Title: Quantum gravity and the Born rule

Speakers: Antony Valentini

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Quantum gravity and the Born rule

Antony Valentini
Theoretical Physics
Imperial College London

Three ideas

1. No such thing as the Born rule in quantum gravity

2. Born rule emerges only in semiclassical regime (quantum relaxation on a classical spacetime)

3. Quantum-gravity corrections to the semiclassical regime can make the Born rule unstable

(AV 2021, 2025)

Three ideas

1. No such thing as the Born rule in quantum gravity

Wheeler-DeWitt equation $\hat{\mathcal{H}}\Psi = 0$

$|\Psi[g_{ij}]|^2$ is not a probability

2. Born rule emerges only in semiclassical regime (quantum relaxation on a classical spacetime)

time-dependent Schrödinger equation $i\frac{\partial \psi}{\partial t} = \hat{H}\psi$

$|\psi|^2$ can be a probability (after relaxation)

3. Quantum-gravity corrections to the semiclassical regime can make the Born rule unstable

$\rho = |\psi|^2$ can evolve to $\rho \neq |\psi|^2$

No such thing as the Born rule in quantum gravity

Wheeler-DeWitt equation $\hat{\mathcal{H}}\Psi = 0$

$\left|\Psi[g_{ij}]\right|^2$ is not a probability

Solutions $\Psi[g_{ij}]$ are non-normalisable

Because: Wheeler-DeWitt is like Klein-Gordon

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0 \longleftrightarrow \left(-\frac{\partial^2}{\partial t^2} + \delta^{ij}\frac{\partial^2}{\partial x^i \partial x^j} - m^2\right)\psi = 0$$

(indefinite metric G_{ijkl} on superspace)

$$\int Dg \ \left|\Psi[g_{ij}]\right|^2 = \infty \longleftrightarrow \int d^3x \int_{-\infty}^{+\infty} dt \ \left|\psi(x,t)\right|^2 = \infty$$

A common explanation

Probability $|\Psi[g_{ij},\phi]|^2$ is naïve because:

time is "hidden" in the 3-metric g_{ij}

For example, quantum cosmology: $\Psi(a,\phi)$

↑

"time"

treat scale factor a as "time"

Controversial:

-- recover standard quantum mechanics?

-- what happens to "time" if a expands and recontracts?

-- maybe time as we know it emerges only approximately and in certain conditions (DeWitt, Rovelli, Barbour)

New explanation

In the deep quantum-gravity regime (Planck scale), *there is simply no such thing as the Born rule*

We can talk about probabilities $P[g_{ij}, \phi, t]$

But they are not tied to the Born rule

In fact, *necessarily,* $P[g_{ij}, \phi, t] \neq |\Psi[g_{ij}, \phi]|^2$ *always*

Quantum gravity is in a perpetual state of 'nonequilibrium'

(true physical significance of non-normalisable $|\Psi[g_{ij}, \phi]|^2$)

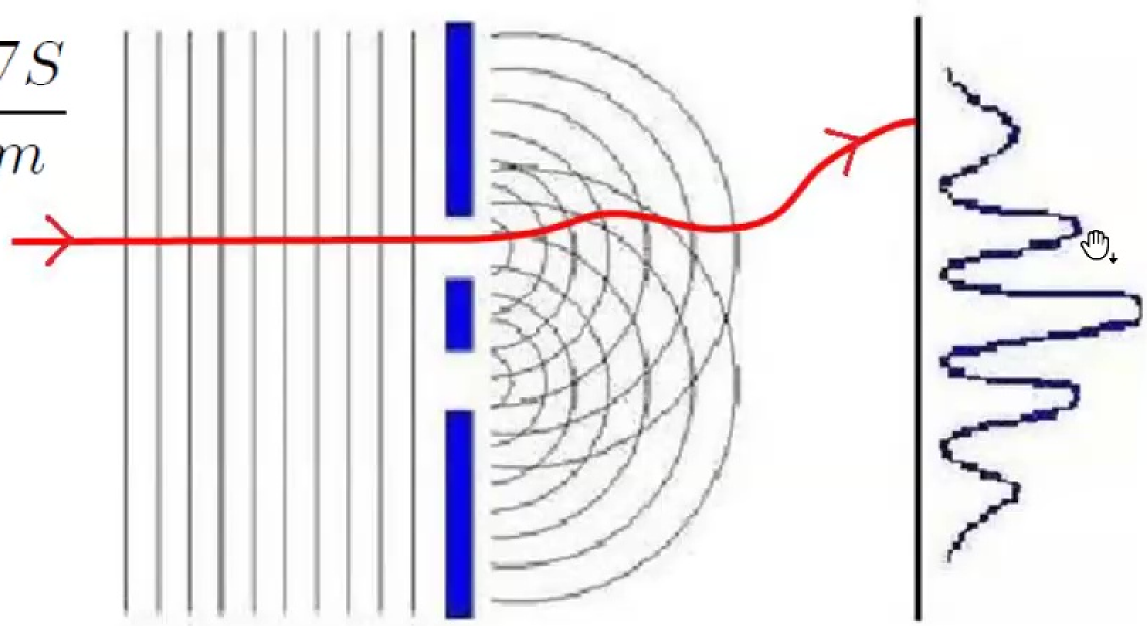
To make sense of this

We need to look at the pilot-wave theory of de Broglie (1927) and Bohm (1952) (interpreted correctly)

The Born rule is not an axiom or law,
but a state of statistical equilibrium
(analogous to classical thermal equilibrium)
(AV 1991, 1992)

Consider the example of the two-slit experiment

$$\frac{d\mathbf{x}}{dt} = \frac{\nabla S}{m}$$



Quantum Equilibrium $\rho(\mathbf{x}, t) = |\psi(\mathbf{x}, t)|^2$

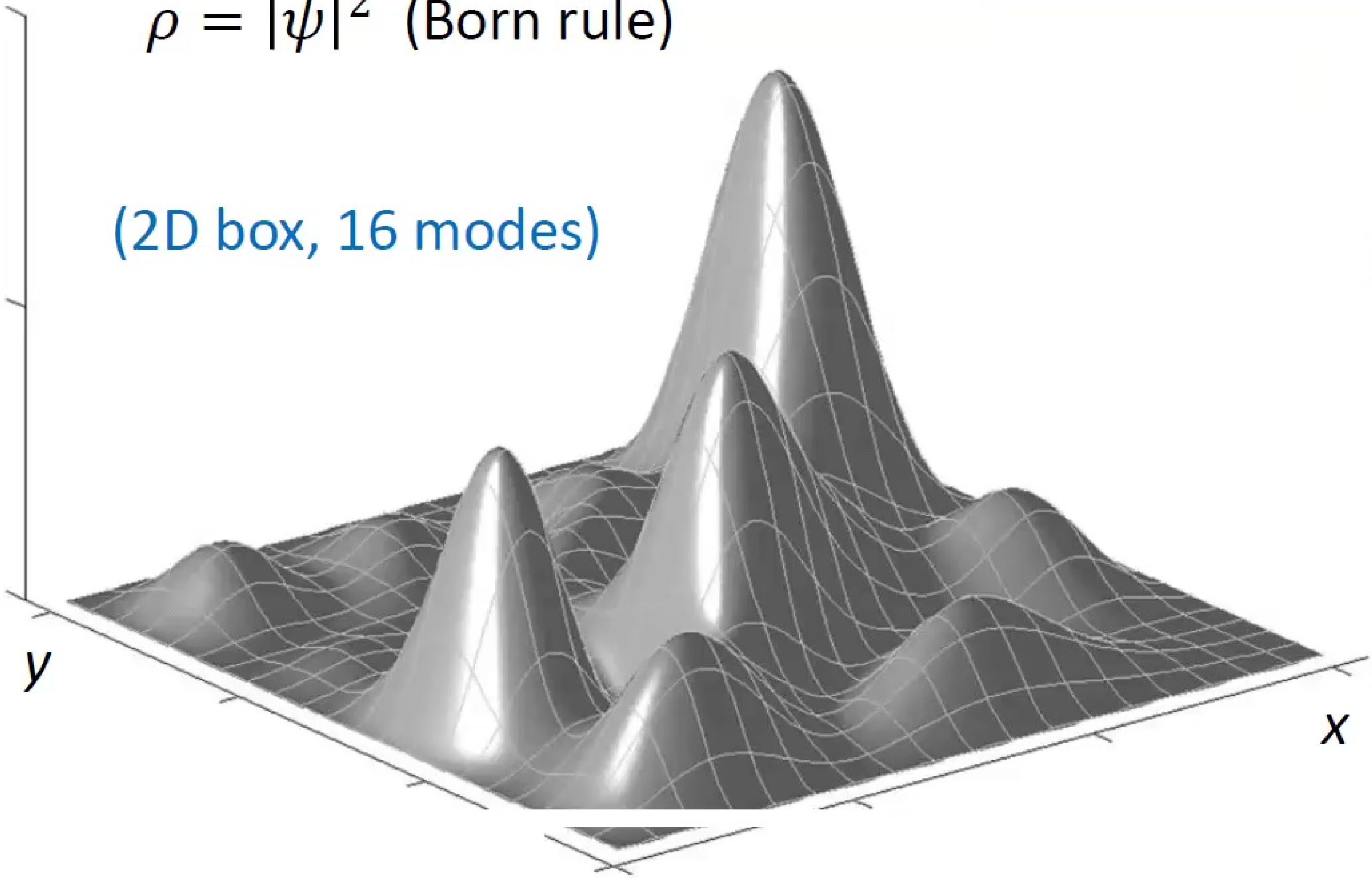
same statistical predictions as quantum mechanics

Quantum Nonequilibrium $\rho(\mathbf{x}, t) \neq |\psi(\mathbf{x}, t)|^2$

statistical *deviations* from quantum mechanics

BUT: *experimentally* we always find the "quantum equilibrium" distribution:

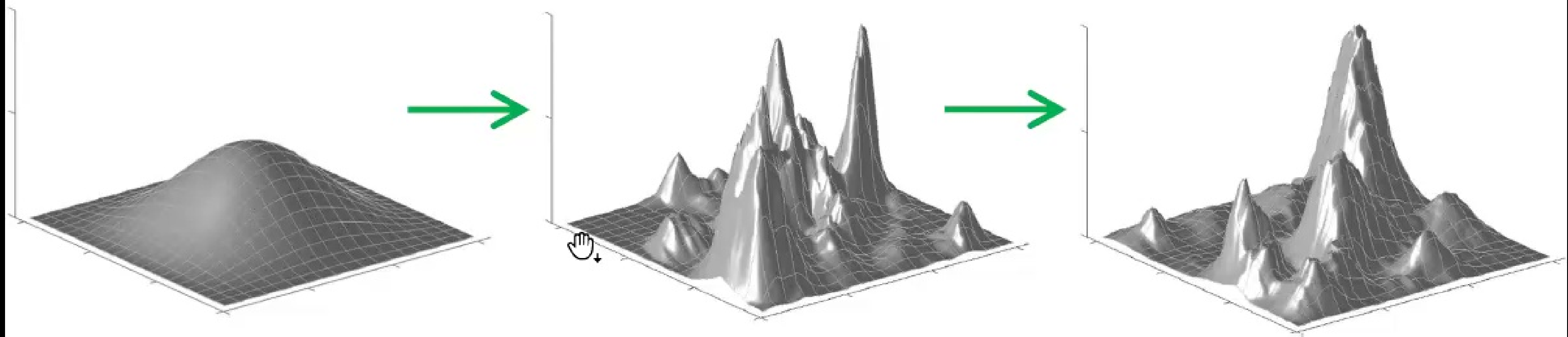
$\rho = |\psi|^2$ (Born rule)



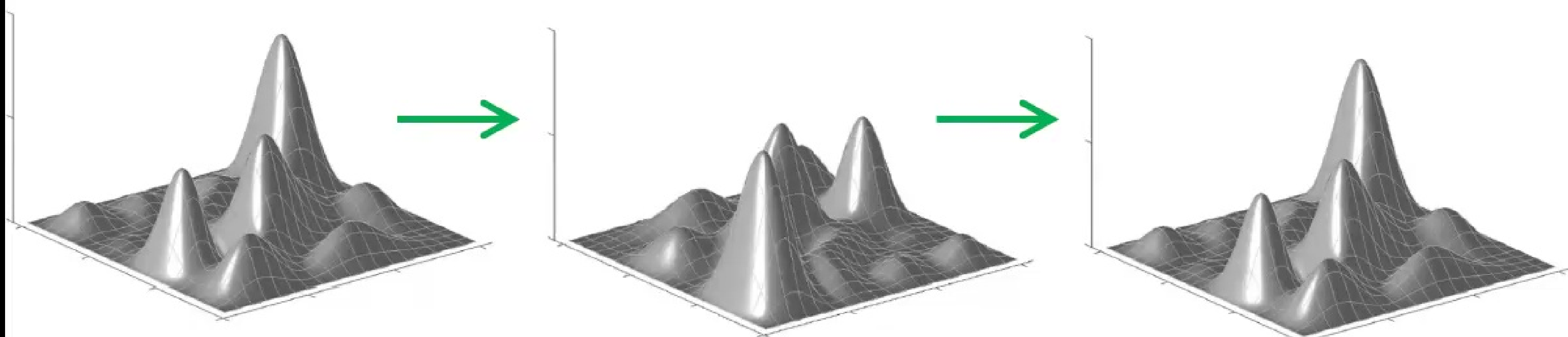
Why?

Quantum relaxation (cf. thermal relaxation)

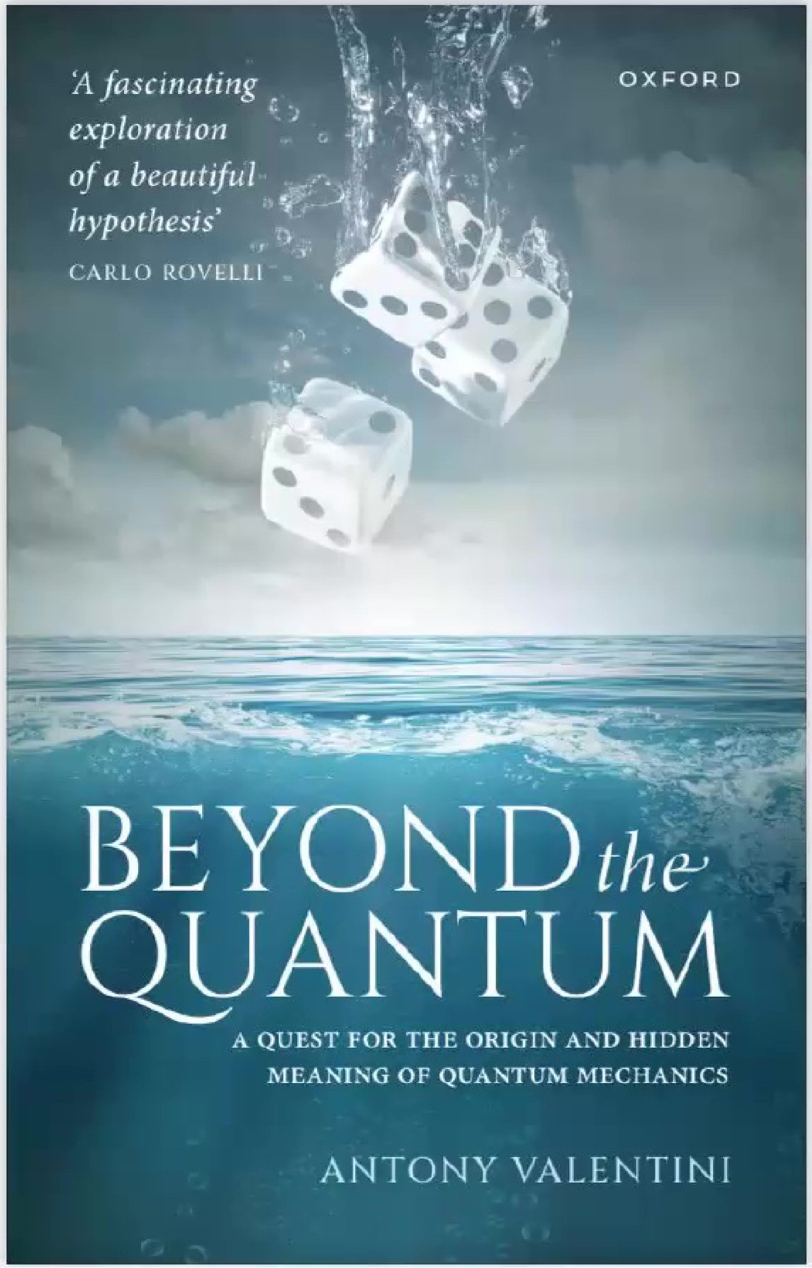
Non-equilibrium ($\rho \neq |\psi|^2$) relaxes to equilibrium



Compare with time evolution of equilibrium $\rho = |\psi|^2$



(Valentini and Westman, Proc. Roy. Soc. A 2005)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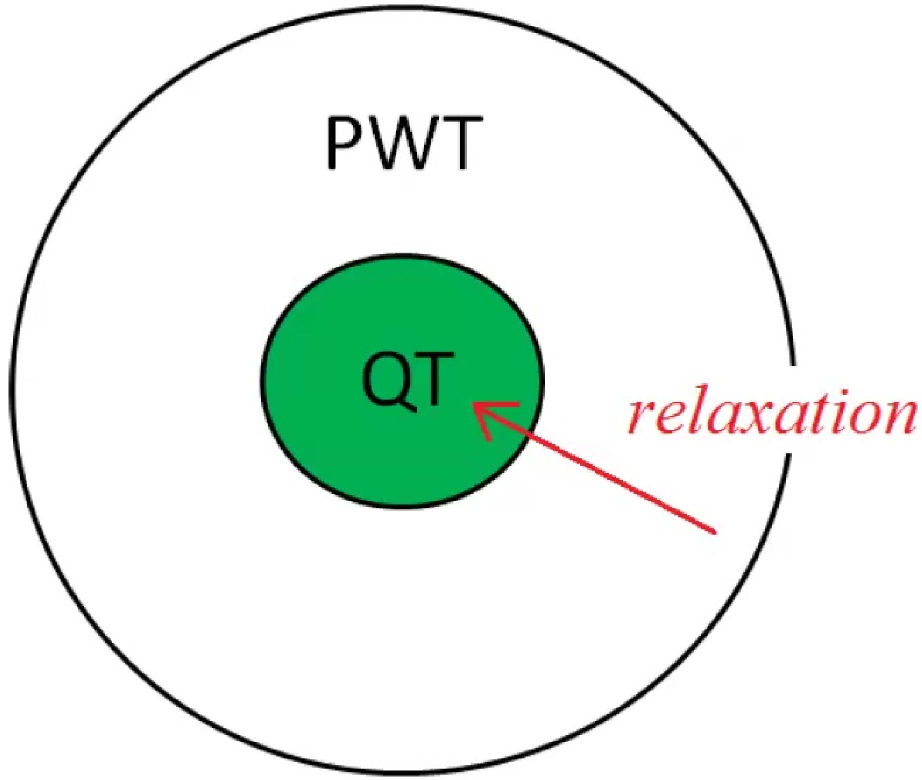
Trapped in quantum equilibrium



Is there a way to escape?

Pilot-wave quantum gravity

(Vink 1992, Horiguchi 1994, Shtanov 1996, Pinto-Neto 2021)

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0 \qquad \Psi = \Psi[g_{ij}]$$

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i\,;\,j} + N_{j\,;\,i} \qquad \Psi = |\Psi|\, e^{iS}$$

(canonical momentum = phase gradient)

-- dynamics of a single system, nonlocal for entangled states

-- effective preferred foliation (Pinto-Neto & Santini 2002)

-- *how can we construct the theory of a quantum equilibrium ensemble?*

(where $|\Psi[g_{ij}]|^2$ is not normalisable)

Theory for a *general* ensemble

$$\left(-G_{ijkl}\frac{\delta^2}{\delta g_{ij}\delta g_{kl}} - g^{1/2}R\right)\Psi = 0$$

(dynamics of a single system,

$$\frac{\partial g_{ij}}{\partial t} = 2NG_{ijkl}\frac{\delta S}{\delta g_{kl}} + N_{i\,;\,j} + N_{j\,;\,i}$$

arbitrary ensemble with distribution P will evolve via

$$\frac{\partial P}{\partial t} + \int d^3x\, \frac{\delta}{\delta g_{ij}}\left(P\frac{\partial g_{ij}}{\partial t}\right) = 0$$

(by construction)

We claim: this theory *has no equilibrium state*

(AV 2021, 2023)

Illustrate with model of quantum cosmology $(\Psi(a,\phi))$

$$\frac{1}{m_{\mathrm{P}}^2}\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial\Psi}{\partial a}\right)-\frac{1}{a^2}\frac{\partial^2\Psi}{\partial\phi^2}+2a^4\mathcal{V}(\phi)\Psi=0$$

Polar decomp. $\Psi=|\Psi|\,e^{iS}$ implies 'pseudo-continuity eqn.'

$$\frac{\partial}{\partial a}\left(a^2\,|\Psi|^2\,\dot a\right)+\frac{\partial}{\partial\phi}\left(a^2\,|\Psi|^2\,\dot\phi\right)=0$$

Non-normalisable 'density' $a^2\,|\Psi|^2$ and de Broglie velocities

$$\dot a=-\frac{1}{m_{\mathrm{P}}^2}\frac{1}{a}\frac{\partial S}{\partial a}\,,\quad \dot\phi=\frac{1}{a^3}\frac{\partial S}{\partial\phi}$$ (can. mom. = phase gradient)

General probability density $P(a,\phi,t)$ evolves by

$$\frac{\partial P}{\partial t}+\frac{\partial}{\partial a}(P\dot a)+\frac{\partial}{\partial\phi}\left(P\dot\phi\right)=0$$

No Born rule in quantum gravity

We *must have* $P \neq a^2 |\Psi|^2$ *always*

(left-hand side is normalisable, right-hand side is not)

Formally: coarse-grained *H*-function

$$\bar{H}(t) = \int \int da d\phi \; \bar{P} \ln(\bar{P} / \overline{a^2 |\Psi|^2})$$

has *no lower bound* (usually bounded below by zero).

'Equilibrium' can never be reached

Quantum gravity is in perpetual nonequilibrium

Quantum gravity is in perpetual nonequilibrium

(AV 2021, 2023)

Alternative approach suggested by Sen (2024):

even for non-normalisable wave functions, quantum relaxation might occur *locally* (in configuration space)

However, numerical simulations show that relaxation does *not* occur, not even locally (Kandhadai & AV 2025)

$$\frac{1}{m_{\mathrm{P}}^{2}}\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial\Psi}{\partial a}\right)-\frac{1}{a^{2}}\frac{\partial^{2}\Psi}{\partial\phi^{2}}+2a^{4}\mathcal{V}(\phi)\Psi=0$$

$$\frac{\partial P}{\partial t}+\frac{\partial}{\partial a}(P\dot{a})+\frac{\partial}{\partial\phi}\left(P\dot{\phi}\right)=0$$

(nodes are static, no relaxation even locally)

Born rule emerges *only* in semiclassical regime (quantum relaxation on a classical spacetime)

time-dependent Schrödinger equation $i\frac{\partial \psi}{\partial t} = \hat{H}\psi$

$|\psi|^2$ can be a probability (after relaxation)

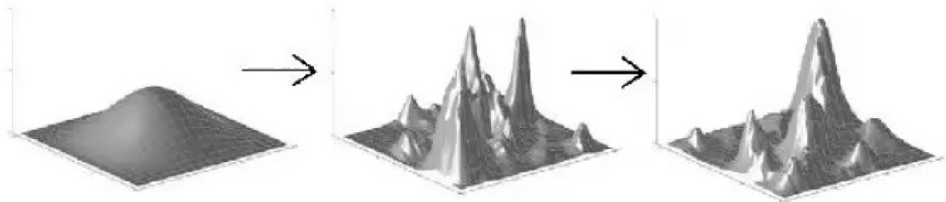
How? …

Semiclassical quantum relaxation

Once we have an effective Schrödinger equation

$$i\frac{\partial\psi[\phi,t]}{\partial t} = \hat{H}_{\text{eff}}\psi[\phi,t]$$

conventional quantum relaxation can take place.



Semiclassical universe begins out of equilibrium,

relaxes to the Born rule only afterwards (cf. AV 1991)

Gravity changes the game?

Tiny quantum-gravitational corrections to the Schrödinger equation can make the Born rule unstable ($\rho = |\psi|^2$ evolves to $\rho \neq |\psi|^2$).

Higher order semiclassical expansion: find small *non-Hermitian terms* in the effective Hamiltonian.

Inconsistent with standard QM (breaks unitarity).

Consistent with pilot-wave theory: non-Hermitian terms generate a small instability of the Born rule

(AV 2021, 2023)

Kiefer and Singh 1991

Semiclassical expansion:

$$\Psi = \exp i\left(\mu S_0 + S_1 + \mu^{-1} S_2 + \cdots\right) \qquad \mu = c^2/32\pi G$$

Corrected Schrödinger equation:

$$i\frac{\partial \psi^{(1)}}{\partial t} = \int d^3x\, N\left(\hat{\mathcal{H}}_\phi + \hat{\mathcal{H}}_a + i\hat{\mathcal{H}}_b\right)\psi^{(1)}$$

$$\hat{\mathcal{H}}_a = \frac{1}{8\mu}\frac{1}{\sqrt{g}R}\hat{\mathcal{H}}_\phi^2 \qquad \hat{\mathcal{H}}_b = \frac{1}{8\mu}\frac{\delta}{\delta\tau}\left(\frac{\hat{\mathcal{H}}_\phi}{\sqrt{g}R}\right)$$

$$\delta/\delta\tau = \dot{g}_{ij}\delta/\delta g_{ij}$$

Gravity changes the game?

Tiny quantum-gravitational corrections to the Schrödinger equation can make the Born rule unstable ($\rho = |\psi|^2$ evolves to $\rho \neq |\psi|^2$).

Higher order semiclassical expansion: find small *non-Hermitian terms* in the effective Hamiltonian.

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Non-Hermitian pilot-wave theory

Semiclassical expansion of Wheeler-DeWitt equation:

$$i\frac{\partial\psi}{\partial t} = (\hat{H}_1 + i\hat{H}_2)\psi$$

Semiclassical expansion of de Broglie equation:

$$v = \frac{j_1}{|\psi|^2} \quad \text{(standard current } j_1 \text{ associated with } \hat{H}_1\text{)}$$

Thus $\frac{\partial\rho}{\partial t} + \partial_q \cdot (\rho v) = 0$ *but* $\frac{\partial|\psi|^2}{\partial t} + \partial_q \cdot (|\psi|^2 v) = s$

with effective 'source' $s = 2\,\mathrm{Re}\left(\psi^* \hat{H}_2 \psi\right)$

Mismatch between the two continuity equations

$\rho = |\psi|^2$ evolves to $\rho \neq |\psi|^2$ on a timescale $\tau_{\text{noneq}} \sim \frac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|}$

Evaporating black holes

Field mode Hamiltonian $\hat{H}_{\mathbf{k}}$ has a non-Hermitian correction $i\hat{H}_2$ with

$$\hat{H}_2 \simeq -\frac{1}{12}\kappa\left(\frac{m_{\rm P}}{M}\right)^4 \hat{H}_{\mathbf{k}}$$

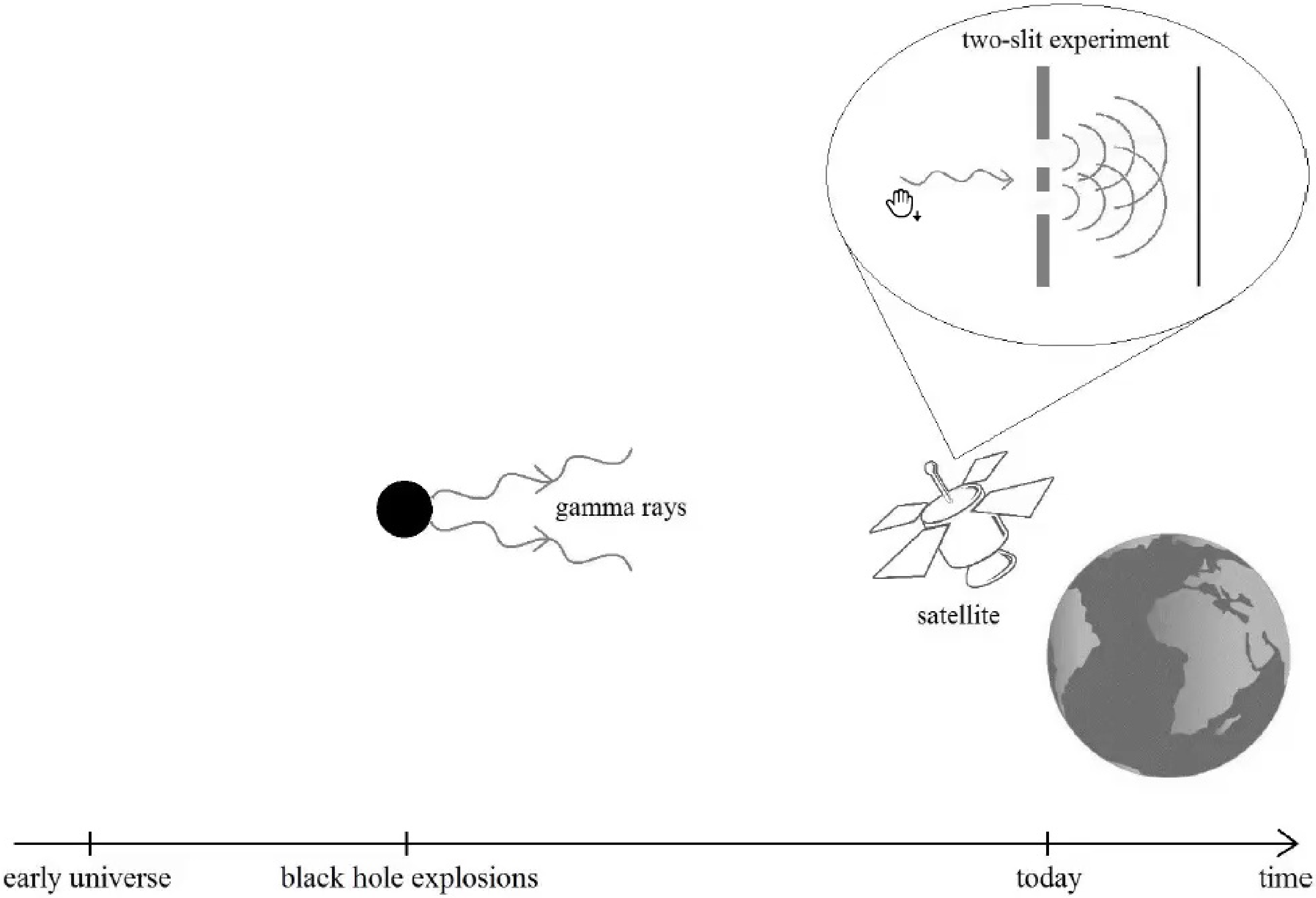
(Kiefer, Müller and Singh 1994).

Significant when mass *M* approaches Planck mass.

$$\tau_{\rm noneq} \sim \frac{1}{2\left|\left\langle \hat{H}_2 \right\rangle\right|} \qquad \tau_{\rm noneq} \sim \frac{48\pi}{\kappa} t_{\rm P}\left(\frac{M}{m_{\rm P}}\right)^5$$

Final burst of Hawking radiation breaks the Born rule

Exploding primordial black holes



QUICK^3 satellite mission (launch in 2025?) has an interferometer to test the Born rule in space

Conclusions

-- quantum gravity is fundamentally a nonequilibrium theory (no Born rule)

-- in the semiclassical regime we find relaxation to the usual Born rule

-- corrections to the semiclassical regime yield a small instability of the Born rule

-- latter effects are very small (except in final burst of an evaporating black hole)

DEDICATED TO LEE SMOLIN

May his adventures never end