Title: Lecture

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Collection/Series: Energy Operators in Particle Physics, QFT, and Gravity - June 6, 9, 11, 2025

Subject: Particle Physics, Quantum Fields and Strings

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Energy Operators (cont.)

$$\mathcal{E}(\vec{n}) = \int_{-\infty}^{\infty} dT \lim_{r\to\infty} r^2 n^i T_{0i}(T, r\hat{n})$$

(Sterman, Lee, ...)

1. Measures "weights" by K^0 of a free particle state.
2. $\mathcal{E}(\vec{n})|0\rangle = 0$
3. $[\mathcal{E}(\vec{n}_1), \mathcal{E}(\vec{n}_2)] = 0$

Lorentz Covariant detector op.

(massless excitations)

$n^\mu = (1, \hat{n})$

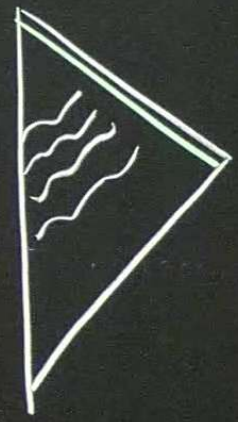
$\bar{n}^\mu = (1, -\hat{n})$

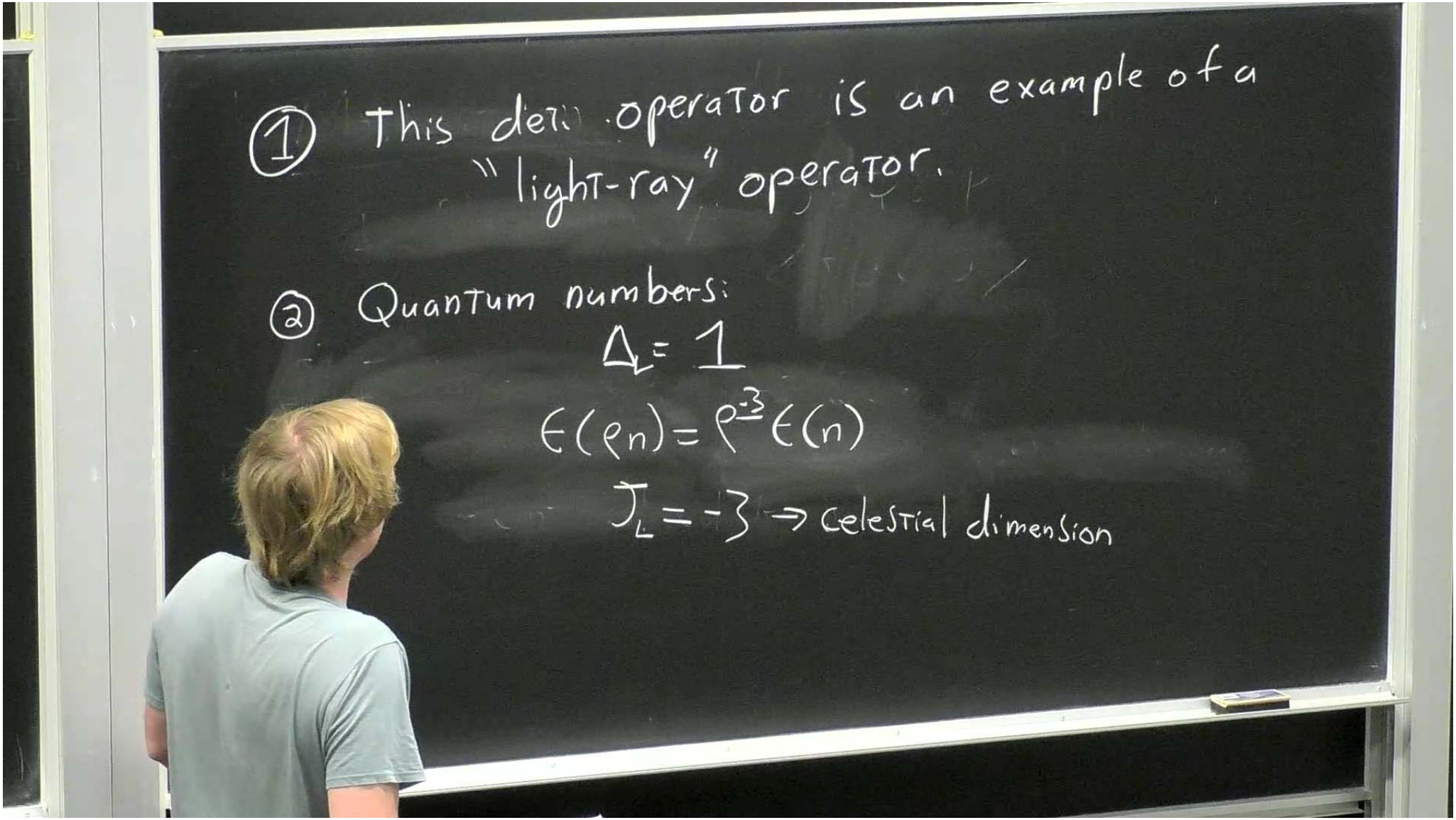
$X_+ = X^- = \bar{n}\cdot x$

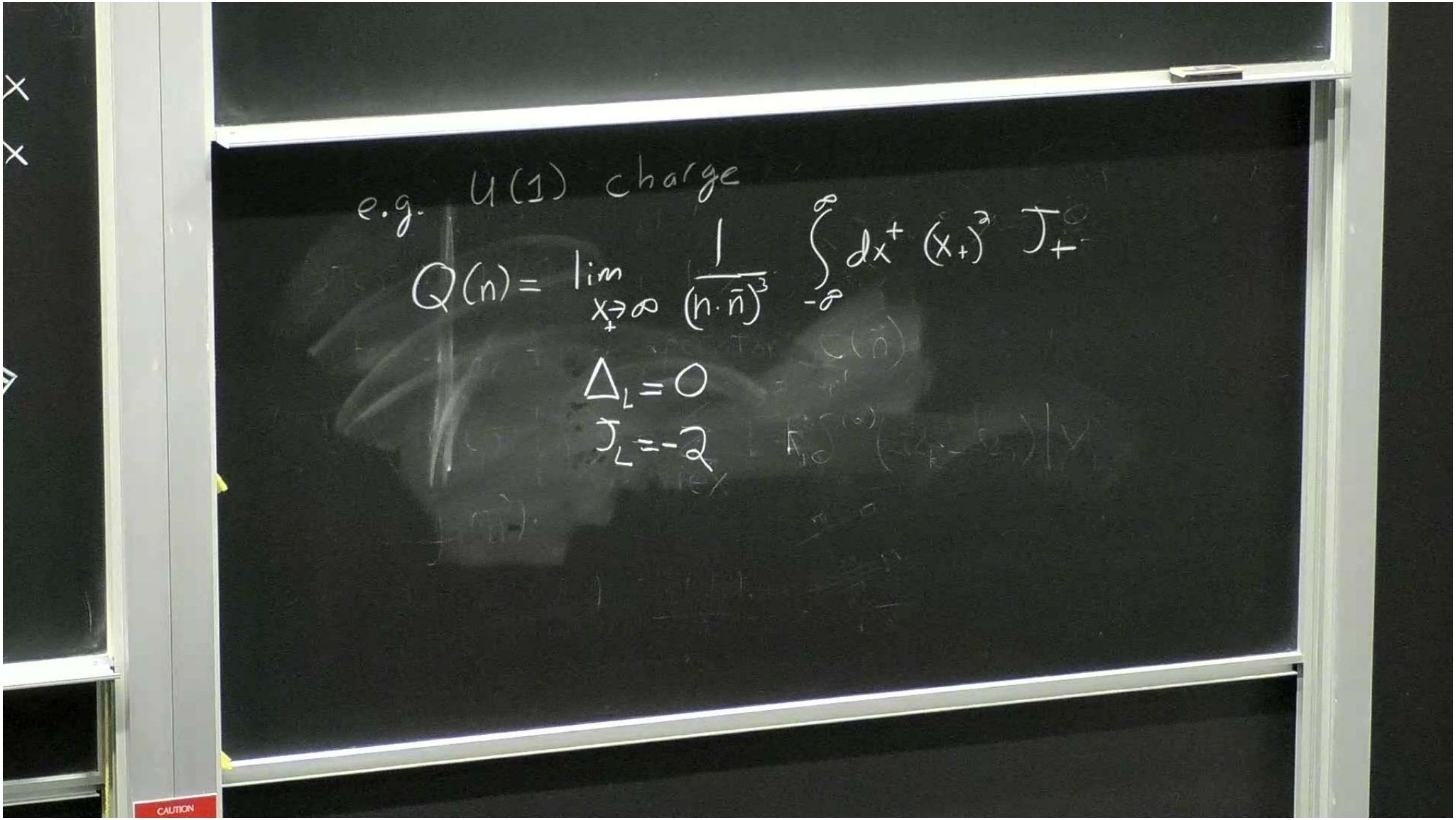
$X_- = x^+ = n\cdot x$

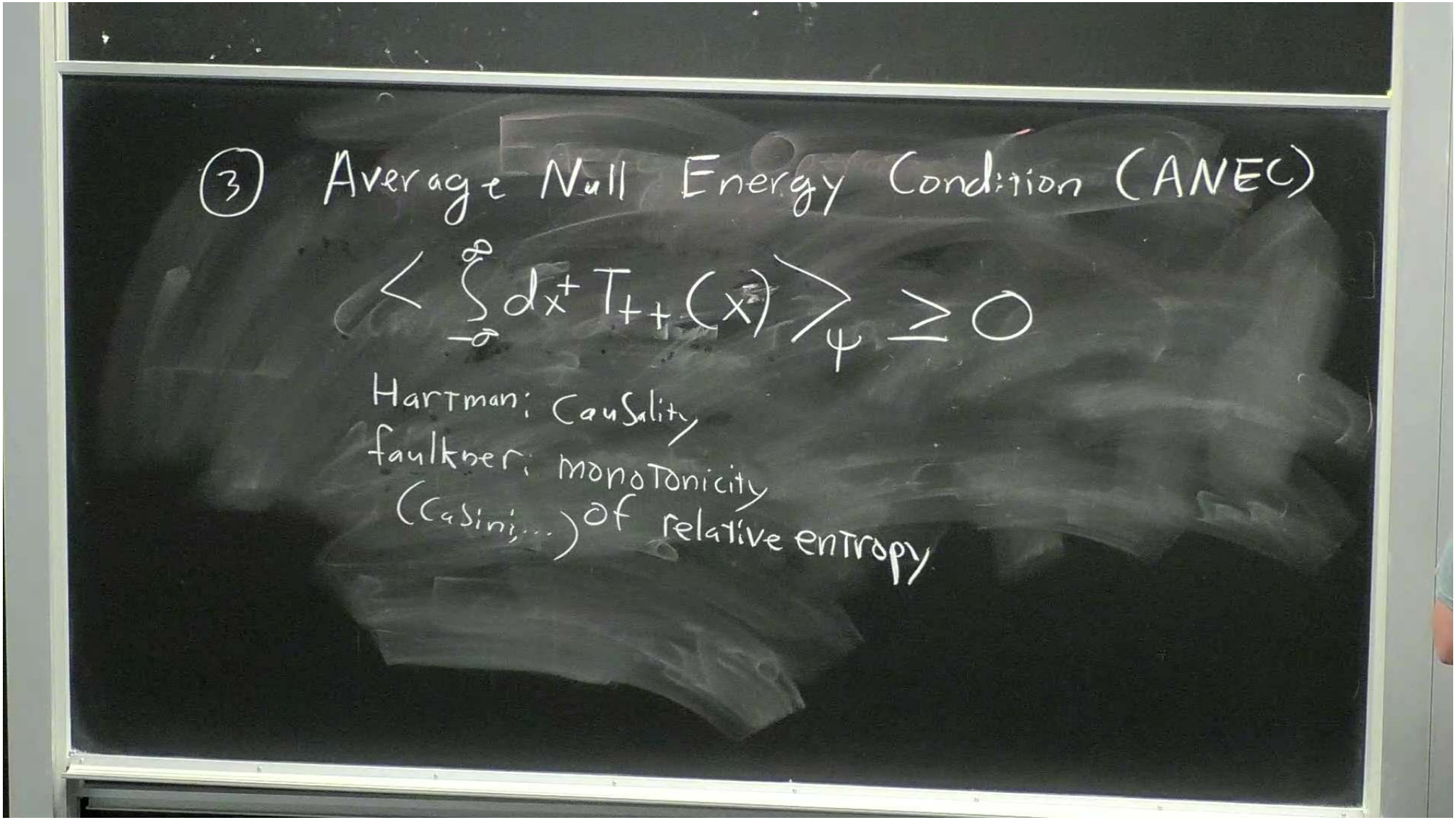
$$x^\mu = X_+ \frac{n^\mu}{n\cdot\bar{n}} + \frac{X_- \bar{n}^\mu}{n\cdot\bar{n}} + x_\perp^\mu$$

$$\epsilon(n) = \lim_{X_+\to\infty} \frac{(X_+)^2}{(n\cdot\bar{n})^4} \int_{-\infty}^{\infty} dx_- \, T_{++}(x)$$




① This deti operator is an example of a "light-ray" operator.
② Quantum numbers:
$\Delta_L = 1$
$\epsilon(\rho n) = \rho^{-3}\epsilon(n)$
$J_L = -3 \Rightarrow$ celestial dimension

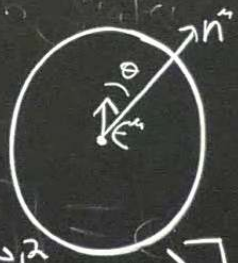

e.g. U(1) charge
$Q(n) = \lim_{x_+ \to \infty} \frac{1}{(n \cdot \bar{n})^3} \int_{-\infty}^{\infty} dx^+ (x_+)^2 J_+$
$\Delta_L = 0$
$J_L = -2$


(3) Average Null Energy Condition (ANEC)
$\langle \int_{-\infty}^{\infty} dx^+ T_{++}(x^+) \rangle_\psi \geq 0$
Hartman: causality
faulkner: monotonicity
(Casini,...) of relative entropy

Ex. ① $\langle \mathcal{E}(n) \rangle_{\phi_Q} = \frac{Q}{4\pi} \longleftrightarrow \langle \phi T \phi \rangle$

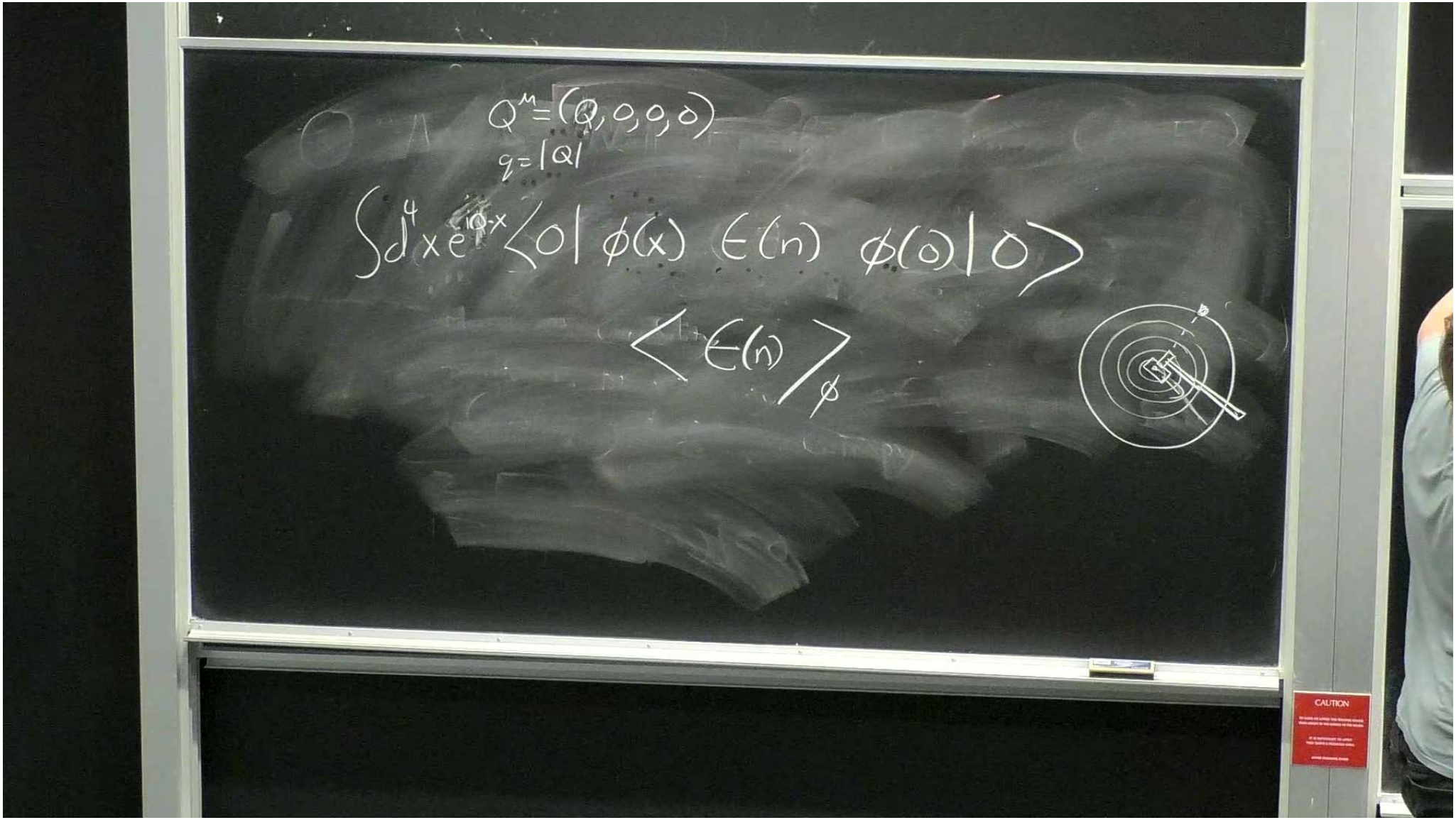
- 1, Tensor check
- Ward identity

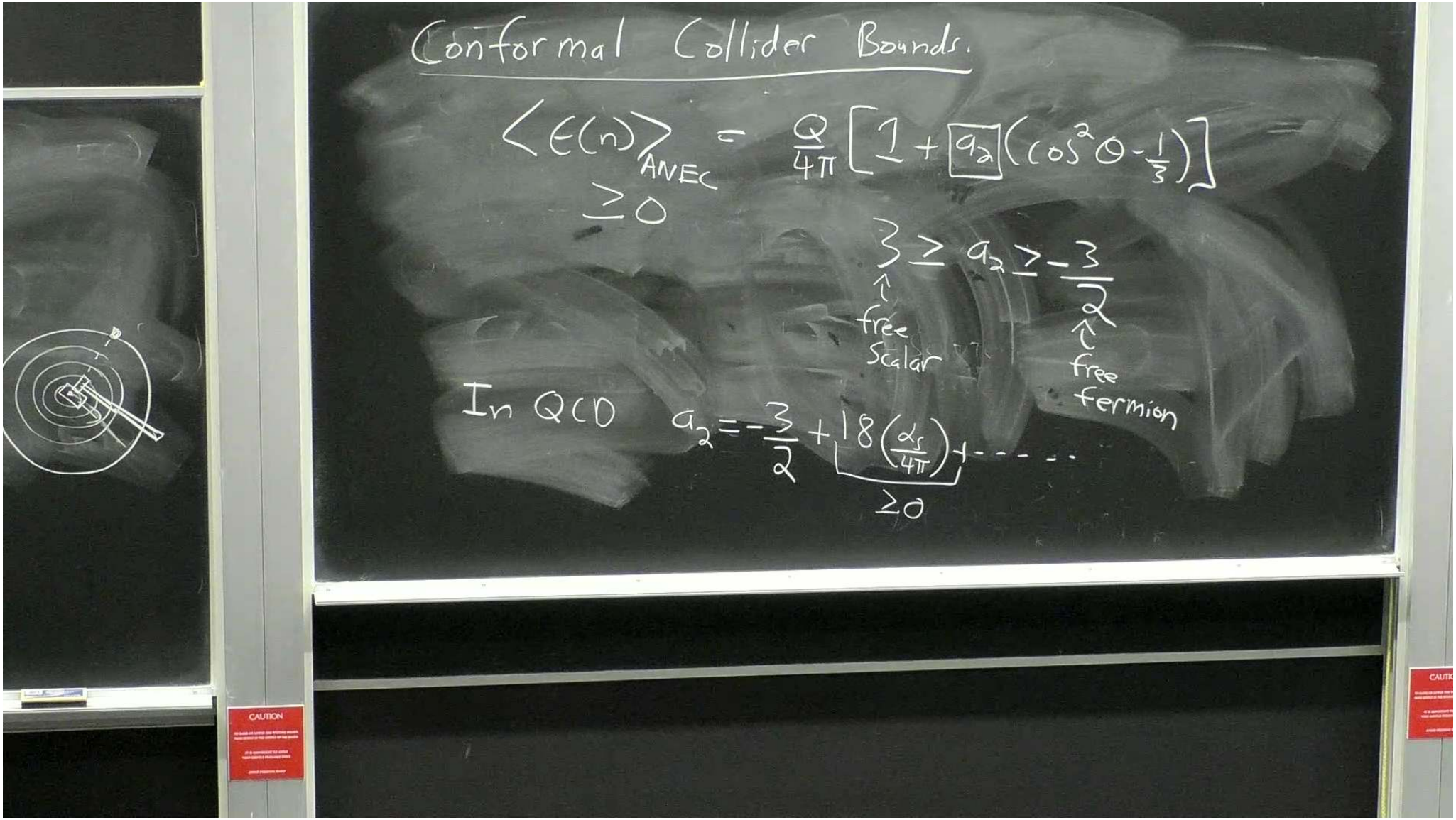
② Conserved current j^μ

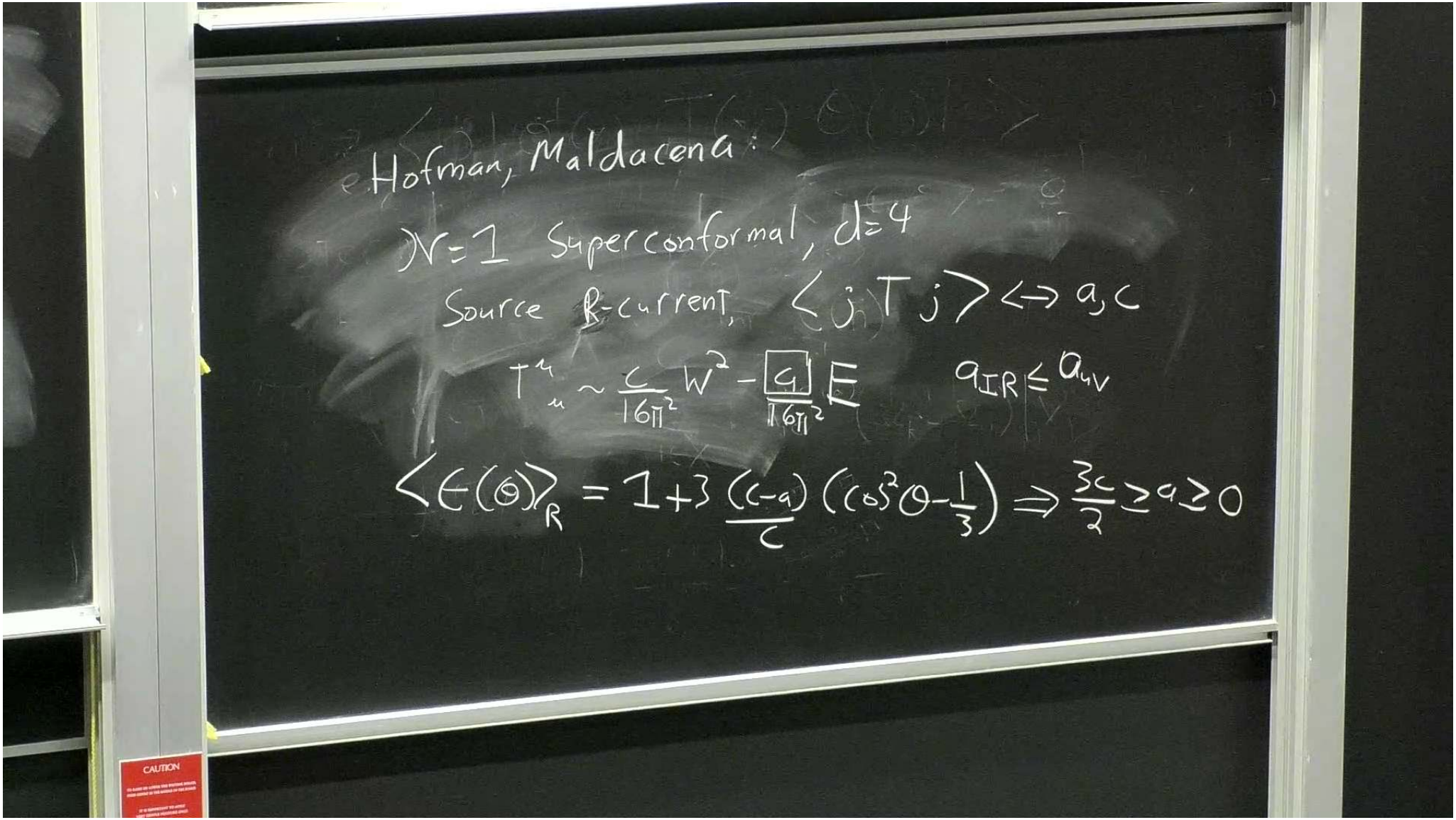


Collider perspective:

$$\langle \mathcal{E}(n) \rangle_{\vec{\epsilon}} = \frac{Q}{4\pi}\left[1 + a_2\left(\frac{|\vec{\epsilon}\cdot\vec{n}|^2}{|\vec{\epsilon}|^2} - \frac{1}{3}\right)\right] = \frac{Q}{4\pi}\left[1 + a_2\left(\cos^2\theta - \frac{1}{3}\right)\right]$$


$Q^\mu = (Q, 0, 0, 0)$
$q = |Q|$
$\int d^4x e^{iQ\cdot x} \langle 0 | \phi(x) \epsilon(n) \phi(0) | 0 \rangle$
$\langle \epsilon(n) \rangle_\phi$
CAUTION

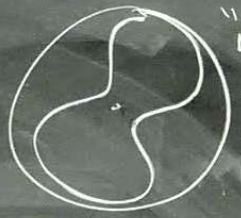

Conformal Collider Bounds.
$\langle \mathcal{E}(n) \rangle_{ANEC} = \frac{Q}{4\pi}\left[1 + \boxed{a_2}\left(\cos^2\theta - \frac{1}{3}\right)\right]$
≥ 0
$3 \geq a_2 \geq -\frac{3}{2}$
free Scalar
free fermion
In QCD $a_2 = -\frac{3}{2} + \underbrace{18\left(\frac{\alpha_s}{4\pi}\right) + \cdots}_{\geq 0}$


Hofman, Maldacena:
$\mathcal{N}=1$ Superconformal, $d=4$
Source R-current, $\langle j\,T\,j\rangle \leftrightarrow a, c$
$T^{\mu}_{\ \mu} \sim \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E$
$a_{IR} \leq a_{UV}$
$\langle \epsilon(\theta) \rangle_R = 1 + 3\frac{(c-a)}{c}\left(\cos^2\theta - \frac{1}{3}\right) \Rightarrow \frac{3c}{2} \geq a \geq 0$
CAUTION

Conformal Collider Bounds.

$$\langle \mathcal{E}(n) \rangle_{\text{ANEC}} = \frac{Q}{4\pi}\left[1 + \boxed{a_2}\left(\cos^2\theta - \frac{1}{3}\right)\right]$$

$$\geq 0$$



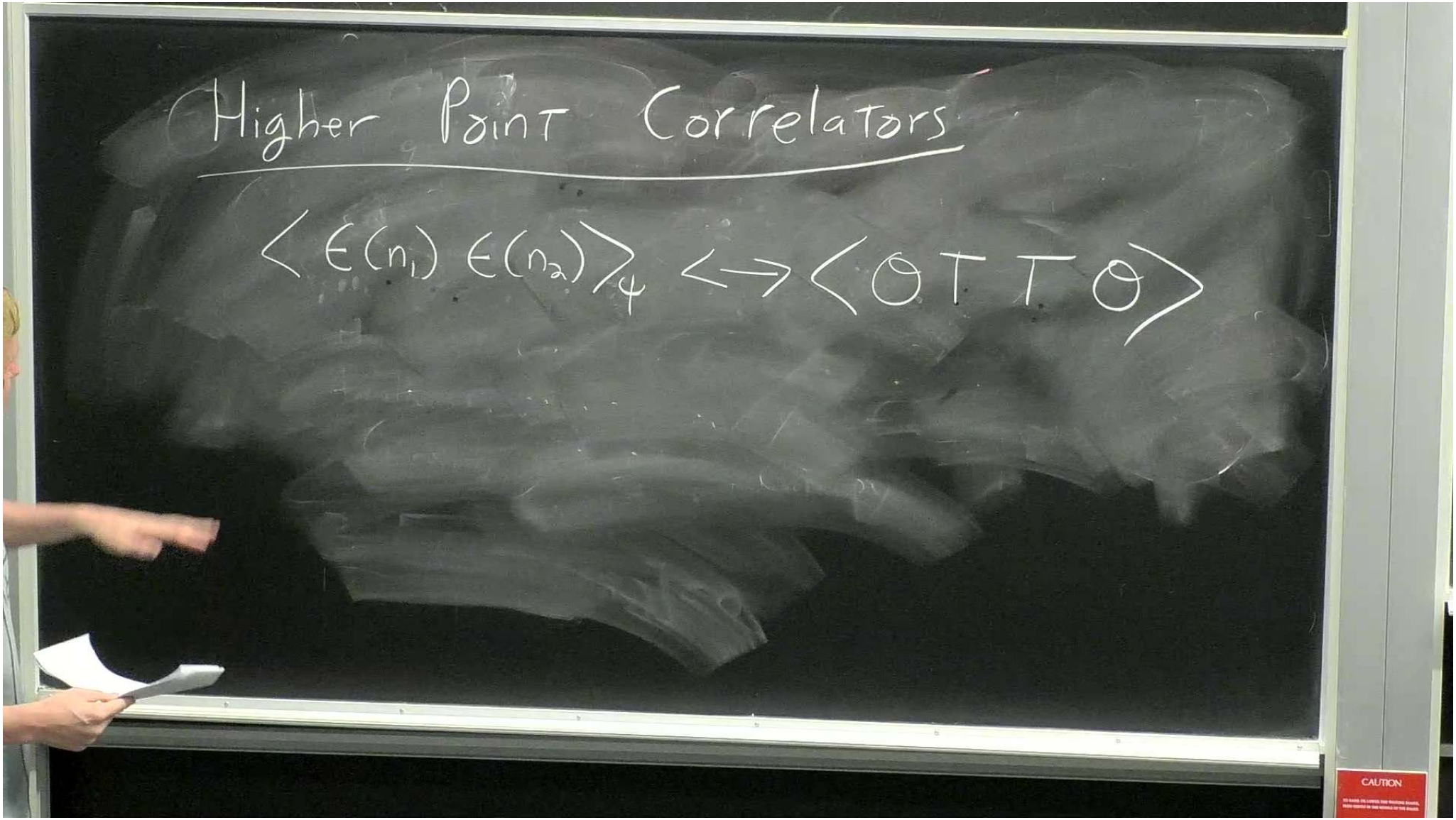
"radiation zeros"

$$\boxed{3 \geq a_2 \geq -\frac{3}{2}}$$

$\phi\partial^4\phi$ — free Scalar — free fermion

In QCD $\quad a_2 = -\frac{3}{2} + \boxed{18\left(\frac{\alpha_s}{4\pi}\right)} + \cdots$

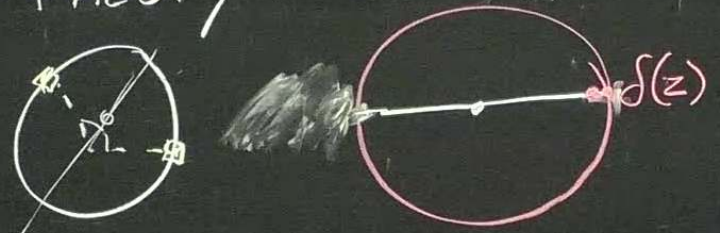
≥ 0


Higher Point Correlators
$\langle \epsilon(n_1)\,\epsilon(n_2)\rangle_\psi \longleftrightarrow \langle \Theta T T \Theta \rangle$
CAUTION

$Q^\mu = (Q,0,0,0)$

free theory: $\mathcal{O} = \phi^2$

$\epsilon(n)|X\rangle = k_i^\mu \zeta(\ )|X\rangle$



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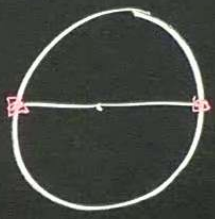
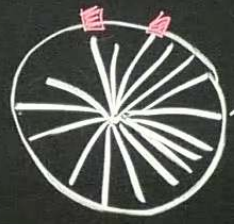


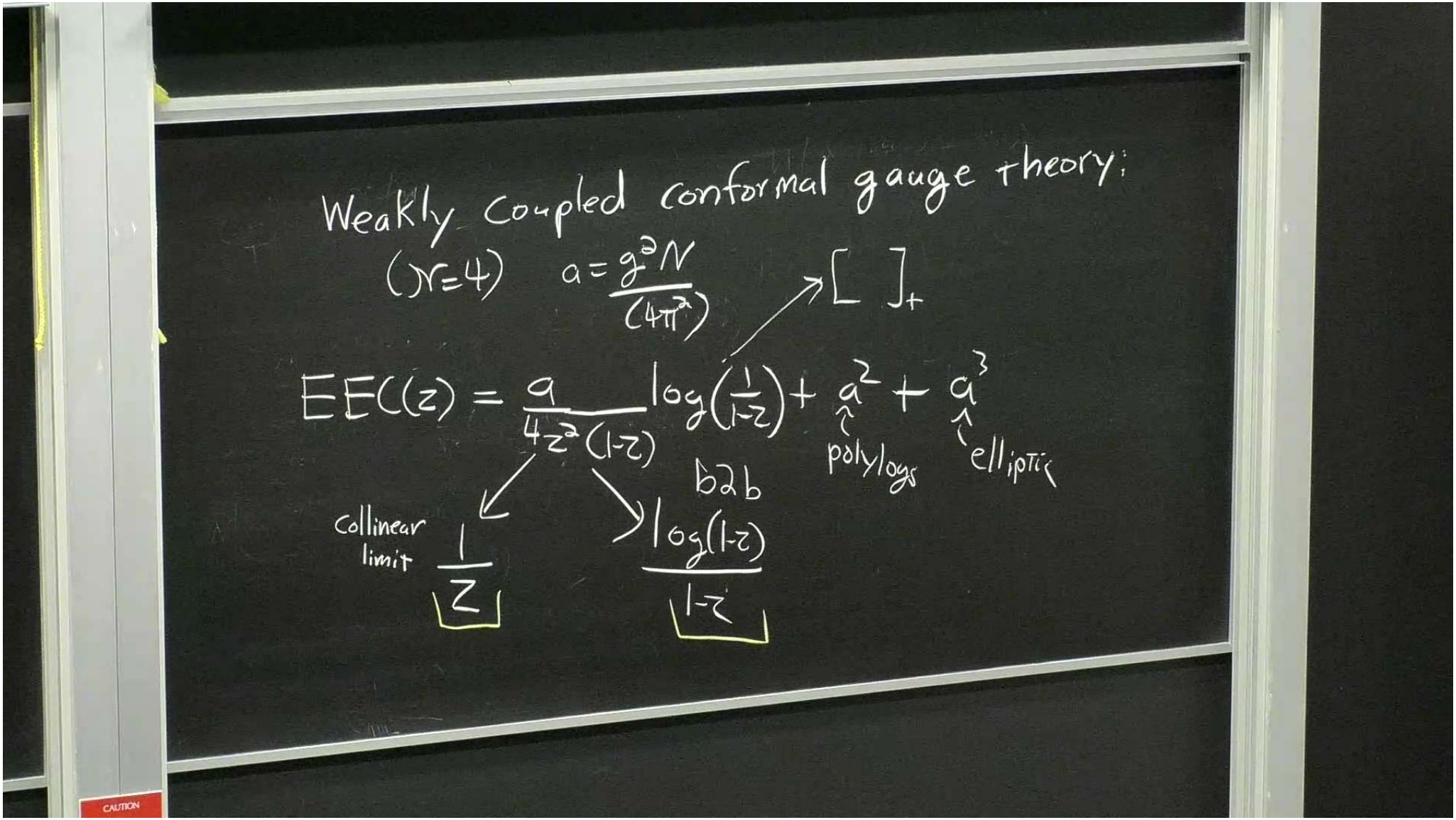
(contact Terms)

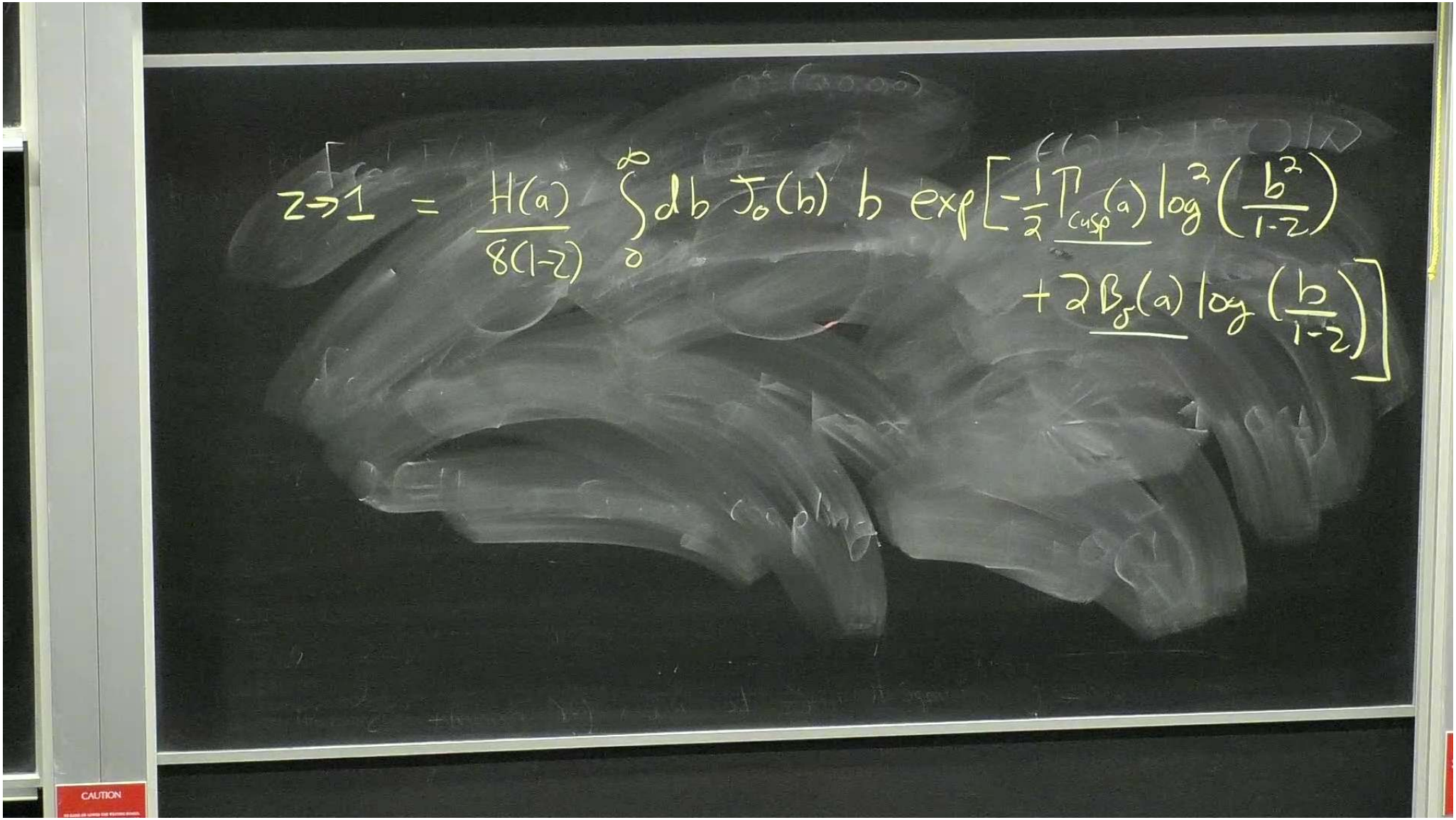
free theory: $\mathcal{O} = \phi^k$, $k \to \infty$

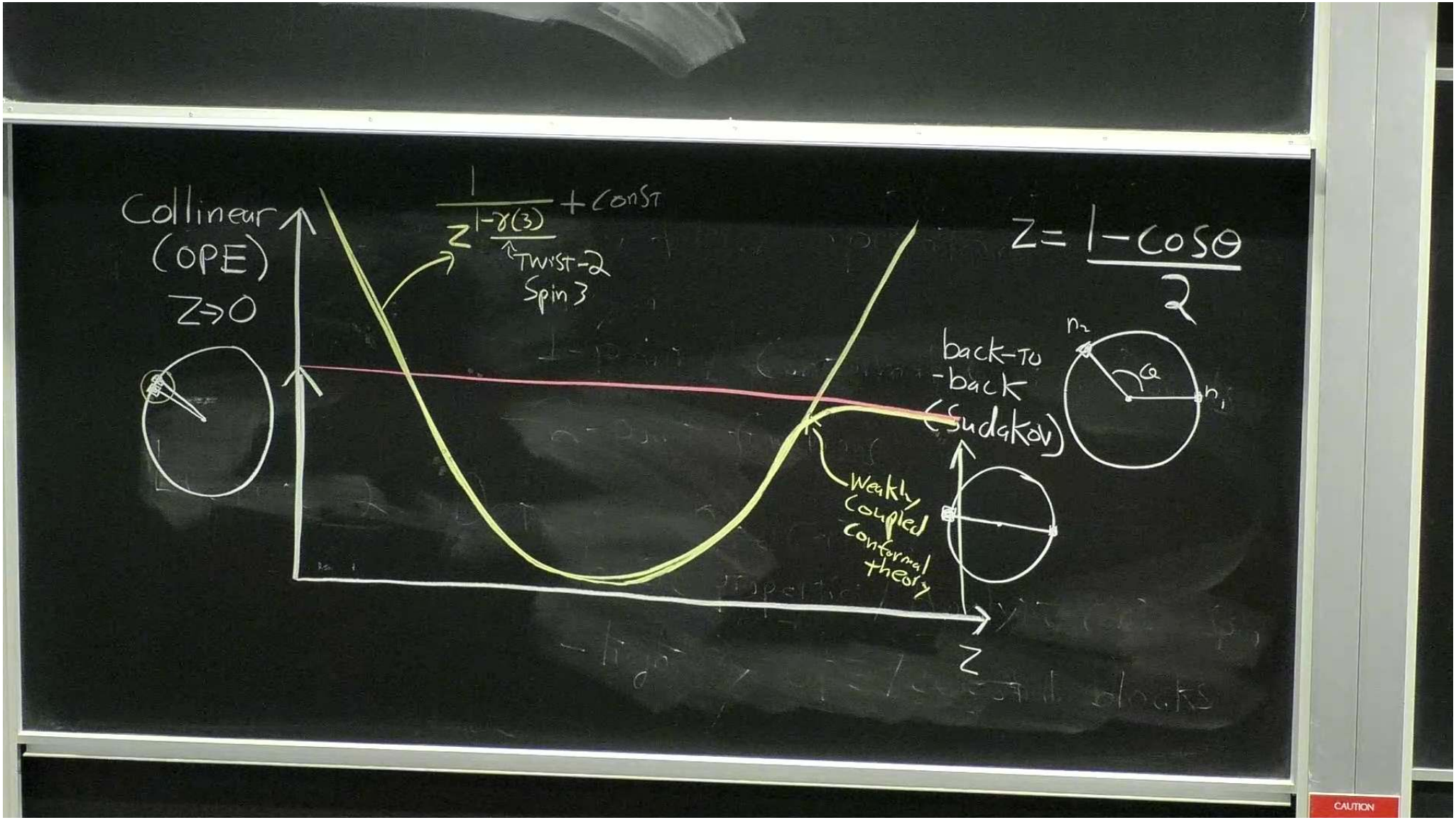


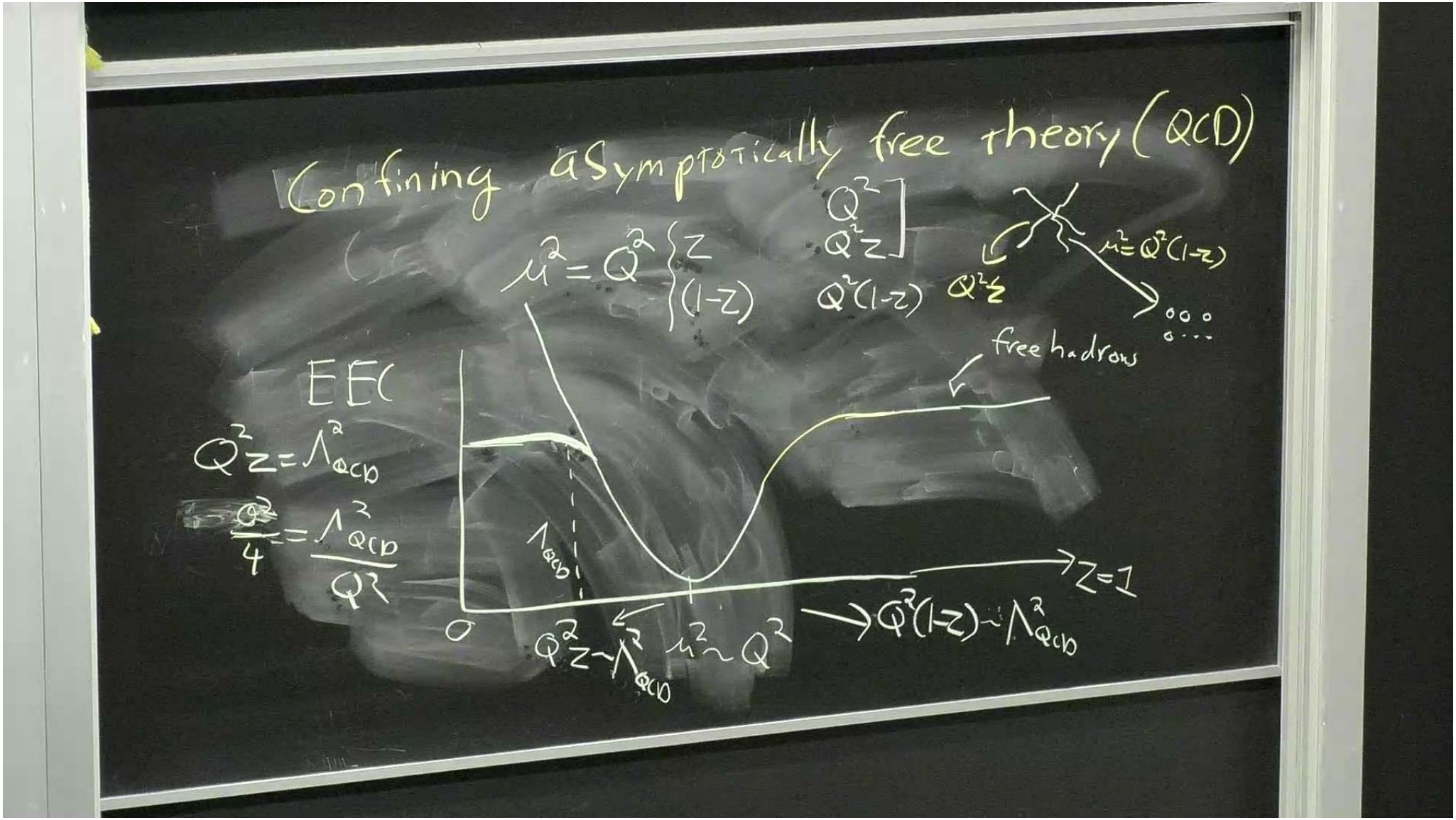
$\sim 1 + \mathcal{O}(\frac{1}{k})$

$\mathcal{N}=4$ at Strong coupling. $\sim 1 + \mathcal{O}(\frac{1}{\lambda})$

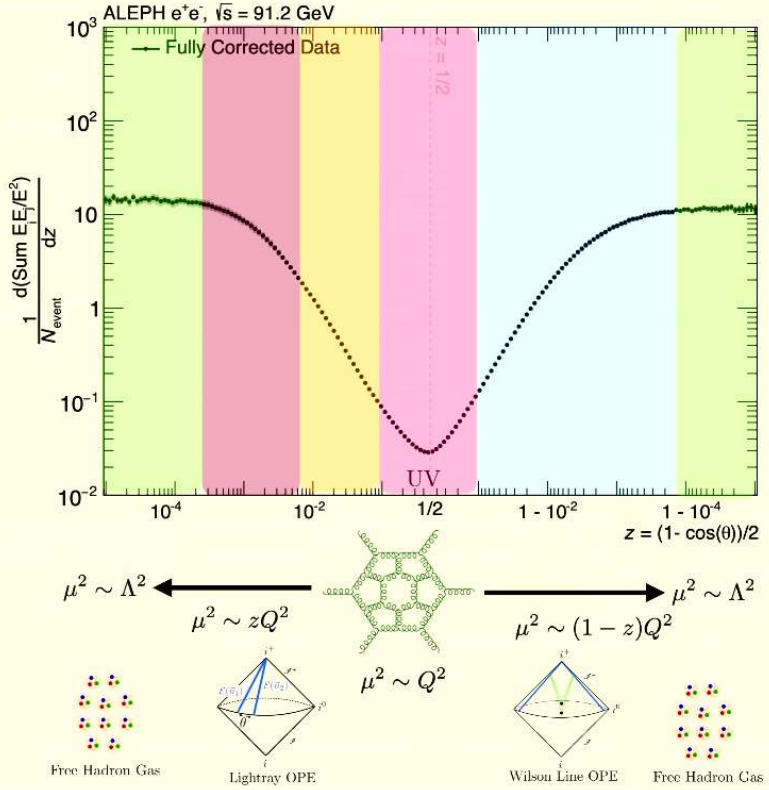
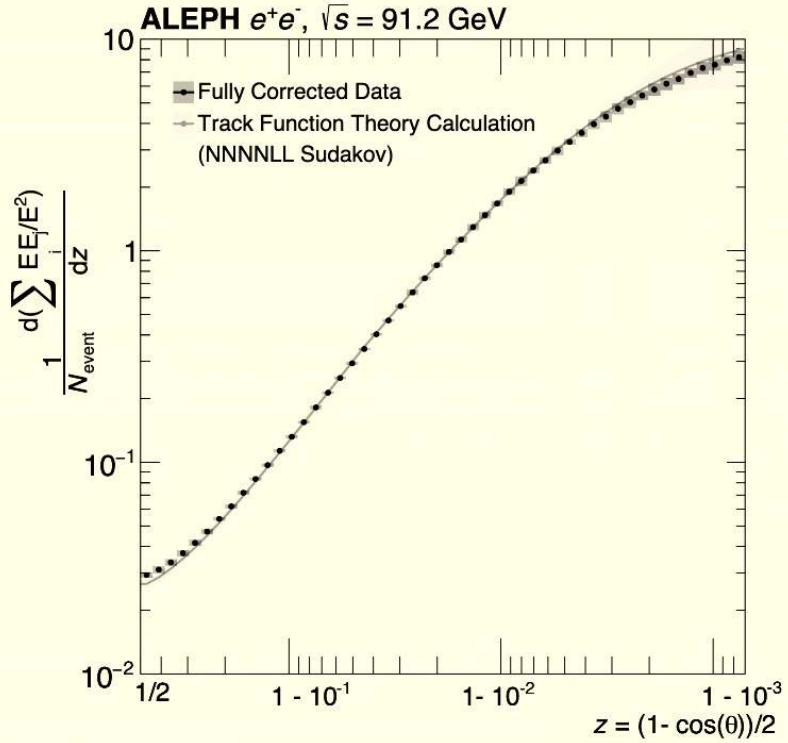
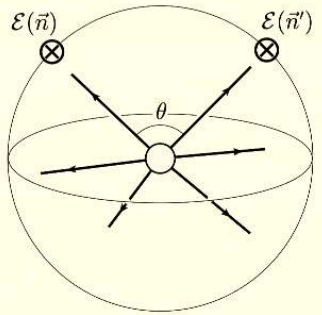

Weakly coupled conformal gauge theory:
(N=4) $a = \frac{g^2 N}{(4\pi^2)}$
$EEC(z) = \frac{a}{4z^2(1-z)} [\log(\frac{1}{1-z}) + a^2 + a^3]_+$
polylogs
elliptic
collinear limit
$\frac{1}{z}$
b2b
$\frac{\log(1-z)}{1-z}$


$z \to 1 = \frac{H(a)}{8(1-z)} \int_0^\infty db\, J_0(b)\, b \exp\left[-\frac{1}{2}\Gamma_{cusp}(a) \log^2\left(\frac{b^2}{1-z}\right) + 2B_g(a) \log\left(\frac{b}{1-z}\right)\right]$


Collinear
(OPE)
$z \to 0$
$\frac{1}{z^{1-\gamma(3)}} + \text{const}$
Twist-2
Spin 3
$z = \frac{1-\cos\theta}{2}$
back-to-back
(Sudakov)
n_2
θ
n_1
Weakly Coupled Conformal theory
z


Confining asymptotically free theory (QCD)
$\mu^2 = Q^2 \begin{cases} z & Q^2 z \\ (1-z) & Q^2(1-z) \end{cases}$
$\mu^2 = Q^2(1-z)$
$Q^2 z$
free hadrons
EEC
$Q^2 z = \Lambda^2_{QCD}$
$\frac{Q^2}{4} = \frac{\Lambda^3_{QCD}}{Q^2}$
Λ_{QCD}
$z=1$
0
$Q^2 z \sim \Lambda^2_{QCD}$
$\mu^2 \sim Q^2$
$Q^2(1-z) \sim \Lambda^2_{QCD}$

ALEPH Data


ALEPH e⁺e⁻, √s = 91.2 GeV
Fully Corrected Data
z = 1/2
UV
$\frac{1}{N_{event}}\frac{d(\text{Sum } EE_i/E^2)}{dz}$
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-4}
10^{-2}
1/2
1 - 10^{-2}
1 - 10^{-4}
$z = (1- \cos(\theta))/2$
$\mu^2 \sim \Lambda^2$
$\mu^2 \sim zQ^2$
$\mu^2 \sim Q^2$
$\mu^2 \sim (1-z)Q^2$
$\mu^2 \sim \Lambda^2$
Free Hadron Gas
Lightray OPE
Wilson Line OPE
Free Hadron Gas

ALEPH Data: Back-to-Back Limit