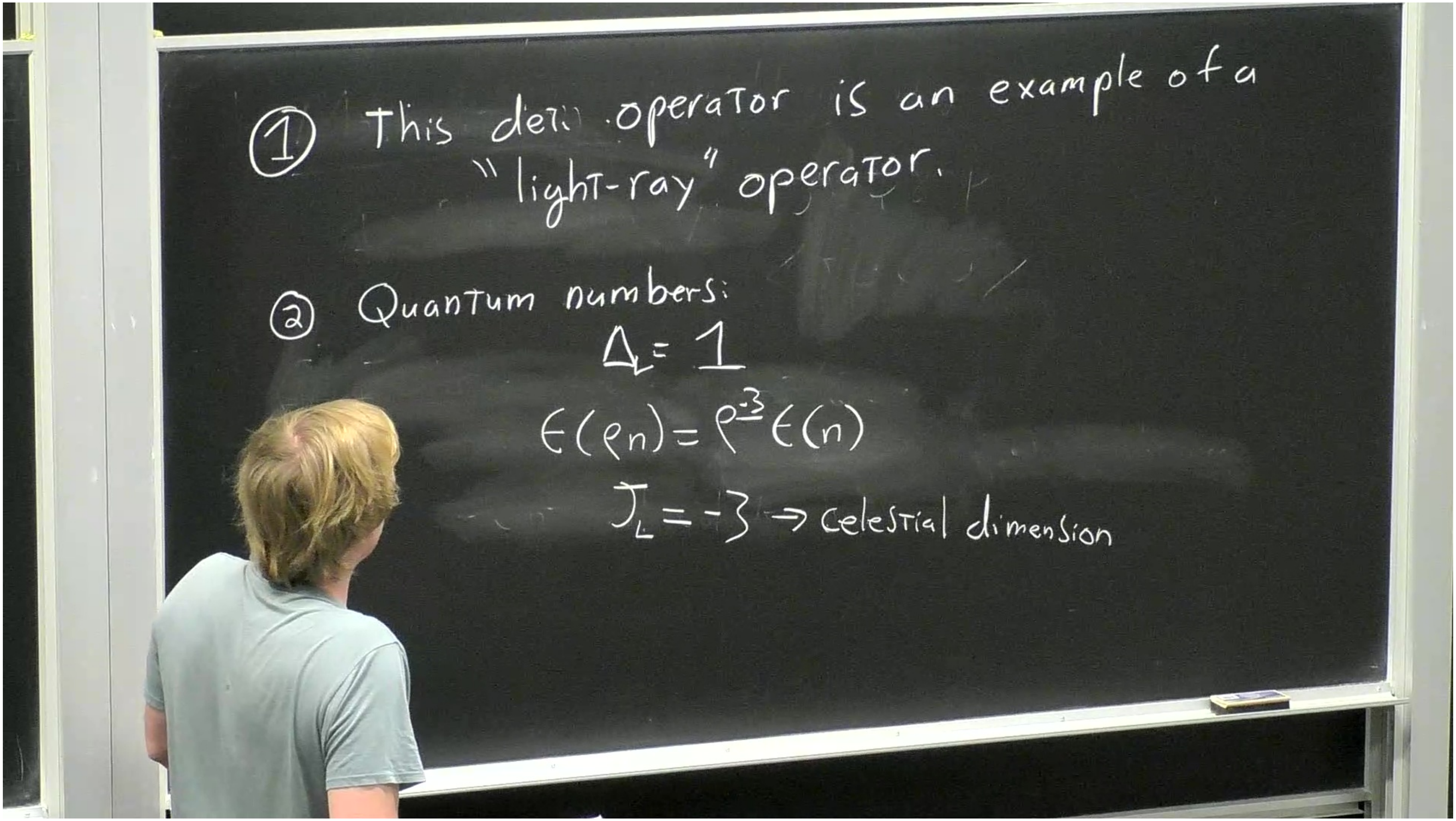
$$n^\mu = (1, \hat{n})$$
$$\bar{n}^\mu = (1, -\hat{n})$$

$$x_+ = x^- = \bar{n}\cdot x$$
$$x_- = x^+ = n\cdot x$$

$$x^\mu = x_+ \frac{n^\mu}{n\cdot\bar{n}} + \frac{x_- \bar{n}^\mu}{n\cdot\bar{n}} + x_\perp^\mu$$

$$\mathcal{E}(n) = \lim_{x_+\to\infty} \frac{(x_+)^2}{(n\cdot\bar{n})^4} \int_{-\infty}^{\infty} dx_-\, T_{++}(x)$$




① This operator is an example of a
"light-ray" operator.
② Quantum numbers:
$\Delta_L = 1$
$\mathcal{E}(\rho n) = \rho^{-3}\mathcal{E}(n)$
$J_L = -3 \rightarrow$ celestial dimension

e.g. $U(1)$ charge

$$Q(n) = \lim_{x_+ \to \infty} \frac{1}{(n\cdot\bar{n})^3} \int_{-\infty}^{\infty} dx^+ \, (x_+)^2 \, J_+$$

$$\Delta_L = 0$$

$$J_L = -2$$

3. Average Null Energy Condition (ANEC)

$$\left\langle \int_{-\infty}^{\infty} dx^+ \, T_{++}(x^{\pm}) \right\rangle_{\psi} \geq 0$$

Hartman: causality

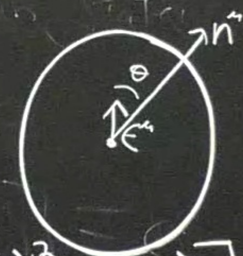
faulkner: monotonicity of relative entropy

((Casini, ...))

Ex. ① $\langle \mathcal{E}(n)\rangle_{\phi_Q} = \frac{Q}{4\pi} \longleftrightarrow \langle \phi T \phi \rangle$

- 1 Tensor check
- Ward identity

② Conserved current j^μ



Collider perspective:

$$\langle \mathcal{E}(n)\rangle_j = \frac{Q}{4\pi}\left[1 + a_2\left(\frac{|\vec{\epsilon}\cdot\vec{n}|^2}{|\vec{\epsilon}|^2} - \frac{1}{3}\right)\right] = \frac{Q}{4\pi}\left[1 + a_2\left(\cos^2\theta - \frac{1}{3}\right)\right]$$

$$Q^\mu = (Q, 0, 0, 0)$$

$$q = |Q|$$

$$\int d^4x \, e^{iQ\cdot x} \langle 0 | \phi(x) \, \mathcal{E}(n) \, \phi(0) | 0 \rangle$$

$$\langle \mathcal{E}(n) \rangle_\phi$$

Conformal Collider Bounds.

$$\langle \mathcal{E}(n) \rangle_{ANEC} = \frac{Q}{4\pi}\left[1 + \boxed{a_2}\left(\cos^2\theta - \frac{1}{3}\right)\right]$$

$$\geq 0$$

$$\underset{\substack{\uparrow \\ \text{free} \\ \text{Scalar}}}{3} \geq a_2 \geq \underset{\substack{\uparrow \\ \text{free} \\ \text{fermion}}}{-\frac{3}{2}}$$

In QCD $\quad a_2 = -\frac{3}{2} + \underbrace{18\left(\frac{\alpha_s}{4\pi}\right)}_{\geq 0} + \cdots$

Hofman, Maldacena:

$\mathcal{N}=1$ Superconformal, $d=4$

Source R-current, $\langle j\, T\, j \rangle \leftrightarrow a, c$

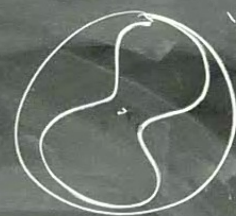
$$T^{\mu}{}_{\mu} \sim \frac{c}{16\pi^2} W^2 - \frac{\boxed{a}}{16\pi^2} E \qquad a_{IR} \leq a_{UV}$$

$$\langle \mathcal{E}(\theta) \rangle_R = 1 + 3\,\frac{(c-a)}{c}\left(\cos^2\theta - \frac{1}{3}\right) \Rightarrow \frac{3c}{2} \geq a \geq 0$$

Conformal Collider Bounds.

$$\langle \mathcal{E}(n)\rangle_{ANEC} = \frac{Q}{4\pi}\left[1 + \boxed{a_2}\left(\cos^2\theta - \frac{1}{3}\right)\right]$$

≥ 0



"radiation zeros"

$\phi\partial^4\phi$

$$\boxed{3 \geq a_2 \geq -\frac{3}{2}}$$

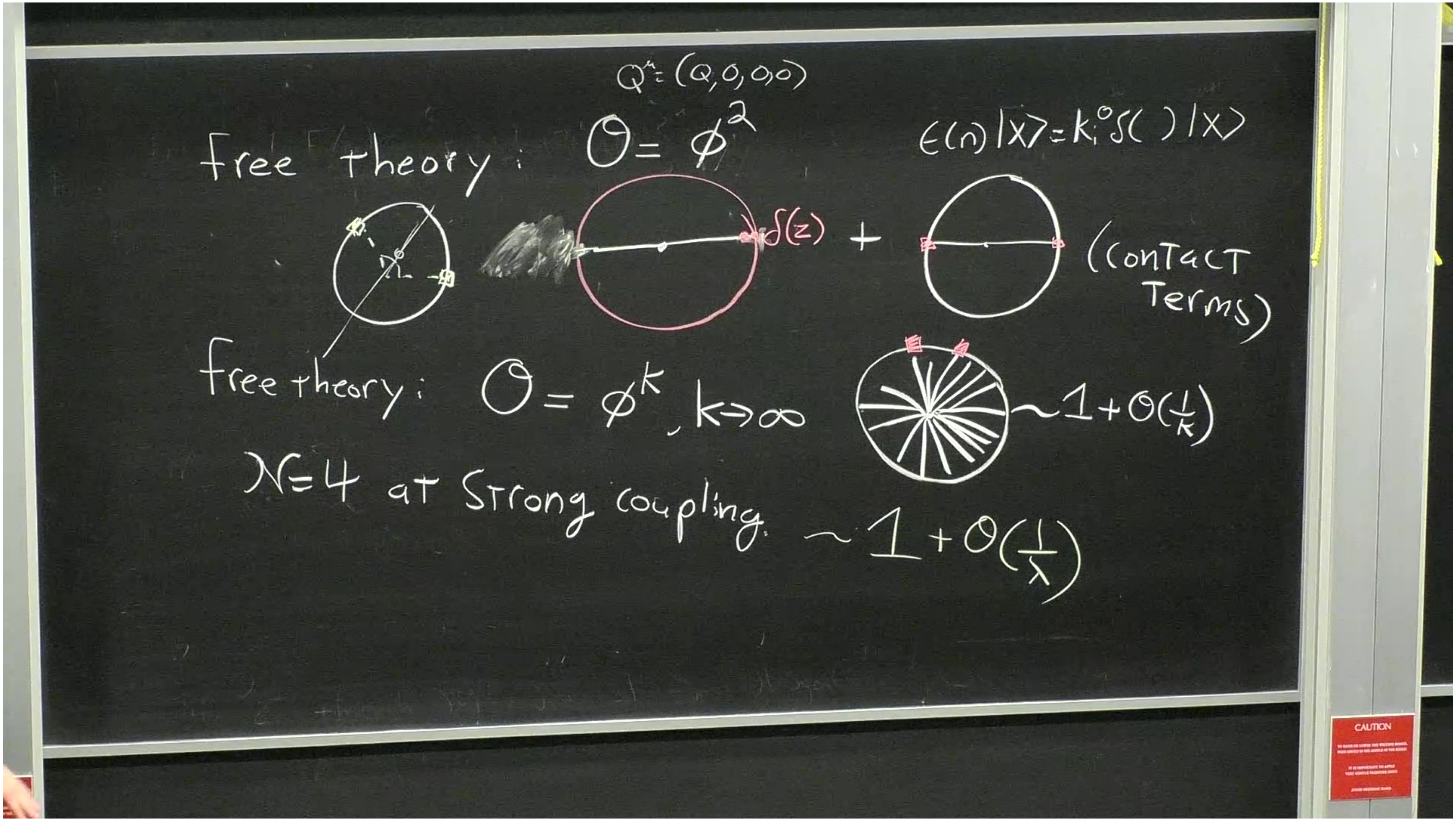
free Scalar

free fermion

In QCD $a_2 = -\dfrac{3}{2} + \underbrace{18\left(\dfrac{\alpha_s}{4\pi}\right)}_{\geq 0} + \cdots$

Higher Point Correlators

$$\langle \mathcal{E}(n_1)\, \mathcal{E}(n_2) \rangle_\psi \longleftrightarrow \langle \mathcal{O}\, T\, T\, \mathcal{O} \rangle$$


$Q^{\mu} = (Q,0,0,0)$
free theory: $\mathcal{O} = \phi^2$
$\mathcal{E}(n)|X\rangle = k_i^0 \delta(\ )|X\rangle$
$\delta(z)$ +
((contact terms)
free theory: $\mathcal{O} = \phi^k$, $k \to \infty$
$\sim 1 + O(\frac{1}{k})$
$\mathcal{N}=4$ at strong coupling $\sim 1 + O(\frac{1}{\lambda})$

Weakly coupled conformal gauge theory:

($\mathcal{N}=4$) $\quad a = \dfrac{g^2 N}{(4\pi^2)}$

$$EEC(z) = \frac{a}{4z^2(1-z)}\log\left(\frac{1}{1-z}\right) + a^2 + a^3$$

$[\ \ ]_+$

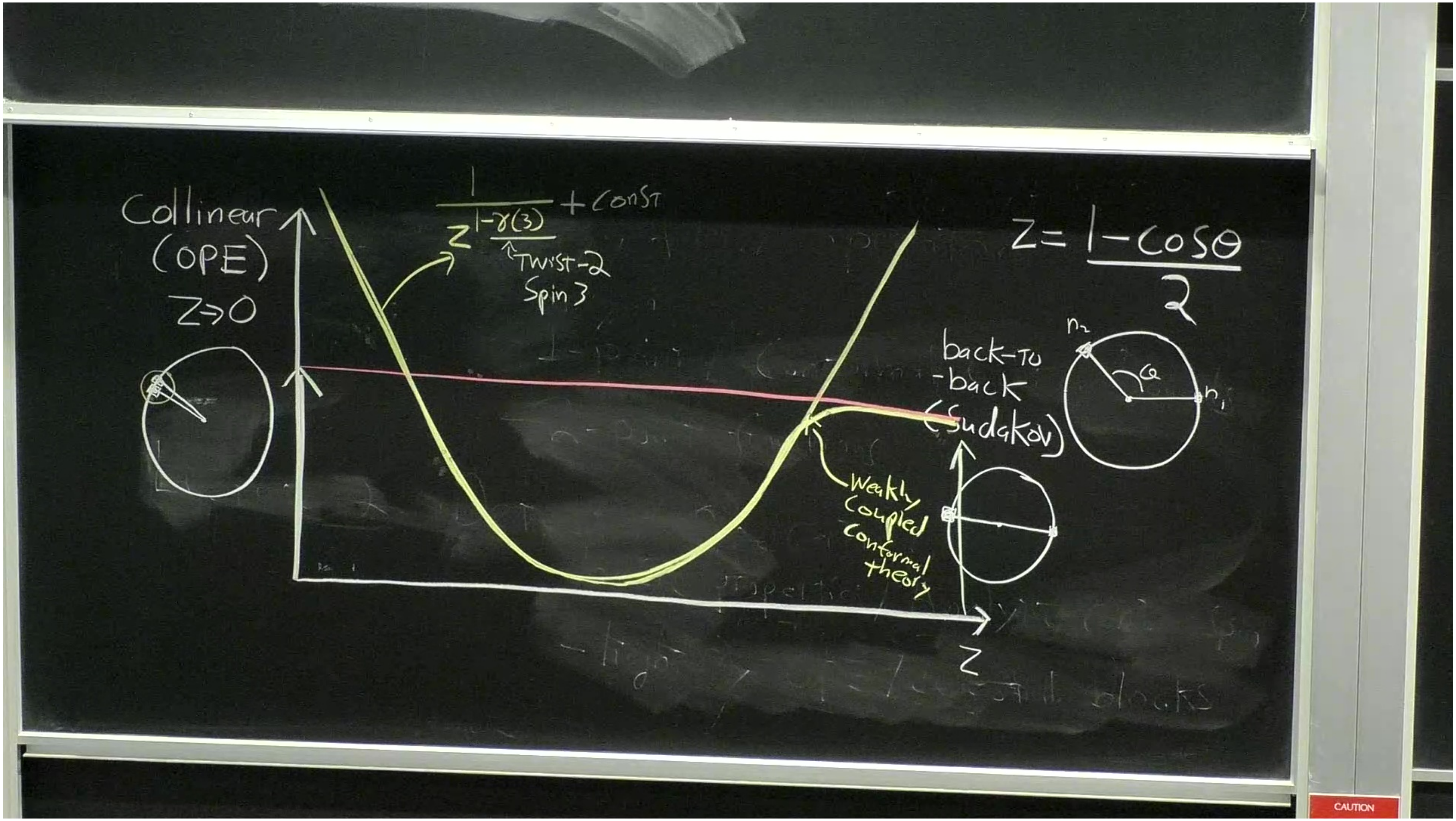
polylogs

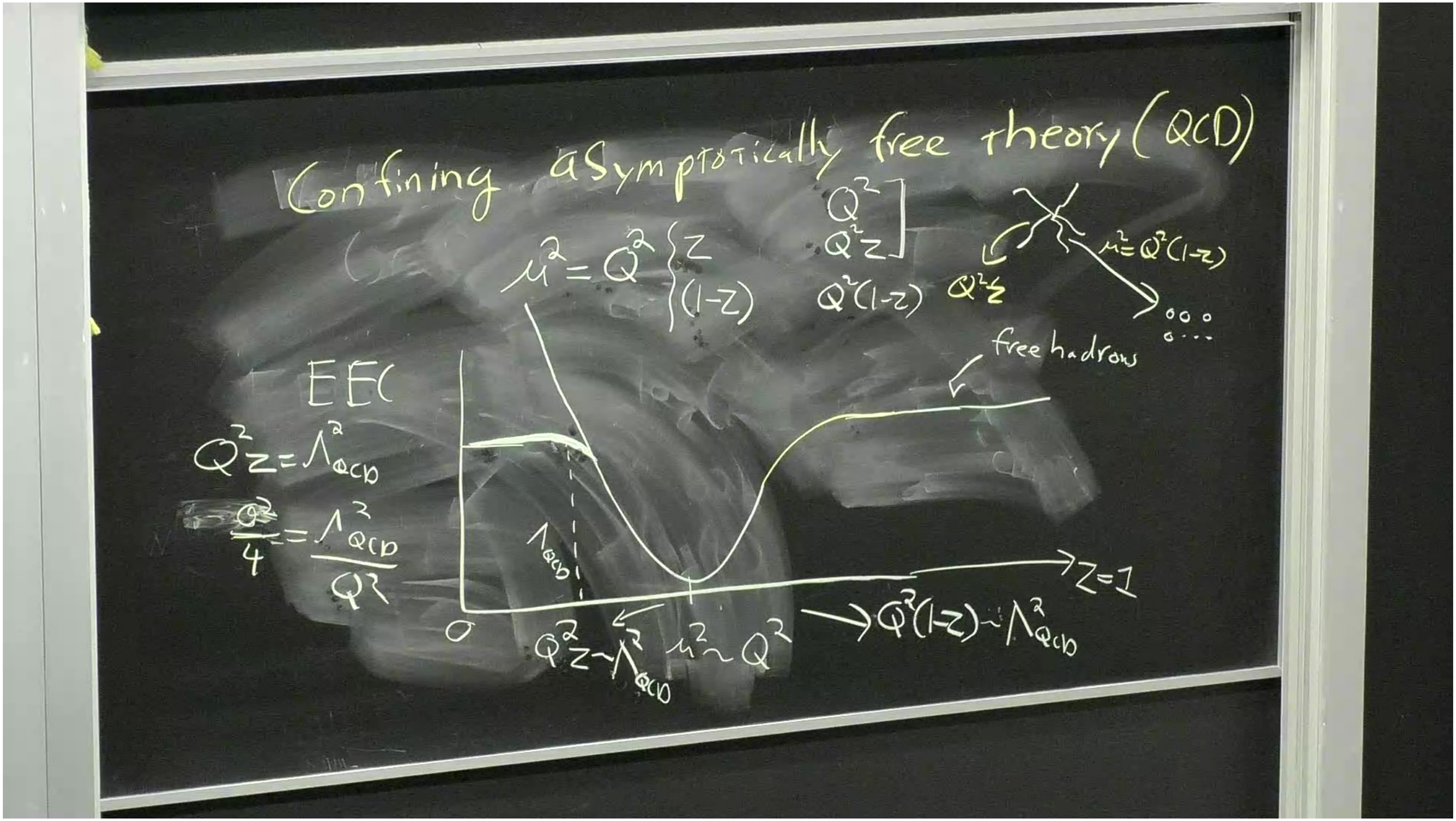
elliptic

collinear limit $\quad \dfrac{1}{z}$

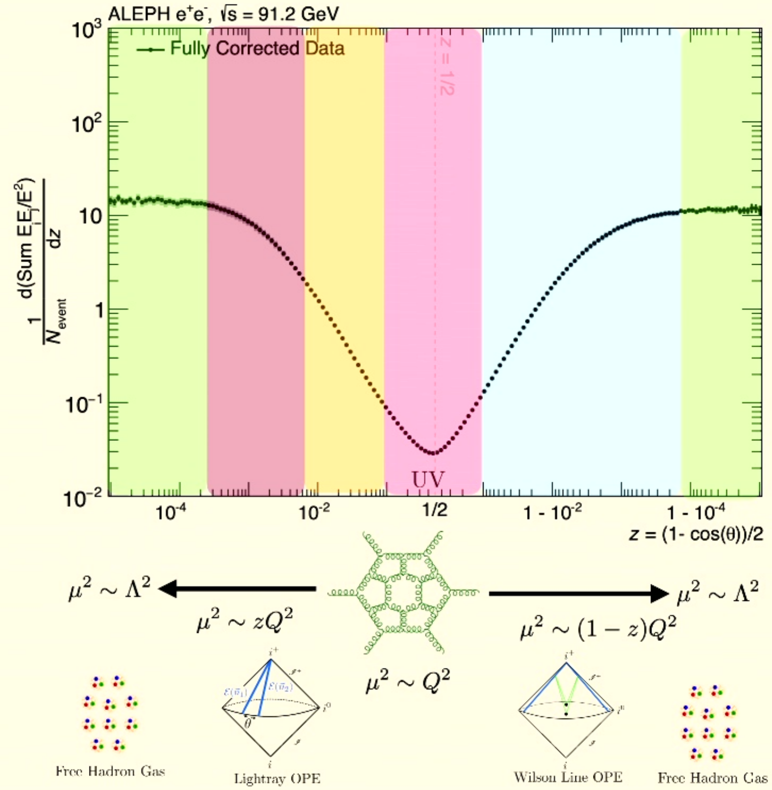
b2b $\quad \dfrac{\log(1-z)}{1-z}$

$$
z \to 1 \;=\; \frac{H(a)}{8(1-z)} \int_0^\infty db\, J_0(b)\, b\, \exp\left[-\frac{1}{2}\,\underline{\Gamma_{cusp}(a)}\,\log^2\left(\frac{b^2}{1-z}\right) + 2\,\underline{B_\delta(a)}\,\log\left(\frac{b}{1-z}\right) \right]
$$


Collinear
(OPE)
Z→0
1/z^{1-γ(3)} + const
Twist-2
Spin 3
Z= (1-cosθ)/2
back-to-back
(Sudakov)
Weakly coupled conformal theory
n₂
θ
n₁
Z


Confining asymptotically free theory (QCD)
$\mu^2 = Q^2 \begin{cases} z \\ (1-z) \end{cases}$
Q^2
$Q^2 z$
$Q^2(1-z)$
$Q^2 z$
$\mu^2 = Q^2(1-z)$
free hadrons
EEC
$Q^2 z = \Lambda^2_{QCD}$
$\frac{Q^2}{4} = \frac{\Lambda^2_{QCD}}{Q^2}$
Λ_{QCD}
$z=1$
0
$Q^2 z \sim \Lambda^2_{QCD}$
$\mu^2 \sim Q^2$
$Q^2(1-z) \sim \Lambda^2_{QCD}$

ALEPH Data



ALEPH e+e-, √s = 91.2 GeV
Fully Corrected Data
z = 1/2
UV
$\frac{1}{N_{event}} \frac{d(\text{Sum } E_i E_j/E^2)}{dz}$
$z = (1 - \cos(\theta))/2$
10^{-4}
10^{-2}
1/2
$1 - 10^{-2}$
$1 - 10^{-4}$
$\mu^2 \sim \Lambda^2$
$\mu^2 \sim zQ^2$
$\mu^2 \sim Q^2$
$\mu^2 \sim (1-z)Q^2$
$\mu^2 \sim \Lambda^2$
Free Hadron Gas
Lightray OPE
Wilson Line OPE
Free Hadron Gas

ALEPH Data: Back-to-Back Limit