

Title: Causality and flow of time in realist quantum models

Speakers: Eliahu Cohen

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

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Abstract:

In a series of papers [Phys. Rev. D 102, 124027 (2020)], [Phys. Rev. D 102, 124028 (2020)] I had the pleasure to explore with Cortês, Elitzur and Smolin realist formulations of quantum theory which obey either retrocausality or disordered causality. After briefly presenting these works, I will attempt to extend them by addressing time as a quantum observable using the Page-Wootters formalism. Within this framework, the clock is treated as a quantum subsystem, endowing the remainder of the system with a notion of time through entanglement. I will show the fruitfulness of this approach by deriving new time-energy uncertainty relations [Quantum 6, 683 (2022)] and by elucidating aspects of dynamical nonlocality [Phys. Rev. A 105, 042207 (2022)]. Furthermore, I will demonstrate that in the post-Newtonian regime, perspective-dependent non-unitarity and a distinct form of time-asymmetry naturally emerge when the clocks experience acceleration or gravitational effects [Commun. Phys. 5, 298 (2022)]. Finally, I will discuss the more recent works viewing all the above as spatiotemporal quantum reference frames [Phys. Rev. A 109, 032205 (2024)], [arXiv:2503.20090].

Causality and flow of time in realist quantum models

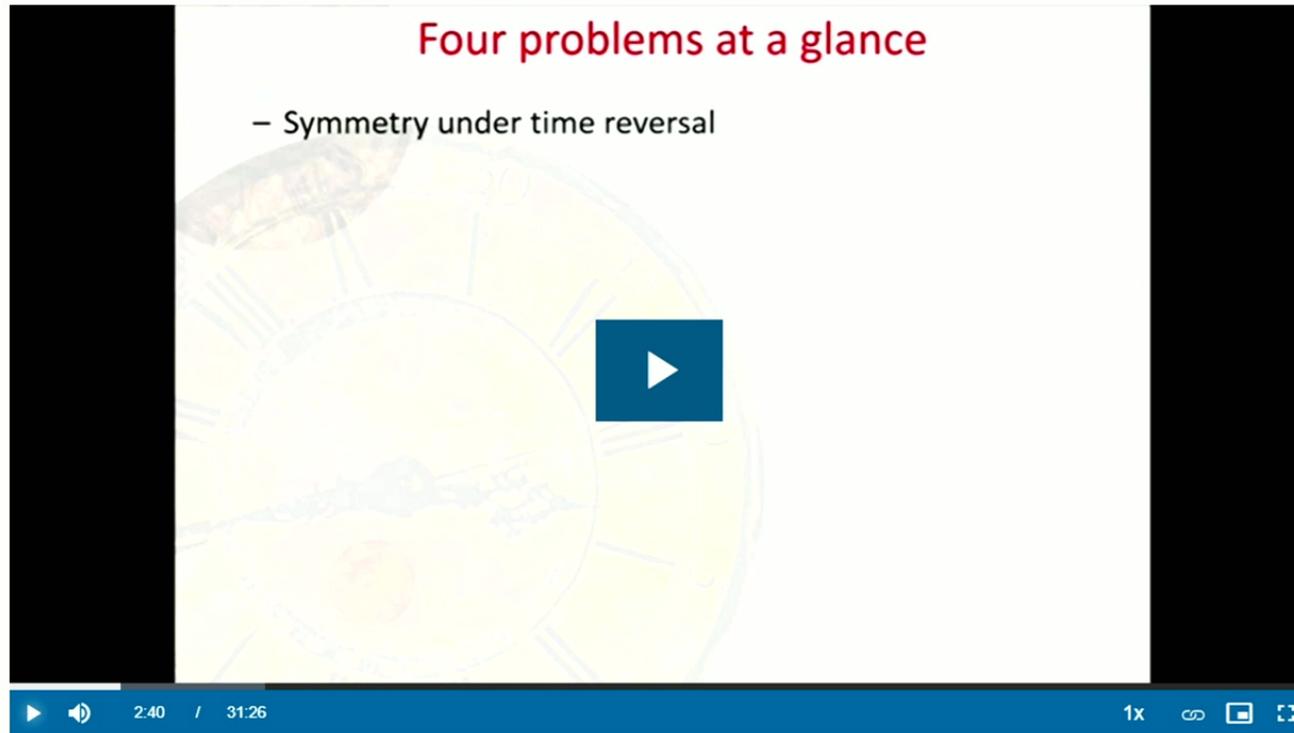
Students, postdocs (especially Ismael and Michael), colleagues and:

Eliahu Cohen, Faculty of Engineering and the Institute of Nanotechnology and Advanced Materials, Bar-Ilan University



Lee's Fest: Quantum Gravity and the Nature of Time, June 2025

Last time at PI with Lee



A Final Boundary Condition: Several Implications for the Universe

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 Eliahu Cohen University of Bristol

June 23, 2016

Talk number: PIRSA:16060065

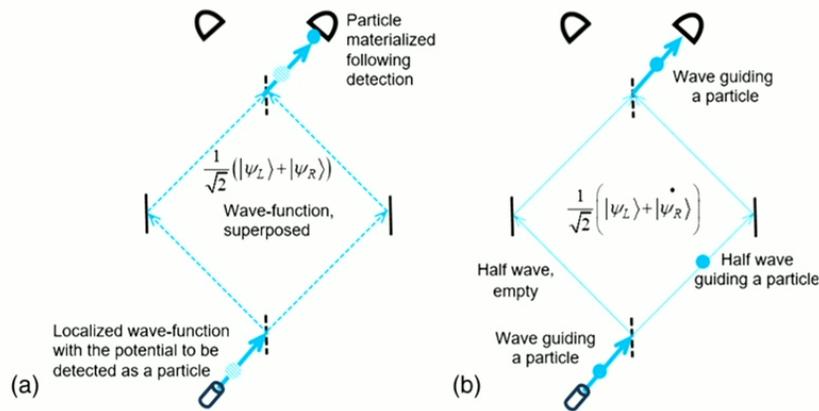
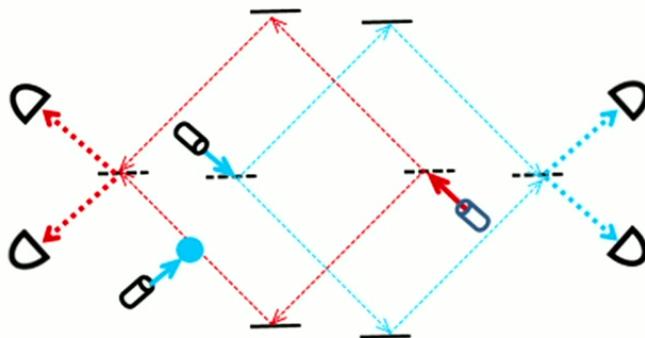
DOI: [10.48660/16060065](https://doi.org/10.48660/16060065)

Nevertheless leading to:



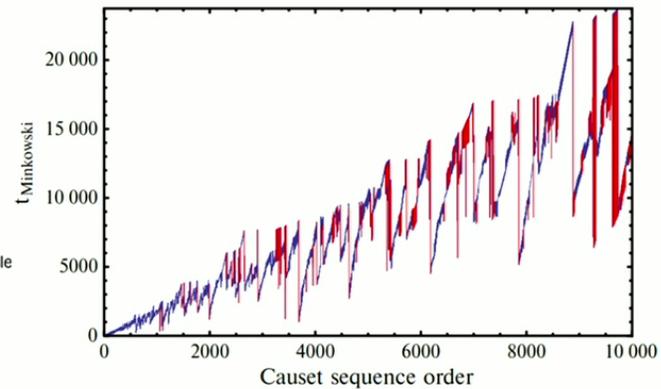
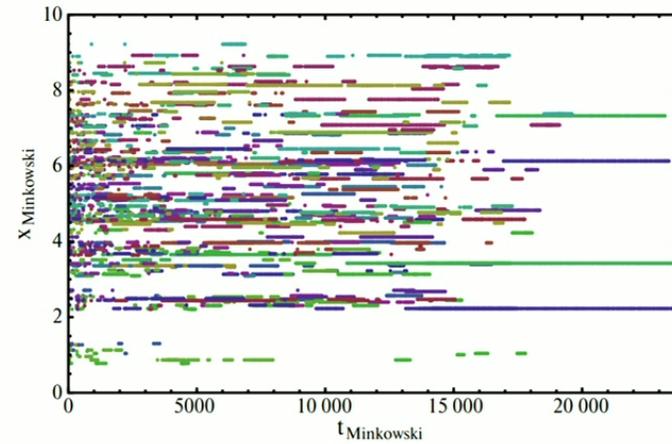
Realism and causality. I. Pilot wave and retrocausal models as possible facilitators

Eliahu Cohen¹, Marina Cortês^{2,3}, Avshalom Elitzur^{4,5} and Lee Smolin³



Realism and causality. II. Retrocausality in energetic causal sets

Eliahu Cohen,¹ Marina Cortês^{2,3}, Avshalom C. Elitzur,^{4,5} and Lee Smolin³





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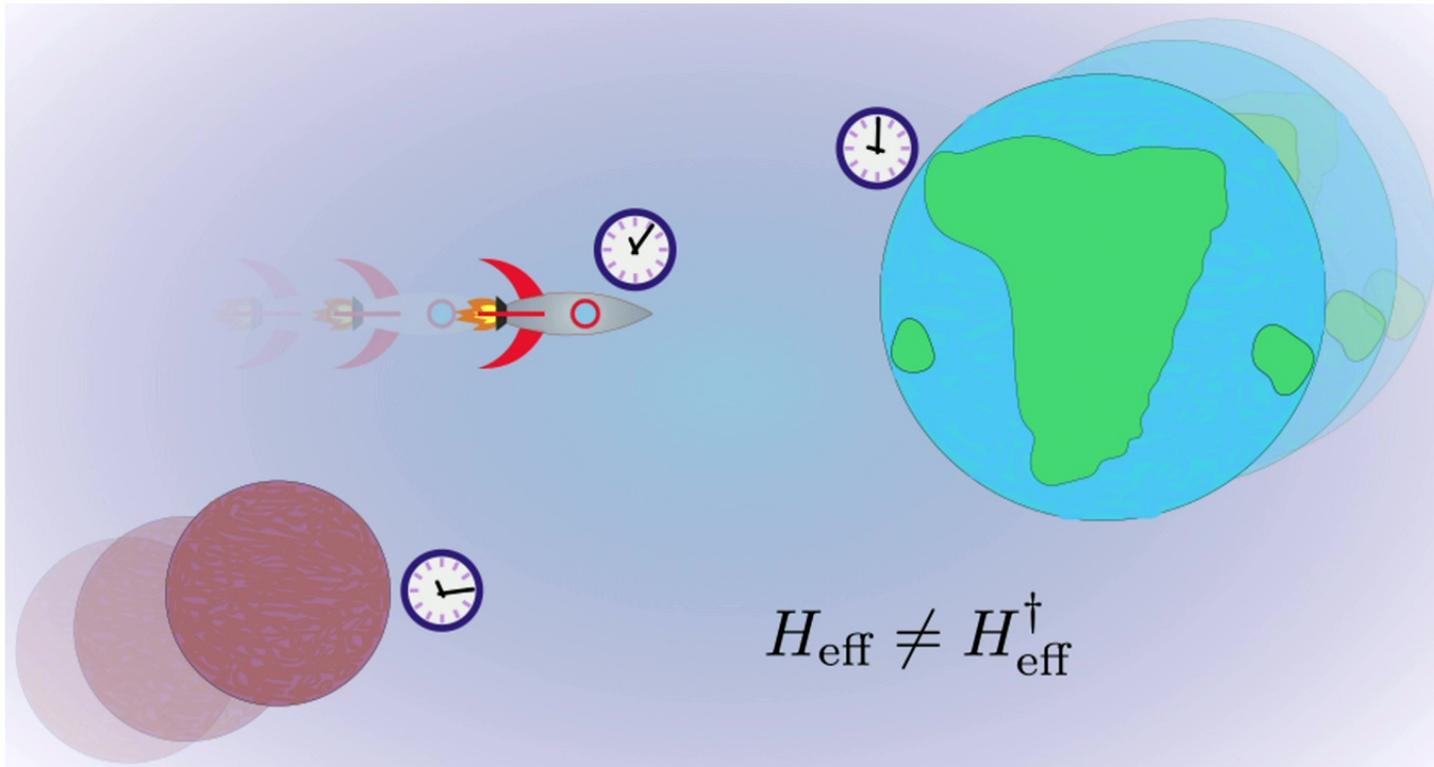
There is a price to pay for realism in entangled systems

**When spacetime is fixed:
Retrocausality**

**When Minkowski spacetime is emergent:
Disordered causality**

But what about the flow of time / “becoming”?

Today



Thanks to entanglement with a “clock”, a globally static system may have some inner (often non-unitary) dynamics from the point of view of an internal clock.

This is a relational description, with time actually flowing and some possible hints for quantum gravity although we only have one post-Newtonian correction

Partially Based on:



- I. L. P., A. C. L., and E. C., Flow of time during energy measurements and the resulting time-energy uncertainty relations, *Quantum* 6, 683 (2022).
- I. L. P., A. T., B.P., E.C., Y.A., Noninertial quantum clock frames lead to non-Hermitian dynamics, *Commun. Phys.* 5, 298 (2023).
- I. L. P., M. N., and E. C., Dynamical nonlocality in quantum time via modular operators, *Phys. Rev. A* 105, 042207 (2022).
- M.S., I. L. P., and E. C., Non-relativistic spatiotemporal quantum reference frames, *Phys. Rev. A* 109, 032205 (2024).
- M.S., A.C. and E.C., Relativity of Quantum Correlations: Invariant Quantities and Frame-Dependent Measures, [arXiv:2503.20090](https://arxiv.org/abs/2503.20090).

Infrastructure in:

- Y.A., E. C., F.C. et al., Finally making sense of the double-slit experiment, *PNAS* 114, 6480 (2017).
- A.C., E. C., Relativistic independence bounds nonlocality, *Sci. Adv.* 5, eaav8370 (2019)/

Problem of time



Apparent **conflict** between the concept of time in **quantum theory** and in diffeomorphism-invariant theories like **general relativity**

Textbook Quantum Mechanics

- Time is a background parameter (external to, and unaffected by, the system)
- Time is universal and absolute

General relativity

- Time is intrinsic, one of the four dimensions, affected by mass, energy and momentum
- Time is dynamical and relative

**Wheeler-DeWitt Equation
in quantum gravity:
(Hamiltonian constraint)**

$$H|\Psi\rangle = 0$$



The Page and Wootters timeless framework



$$|\psi\rangle\rangle = \int dt_A |t_A\rangle \otimes |\psi(t_A)\rangle$$

Wheeler-DeWitt equation

$$H_T |\Psi\rangle\rangle = 0$$

Page and Wootters

$$[H_A + H_R + H_{int}(T_A)] |\Psi\rangle\rangle = 0$$

$$i\hbar \frac{\partial}{\partial t_A} |\psi(t_A)\rangle = [H_R + H_{int}(t_A)] |\psi(t_A)\rangle$$

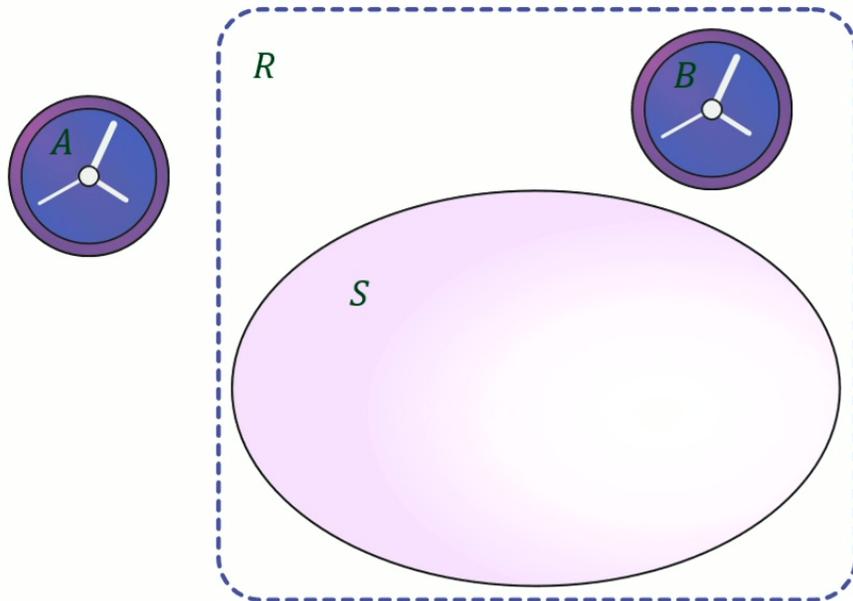


$$H_{eff}^A = H_R + H_{int}(t_A)$$

D. N. P. and W. K. W., Phys. Rev. D 27, 2885 (1983)



The Page and Wootters timeless framework



Time arises from entanglement and correlations

D. N. P. and W. K. W., Phys. Rev. D 27, 2885 (1983)

E. C.-R., F. G., A. B., and Č. B., Nat. Commun. 11, 2672 (2020)

$$|\psi\rangle\rangle = \int dt_A |t_A\rangle \otimes |\psi(t_A)\rangle$$

Wheeler-DeWitt equation

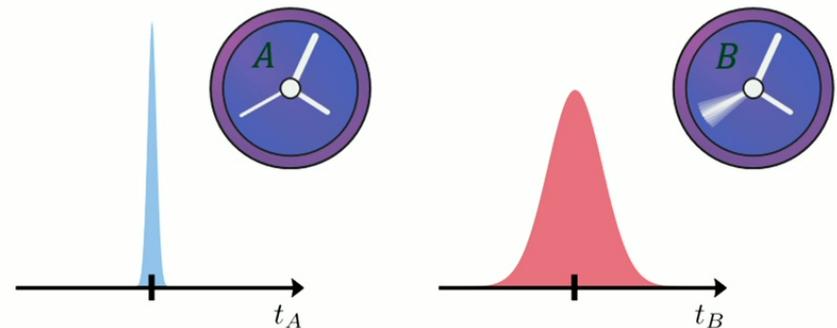
$$H_T |\Psi\rangle\rangle = 0$$

Page and Wootters

$$[H_A + H_B + H_S + H_{int}(T_A, T_B)] |\Psi\rangle\rangle = 0$$

$$H_{eff}^A = H_B + H_S + H_{int}(t_A, T_B)$$

$$H_{eff}^B = H_A + H_S + H_{int}(T_A, t_B)$$



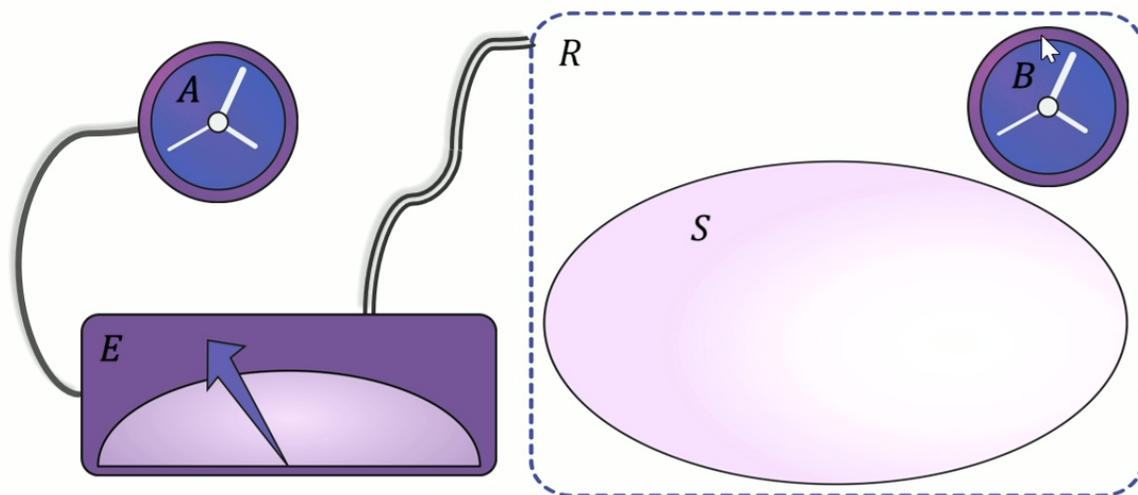


Formal details

- The clock should have an observable T_A associated with its time
- It obeys $|t_0 + t_A\rangle = e^{-iH_A t_A/\hbar} |t_0\rangle$
- The wavefunction obeys $|\psi\rangle\rangle = \int dt_A |t_A\rangle \otimes |\psi(t_A)\rangle$
- T_A needn't be a self-adjoint operator, canonically conjugated to H_A
- It can be constructed as a POVM
- However, it is common to assume an *ideal* clock:



Energy measurement



Non-Hermitian!

Carried out by external system

$$H_R = H_B + H_S + H_{int}(T_B)$$

$$H_{VN} = g(T_A)H_R P_E$$

$$H_T = H_A + H_R + H_{VN}$$

$$H_{eff}^A = H_R + g(t_A)H_R P_E$$

$$\frac{d}{dt_A} T_B = I + g(t_A)P_E$$

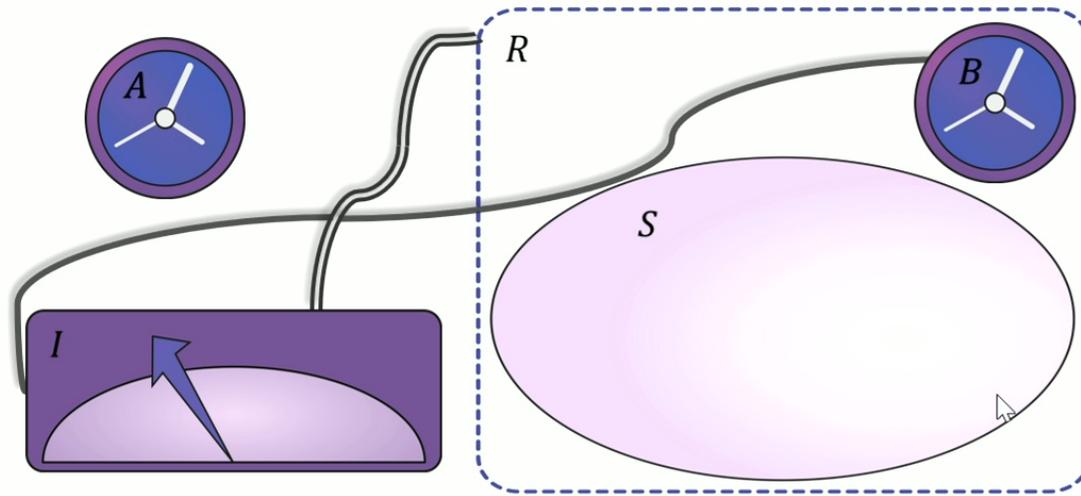
$$\langle T_B(\tau) - T_B(0) \rangle \geq K \langle P_E \rangle$$

$$H_{eff}^B = [I + g(T_A)P_E]^{-1} H_A + H_S + H_{int}(t_B)$$





Energy measurement



Carried out by internal system

Carried out by external system

$$H_{eff}^A = H_R + g(t_A)H_R P_E \quad H_{eff}^B = [I + g(T_A)P_E]^{-1}H_A + H_S + H_{int}(t_B)$$

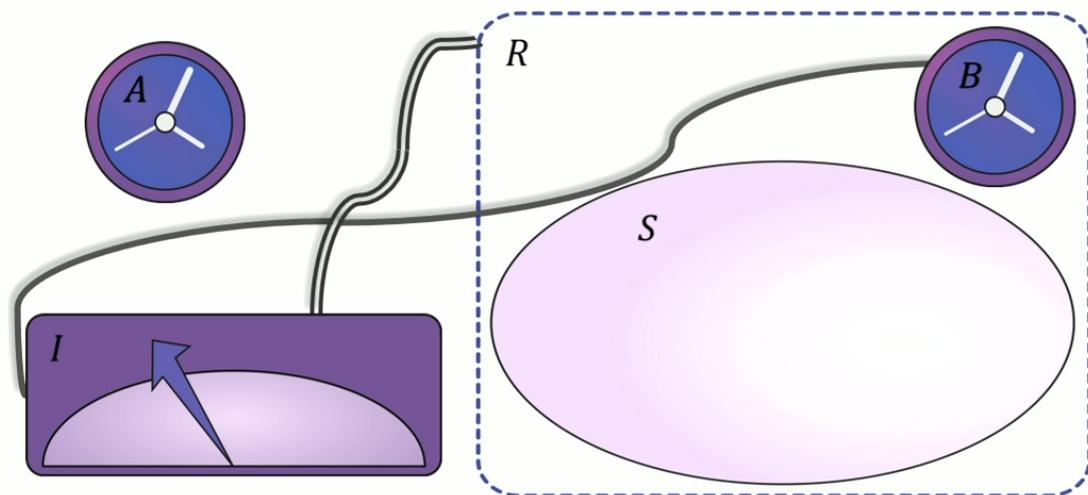
$$\frac{d}{dt_A} T_B = I + g(t_A)P_E \quad \frac{d}{dt_B} T_A = [I + g(T_A)P_E]^{-1}$$

$$\langle T_B(\tau) - T_B(0) \rangle \geq K \langle P_E \rangle$$





Energy measurement



Carried out by internal system

$$H_R = H_B + H_S + H_{int}(T_B)$$

$$H_{VN} = \frac{1}{2} [g(T_B)H_R + H_R g(T_B)] P_I$$

$$H_T = H_A + H_R + H_{VN}$$

$$H_{eff}^A = H_R + \frac{1}{2} [g(T_B)H_R + H_R g(T_B)] P_I$$

$$\frac{d}{dt_A} T_B = I + g(T_B) P_I$$

$$H_{eff}^B = [I + g(t_B) P_I]^{-1} \left[H_A - \frac{i\hbar}{2} g'(t_B) P_I \right] + H_S + H_{int}(t_B)$$

Carried out by external system

$$H_{eff}^A = H_R + g(t_A) H_R P_E$$

$$H_{eff}^B = [I + g(T_A) P_E]^{-1} H_A + H_S + H_{int}(T_A)$$

$$\frac{d}{dt_A} T_B = I + g(t_A) P_E$$

$$\frac{d}{dt_B} T_A = [I + g(T_A) P_E]^{-1}$$

Non-Hermitian!

$$\langle T_B(\tau) - T_B(0) \rangle \geq K \langle P_E \rangle$$



Non-unitarity in non-inertial systems

(with Ismael de Paiva,
Amit Te'eni, Bar Peled
and Yakir Aharonov)



Non-unitarity from acceleration

- Hamiltonian of a free particle: H
- How is it modified if the particle starts accelerating?

$$H \mapsto H + V(x) \quad V(x) = -m \int_{x_0}^x a(x') dx' = mf(x)$$

$$H \mapsto H + mf(X)$$

- Relativistic correction: $m \mapsto m + H/c^2$
- Then: $H = H_A + H_M + H_A f(X_M)$



Non-unitarity from acceleration

- Hamiltonian of a free particle: H
- How is it modified if the particle starts accelerating?

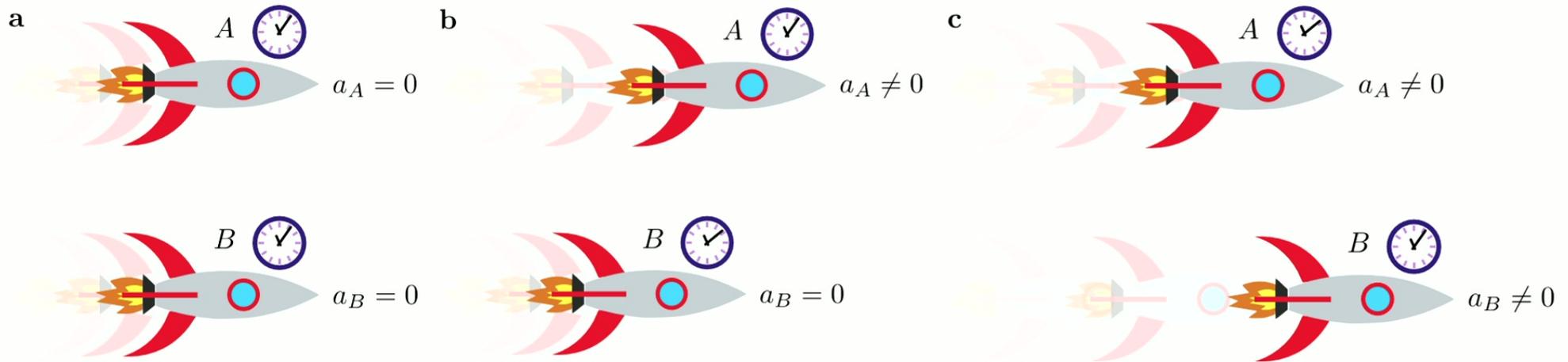
$$H \mapsto H + V(x) \quad V(x) = -m \int_{x_0}^x a(x') dx' = mf(x)$$

$$H \mapsto H + mf(X)$$

- Relativistic correction: $m \mapsto m + H/c^2$ Moreover, $\frac{d}{dt_B} T_A = \frac{i}{\hbar} [H_{\text{eff}}^B, T_A] = \frac{i}{\hbar} [I + f(X_M)] [H_A, T_A]$
- Then: $H = H_A + H_M + H_A f(X_M)$ is remarkably close to: $d\tau \approx \left(1 + \frac{V}{c^2}\right) dt$
- Leading to: $i\hbar \frac{\partial}{\partial t_A} |\psi(t_A)\rangle = H_{\text{eff}}^A |\psi(t_A)\rangle$, $H_{\text{eff}}^A \equiv [I + f(X_M)]^{-1} H_M$

Non-Hermitian!

Non-unitarity from acceleration

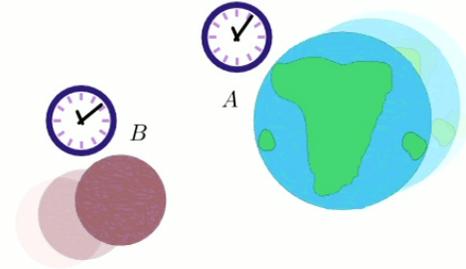




Gravitationally interacting clocks

$$V(x_N - x_M) = -G \frac{m_M m_N}{|x_N - x_M|}$$

$$V(X_N) = -GH_A H_B |X_N|^{-1} / 2c^4$$



$$H = [I + f(X_N, H_B)]H_A + H_B + H_M + H_N \quad f(X_N, H_B) = -GH_B |X_N|^{-1} / 2c^4$$

$$H_{eff}^A = [I + f(X_N, H_B)]^{-1} (H_B + H_M + H_N) \quad \text{Non-Hermitian!}$$

Is this a feature of the framework or a general property shared also by quantum gravity?



Spatiotemporal reference frames

(with Michael Suleymanov)

Spatial Quantum Reference Frames (SQRF)

Hilbert space: $\mathcal{H}^{\text{kin}} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$

Translation invariance constraint

$$\hat{P}_T |\Psi\rangle^{\text{phys}} = 0 \quad \hat{P}_T = \hat{p}_A + \hat{p}_B + \hat{p}_C$$

A

B

$$|\Psi\rangle^{\text{kin}} = \int dp_A dp_B dp_C |p_A, p_B, p_C\rangle \Psi^{\text{kin}}(p_A, p_B, p_C)$$

C

$$|\Psi\rangle^{\text{phys}} = \delta(\hat{P}_T) |\Psi\rangle^{\text{kin}} = \int dp_A dp_B dp_C |p_A, p_B, p_C\rangle \delta(P_T) \Psi^{\text{kin}}(p_A, p_B, p_C)$$

$$\int d(p)_{\bar{A}} (\dots)_{\bar{A}} = \int d(p)_{\bar{B}} (\dots)_{\bar{B}} = \int d(p)_{\bar{C}} (\dots)_{\bar{C}} \quad d(p)_{\bar{A}} = dp_B dp_C$$

Spatiotemporal Quantum Reference Frames (STQRFs)

$$\mathcal{H}^{kin} = \otimes_{I=A,B,C} (\mathcal{H}_{C_I} \otimes \mathcal{H}_{S_I})$$

Translation invariance: $\hat{P}_T |\Psi\rangle^{phys} = 0$

Wheeler DeWitt: $\hat{H}_T |\Psi\rangle^{phys} = 0$

$$\hat{H}_T = \sum_I \frac{\hat{p}_I^2}{2m_I} + \sum_I \hat{\omega}_I$$

$$|\Psi\rangle^{kin} = \int d(\omega, p) |\omega, p\rangle \Psi^{kin}(\omega, p)$$

$$|\Psi\rangle^{phys} = \delta(\hat{P}_T) \delta(\hat{H}_T) |\Psi\rangle^{kin}$$



x_A, t_A



x_B, t_B



x_C, t_C



Transformations of the variance and covariance (momentum constraint, no interactions)

$$\sigma^2(\hat{p}_I)_J = -\text{cov}(\hat{p}_I, \hat{p}_J)_M - \sum_{L \in \mathcal{I} \setminus \{I, J\}} \text{cov}(\hat{p}_I, \hat{p}_L)_J$$

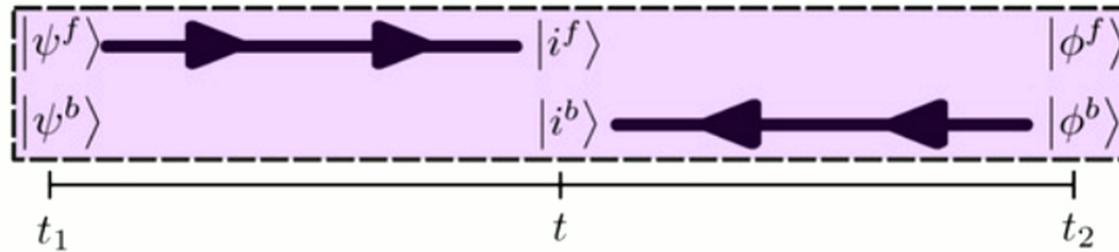
$$\begin{aligned} \sigma^2(\hat{x}_I)_J(t_J) &= \sigma^2(\hat{x}_I)_J(t_J = 0) - t_J \widetilde{\text{cov}}(\hat{x}_I, \hat{v}_{I|J})_J(t_J = 0) \\ &\quad + t_J^2 \sigma^2(\hat{v}_{I|J})_J(t_J = 0), \end{aligned}$$

$$\begin{aligned} \sigma^2(\hat{x}_I)_J(t_J) &= \sigma^2(\hat{x}_I)_J(t_J = 0) - t_J \widetilde{\text{cov}}(\hat{x}_I, \hat{v}_{I|J})_J(t_J = 0) \\ &\quad + t_J^2 \sigma^2(\hat{v}_{I|J})_J(t_J = 0), \end{aligned}$$

But for any quadratic Hamiltonian: $\det \left(\mathbf{cov}_{BC\dots N|A}^{(x,p)}(t_A) \right) = \det \left(\mathbf{cov}_{AC\dots N|B}^{(x,p)}(t_B) \right)$

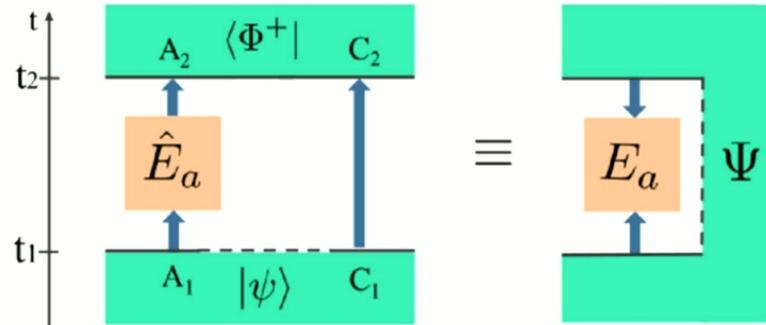
Combining everything?

With Michael Ridley



$$\frac{\partial}{\partial t_F} |t_F\rangle = -\frac{i}{\hbar} H_F |t_F\rangle \quad \frac{\partial}{\partial t_B} |t_B\rangle = \frac{i}{\hbar} H_B |t_B\rangle$$

$$\Psi = {}_{\mathcal{A}_2} \langle \phi | \otimes | \psi \rangle_{\mathcal{A}_1} \in \mathcal{H}_{\mathcal{A}_2} \otimes \mathcal{H}_{\mathcal{A}_1}$$



R. Silva et al. "Connecting processes with indefinite causal order and multi-time quantum states." *New J. Phys.* 19, 103022 (2017)

Thank you, Lee, for the honesty, vision and inspiration!

Thank you

For more details:

I. L. P., A. C. L., and E. C., *Flow of time during energy measurements and the resulting time-energy uncertainty relations*, [Quantum 6, 683 \(2022\)](#)

I.L.P., A. T., B.P., E.C., Y.A., *Noninertial quantum clock frames lead to non-Hermitian dynamics*, [Commun. Phys. 5, 298 \(2022\)](#)

I.L.P., M.N., and E.C., *Dynamical nonlocality in quantum time via modular operators*, [Phys. Rev. A 105, 042207 \(2022\)](#)

M.S., I.L.P., and E.C., *Non-relativistic spatiotemporal quantum reference frames*, [Phys. Rev. A 109, 032205 \(2024\)](#)

M.S., A.C. and E.C., *Relativity of Quantum Correlations: Invariant Quantities and Frame-Dependent Measures*, [arXiv:2503.20090](#)

Y.A., E. C., F.C. et al., *Finally making sense of the double-slit experiment*, [PNAS 114, 6480 \(2017\)](#)

A.C., E. C., *Relativistic independence bounds nonlocality*, [Sci. Adv. 5, eaav8370 \(2019\)](#)

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