Title: Lorentzian Quasicrystals and the Irrationality of Spacetime

Speakers: Sotirios Mygdalas

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

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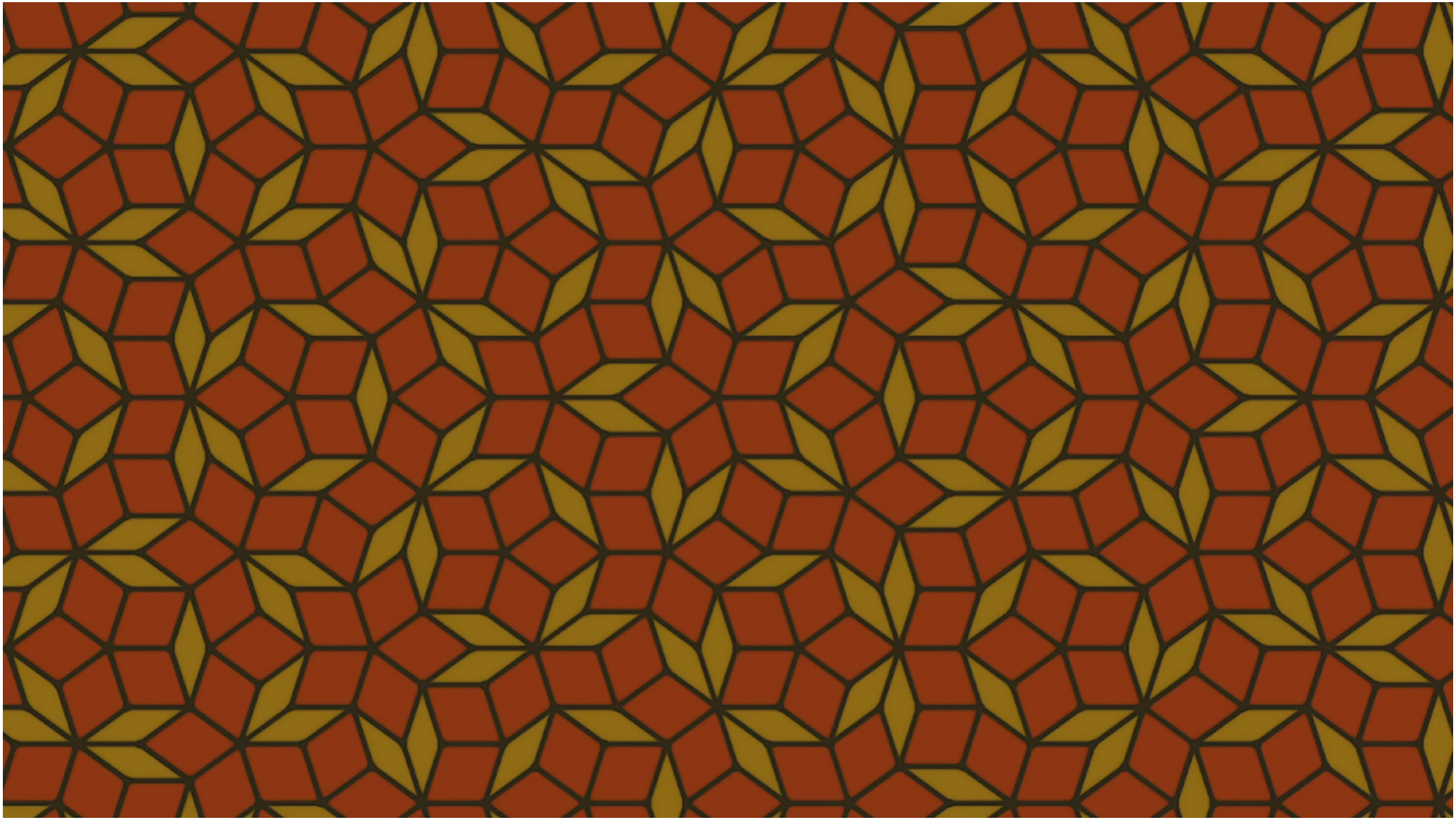
Abstract:

Ordered structures that tile the plane in an aperiodic fashion - thus lacking translational symmetry - have long been considered in the mathematical literature. A general method for the construction of quasicrystals is known as *cut-and-project* (CNP for short), where an irrational slice "cuts" a higher-dimensional space endowed with a lattice and suitably chosen lattice points are further "projected down" onto the subspace to form the vertices of the quasicrystal. However, most of the known examples of CNP quasi-tilings are Euclidean. In this talk, after presenting the main ingredients of the Euclidean prescription, we will extend it to Lorentzian spacetimes and develop Lorentzian CNP. This will allow us to discuss the first ever examples of Lorentzian quasicrystals, one in $(1+1)$- and another in $(1+3)$-dimensional spacetime. Finally, we will argue why the latter construction might be relevant for *our Lorentzian spacetime*. In particular, we shall appreciate how the picture of a quasi-crystalline spacetime could provide a potentially new string-compactification scheme that can naturally accommodate for the hierarchy problem and the smallness of our cosmological constant. Lastly, we will comment on its relevance to quantum geometry and quantum gravity; first, as a conformal Lorentzian structure of no intrinsic scale, and second through the connection of quasicrystals to quantum error-correcting codes.




Can we do that in
Lorentzian Spaces?

Lorentzian Quasicrystals and the *irrationality* of Spacetime

UNIVERSITY OF WATERLOO

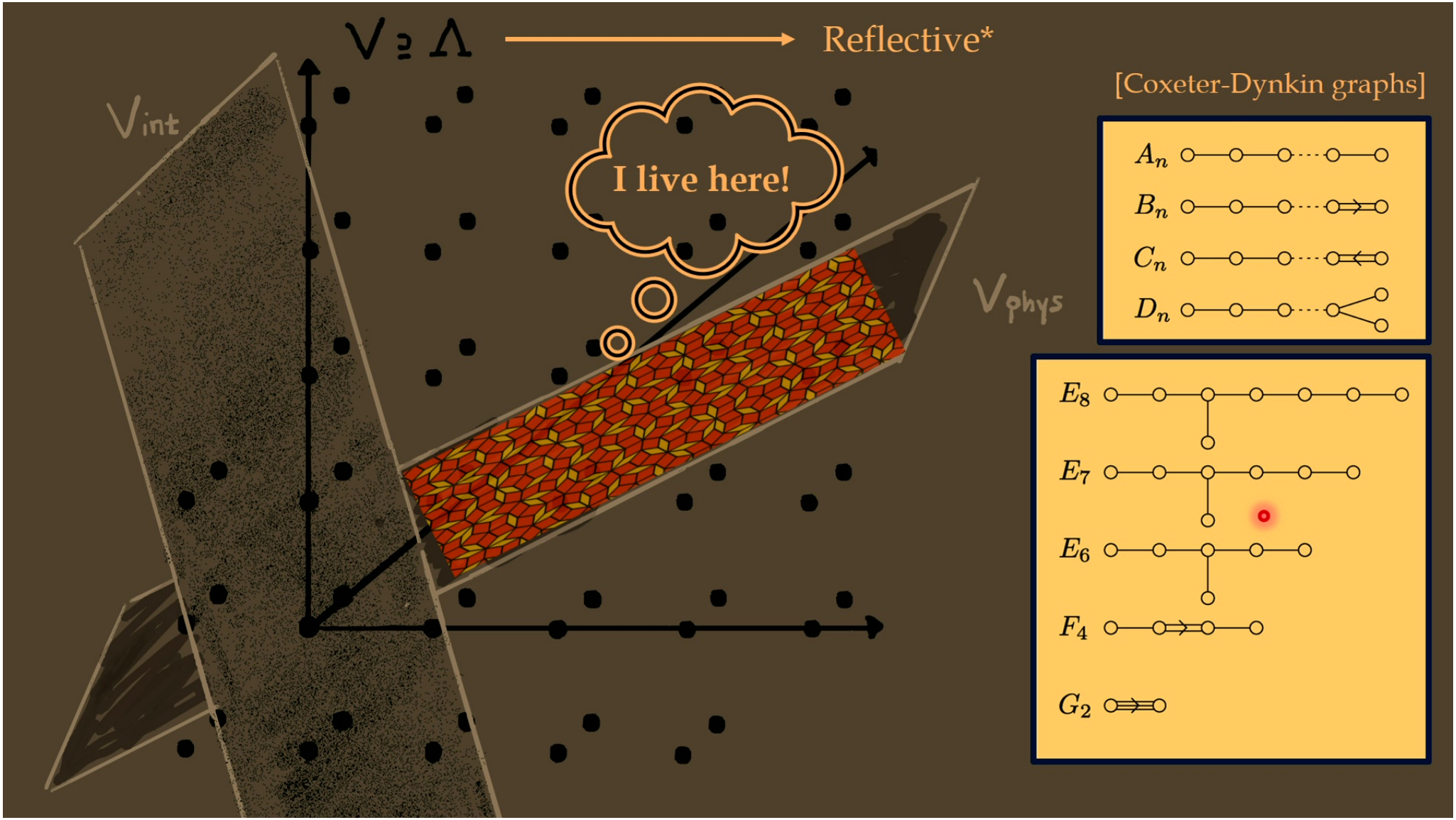
Sotiris Mygdalas

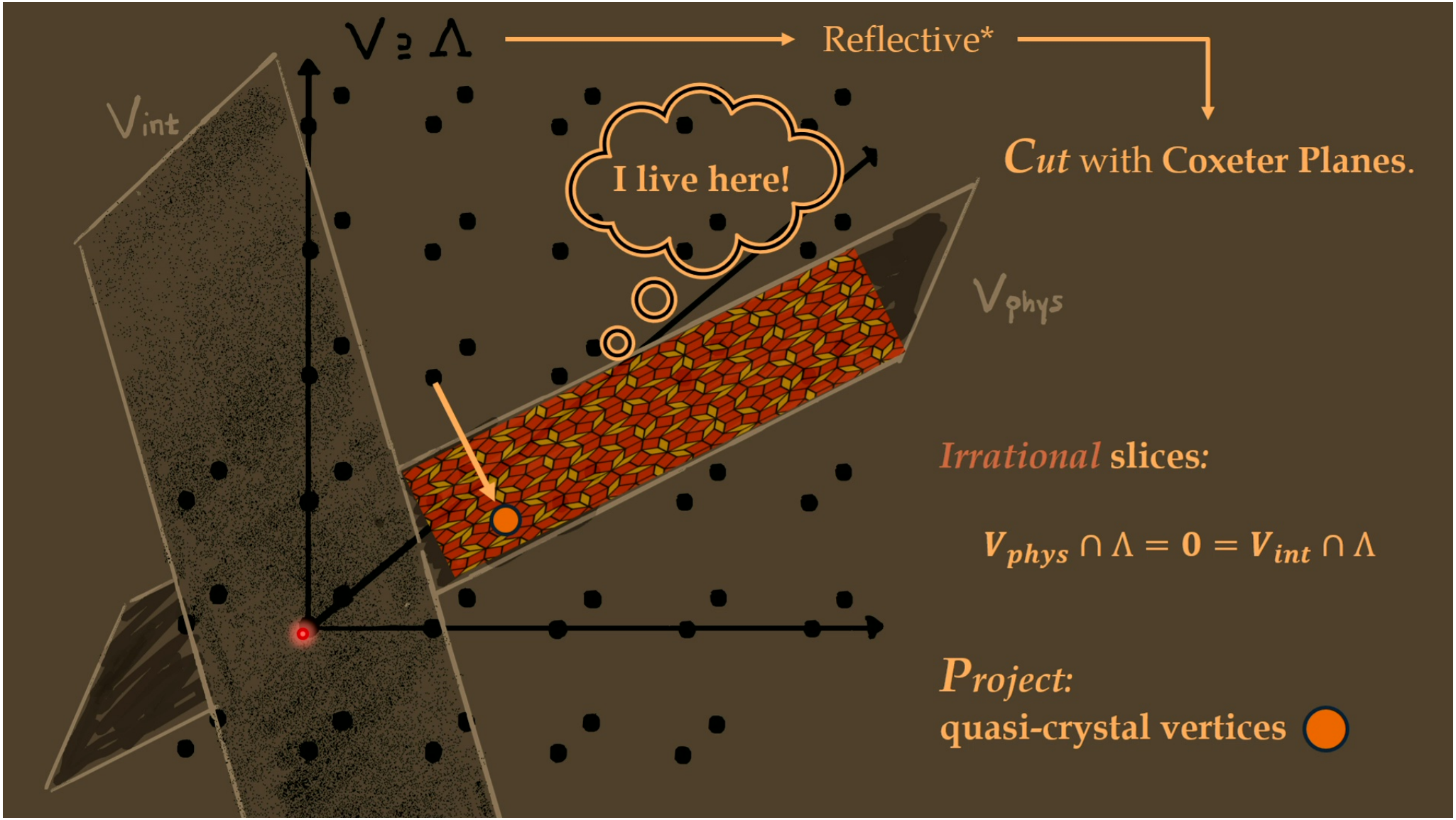
Advisor: Latham Boyle (work in progress)

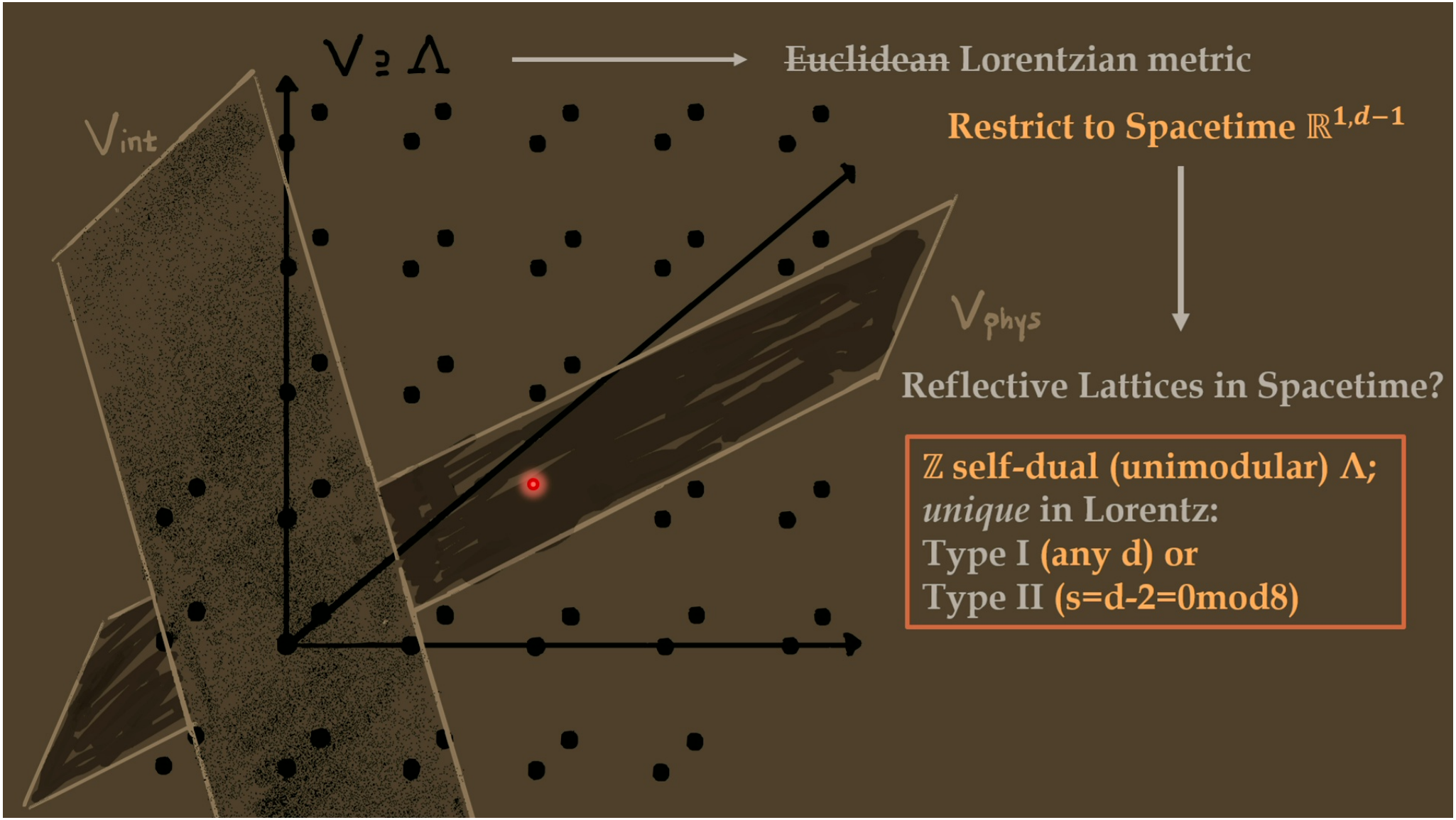
In Celebration of **Lee Smolin – Lee's Fest** @ PI – June 6, 2025

Quasicrystals through Cut-and-Project (CNP)

(Euclidean Setting)


V ∋ Λ
Reflective*
[Coxeter-Dynkin graphs]
V_{int}
I live here!
V_{phys}
A_n
B_n
C_n
D_n
E_8
E_7
E_6
F_4
G_2

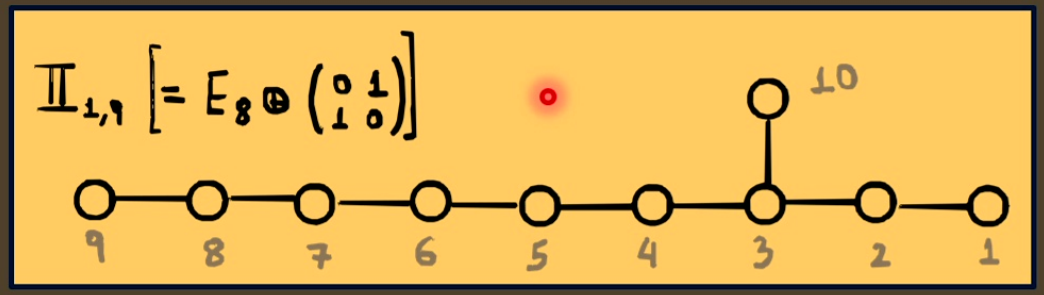

V ∋ Λ
Reflective*
Cut with Coxeter Planes.
V_int
I live here!
V_phys
Irrational slices:
$V_{phys} \cap \Lambda = 0 = V_{int} \cap \Lambda$
Project:
quasi-crystal vertices


V ⊇ Λ
~~Euclidean~~ Lorentzian metric
Restrict to Spacetime $\mathbb{R}^{1,d-1}$
V_{int}
V_{phys}
Reflective Lattices in Spacetime?
$\mathbb{Z}$ self-dual (unimodular) Λ;
unique in Lorentz:
Type I (any d) or
Type II (s=d-2=0mod8)

Hyperbolic Coxeter Groups [up to d=10]

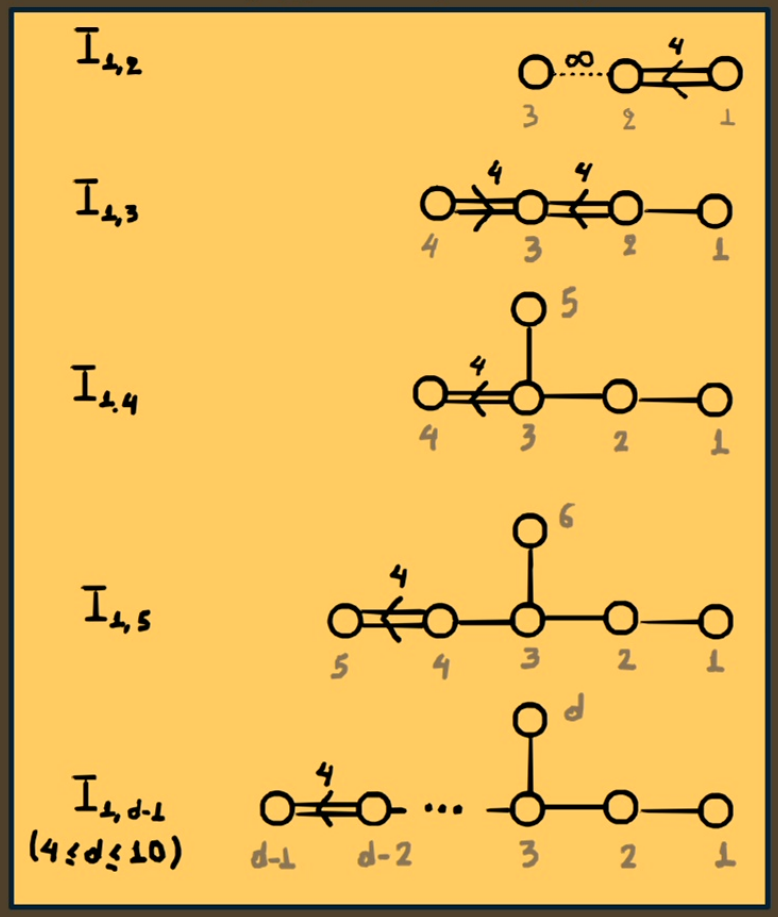
[Vinberg's algorithm; works in the 70s]

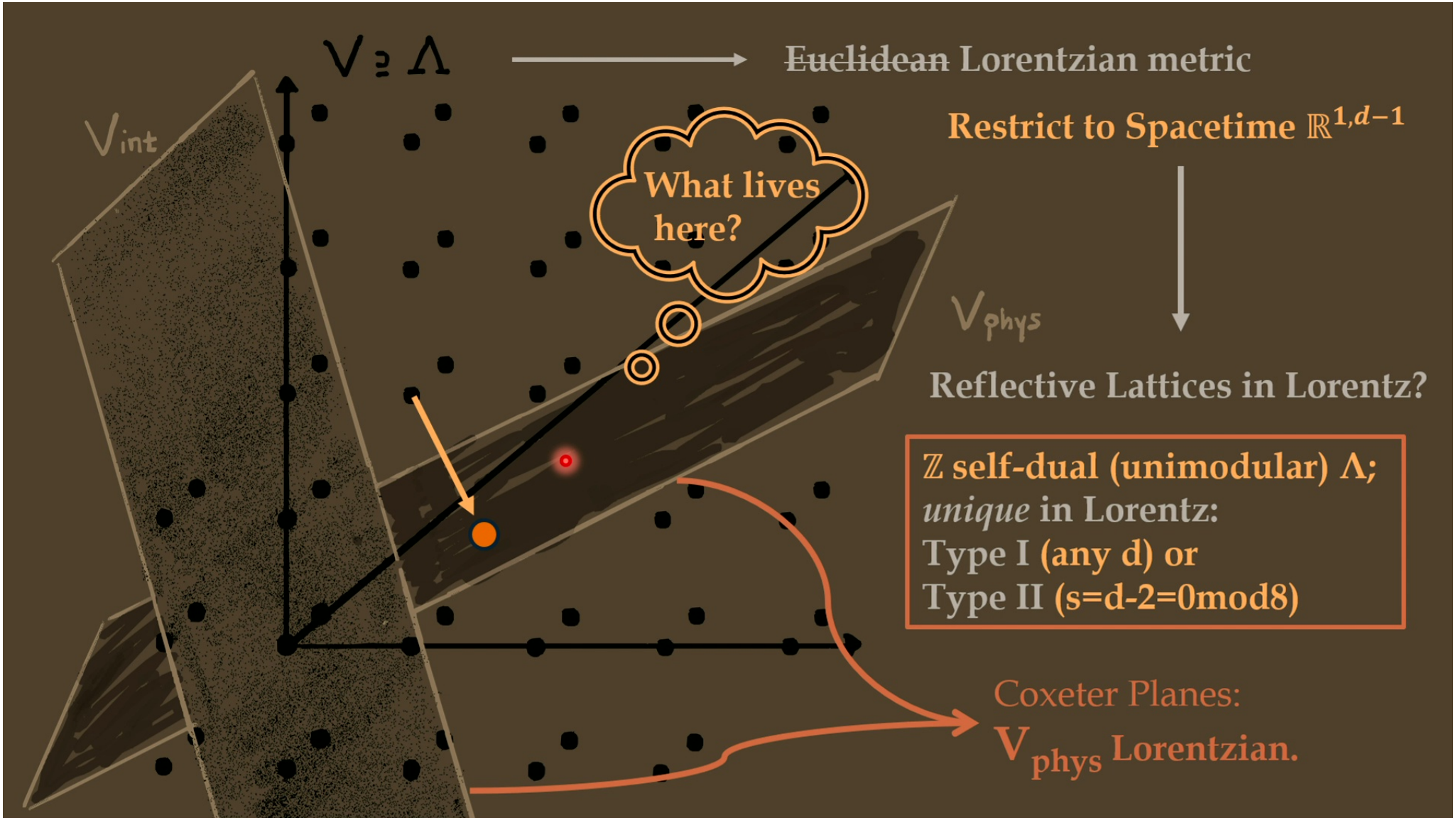
Type II (Even)



(+ more graphs with finite #simple roots up to d=20.)

Type I (Odd)




V ⊇ Λ
~~Euclidean~~ Lorentzian metric
Restrict to Spacetime $\mathbb{R}^{1,d-1}$
V_{int}
What lives here?
V_{phys}
Reflective Lattices in Lorentz?
ℤ self-dual (unimodular) Λ;
unique in Lorentz:
Type I (any d) or
Type II (s=d-2=0mod8)
Coxeter Planes:
V_{phys} Lorentzian.

Coxeter Eigenbasis *is* a NP Frame!

Coxeter element (Point Set) = Point Set

→ infinite symmetry! (discrete Lorentz)

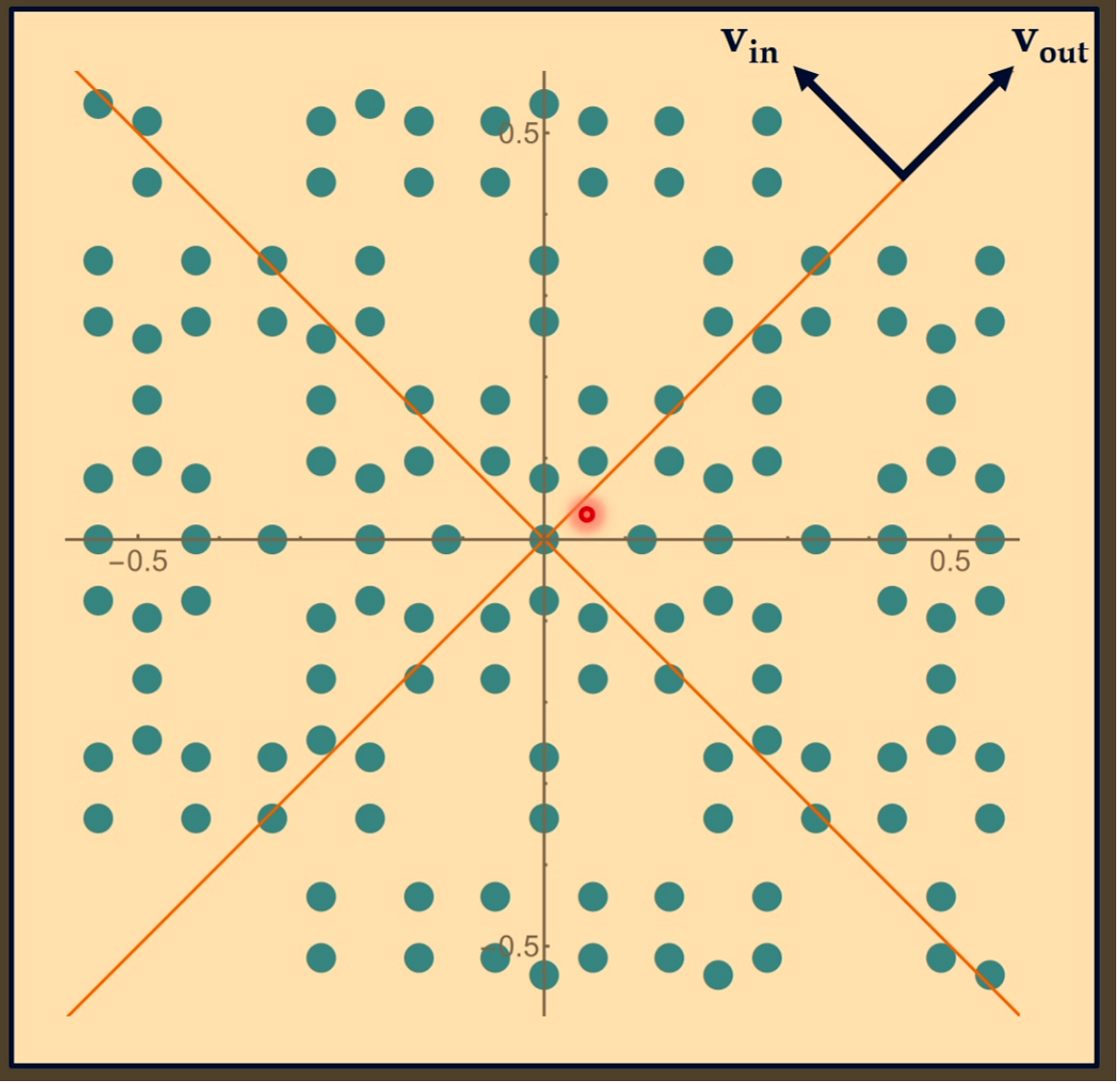
Inflation (scaling) symmetry.

Irrationality:
No null-separated vertices.

… Causal Sets?

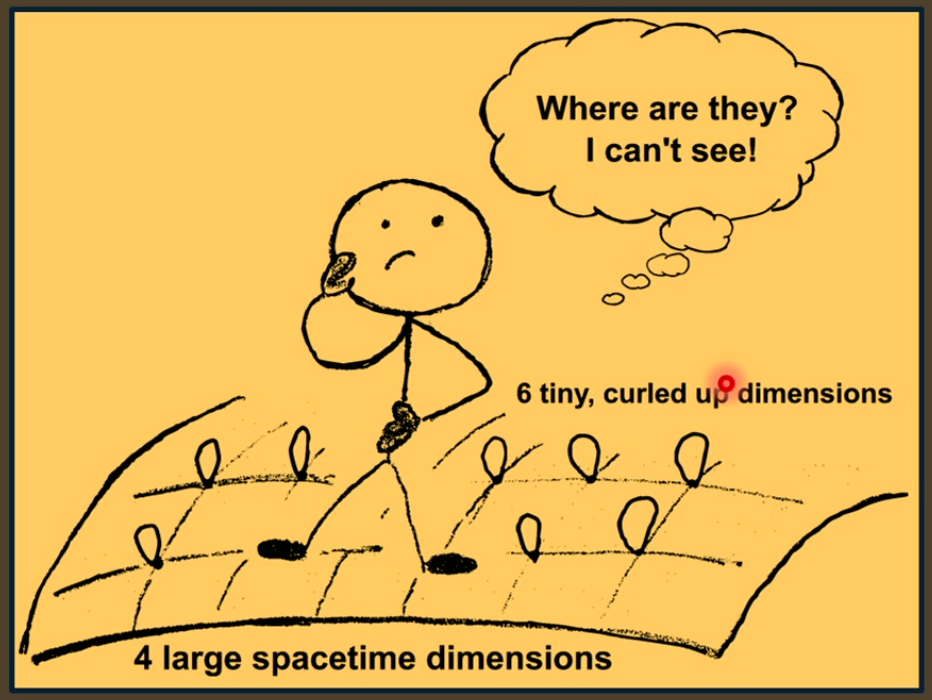
Trade *Randomness* with *Symmetry*!

… Q error-correcting codes?



1 Fitting the Universe in a Nutshell

String Theory:
"We *really* live in 10 dimensions."



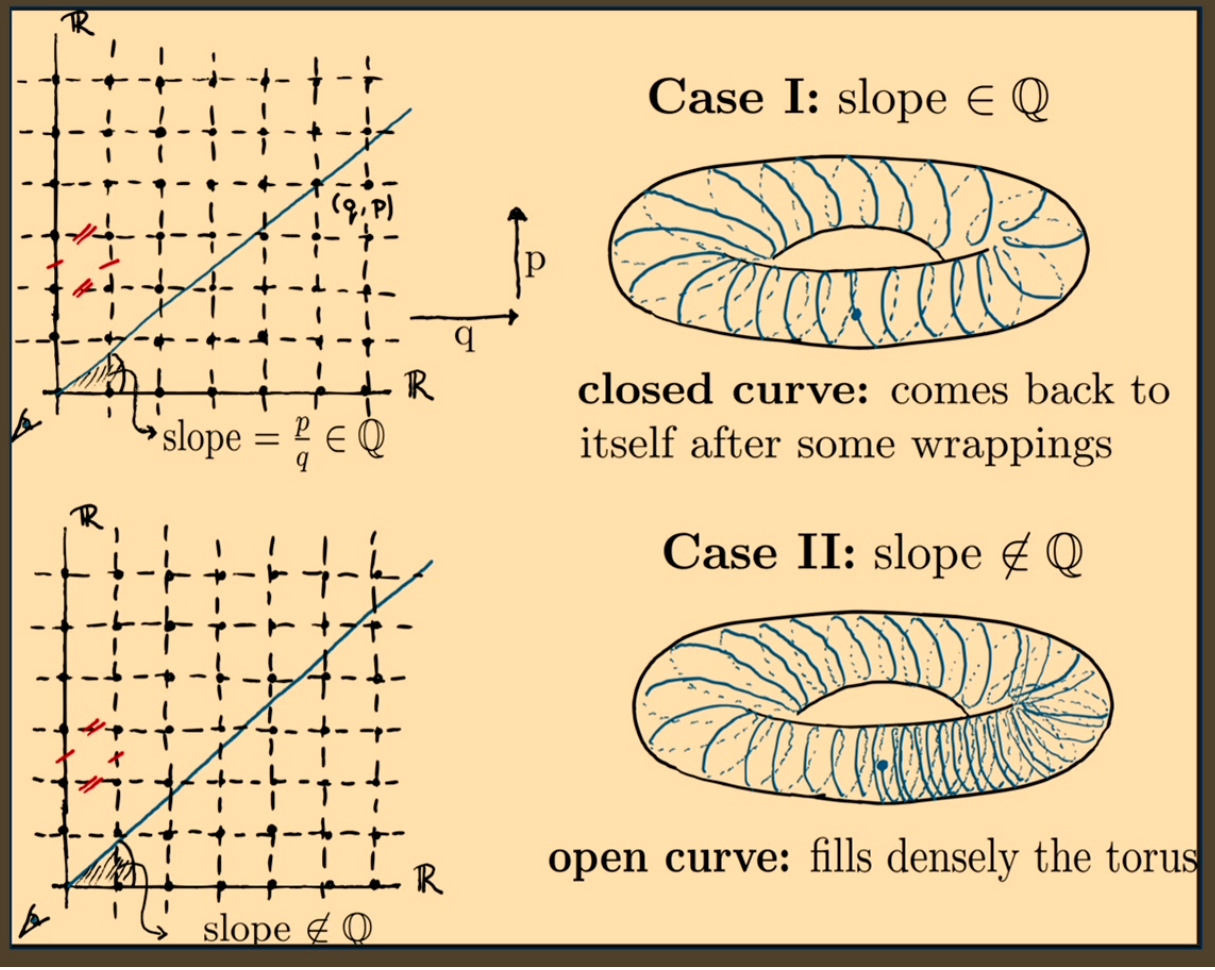
1 Fitting the Universe in a Nutshell

"Toroidal" Compactification

$$\mathbb{T}^{1,9} = \mathbb{R}^{1,9}/\Pi_{1,9}$$

Moore's "most symmetric" compactification
[Finite in all Directions, hep-th/9305139]

… what about causality violations and CTCs?



1 Fitting the Universe in a Nutshell

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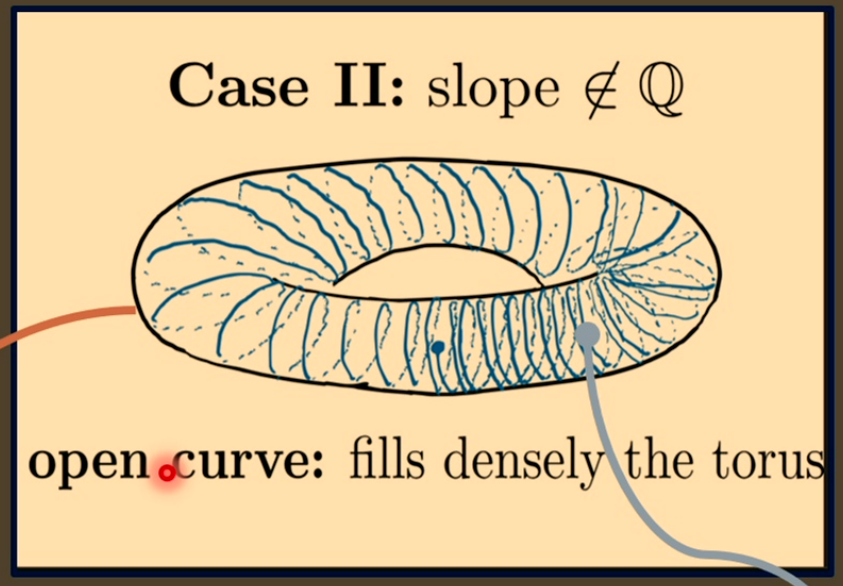
… what about causality violations and CTCs?

Enter Irrationality:
no CTCs in 4D!



1+1 The Hierarchy Problem

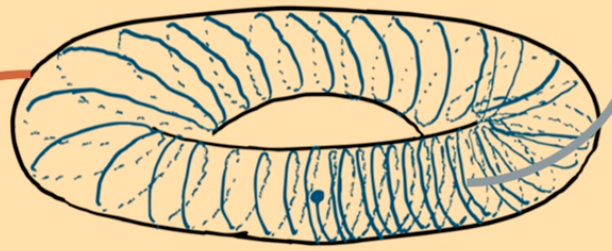
$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}\left[\underbrace{M_{Pl}^2\mathcal{R}}_{\text{Bending}} + \underbrace{M_H^2(h^\dagger h)}_{\text{Higgs}} + \underbrace{M_{vac}^4}_{\text{Stretching}}\right]$$

$M_{Pl} \sim 10^{19}\,\text{GeV}$

$M_{EW} \sim 10^{2}\,\text{GeV}$

$M_{vac} \sim 10^{-12}\,\text{GeV}$

Case II: slope $\notin \mathbb{Q}$



open curve: fills densely the torus

$\mathbb{T}^{1,9}$

4D Spacetime

Still Feels *Irrational?*
3+1 reasons to think over lunch:

- ✓ **Highly Symmetric** Discrete Point Set in Spacetime with **no intrinsic scale!**
- ✓ **Q-Error Correcting** Quasi-crystalline Spacetime?
- ✓ "Most symmetric" **string compactification** possible!
- ✓ **Geometric Explanation** of the Hierarchy of Scales.