Title: The problem of time and evolving constants of motion: the cosmological case

Speakers: Saeed Rastgoo

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Abstract:

I present a cosmological toy model of the resolution of the problem of time based on the Page-Wootters formalism but written in terms of evolving constants of motion. The use of these quantities resolves the issues, e.g., the incorrect propagators, etc., of the Page-Wootters formalism, and points to some interesting preliminary results.

The Problem Of Time And Evolving Constants Of Motion: The Cosmological Case

Saeed Rastgoo (University of Alberta)
with J. Roberts

UNIVERSITY OF ALBERTA

Lee's Fest

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The Problem of Time

- Dirac observables O are frozen in t

$$\dot{O} = \{O, H\} = \delta O = 0$$

- In the quantum regime: $\hat{H}\Psi = 0 \Rightarrow$ no Schrodinger equation with $\frac{\partial \Psi}{\partial t}$ on the RHS

The Method: Big Picture

Combination of [Gambini, Porto, 2001; Gambini, Porto, Pullin, Torterolo, 2009]

1. Page-Wootters relational formalism [Page-Wootters, 1983]

2. Evolving constants of motion (parametrized Dirac observables) [Rovelli, 1991]

Page-Wootters Formalism

Page-Wootters formalism: Quantum subsystem $Q \subset U$ evolves w.r.t. quantum clock subsystem $T \subset U$

- Construct $\hat{Q}$, $\hat{T}$ as operators on a Hilbert space $\mathscr{H} = \mathscr{H}_T \otimes \mathscr{H}_Q$,
- Quantum conditional probabilities (using Lüder's rule):

$$P\left(Q = Q_0 \middle| T = T_0\right) = \frac{\mathrm{Tr}\left[\hat{\mathcal{P}}_{T_0}\hat{\rho}\hat{\mathcal{P}}_{T_0}\hat{\mathcal{P}}_{Q_0}\right]}{\mathrm{Tr}\left(\hat{\rho}\hat{\mathcal{P}}_{T_0}\right)}$$

- $\hat{\rho}$: **density operator** of the whole system U
- $\hat{\mathcal{P}}_{Q_0} = |Q_0\rangle\langle Q_0|$: **projection operator** corresponding to eigenvalue Q_0
- $\hat{\mathcal{P}}_{T_0} = |T_0\rangle\langle T_0|$: **projection operator** corresponding to eigenvalue T_0

Problems with Page-Wootters Formalism

- **Wrong propagator** of a single-particle: the Page-Wootters formalism yields **no motion**! [Kuchar, 2011]

$$K_{x_f,T_f;x_i,T_i} = \delta\left(T - T'\right)\delta\left(x - x'\right)$$

instead of the correct one

$$K_{x_f,T_f;x_i,T_i} = \left[\frac{2\pi i\left(T_f - T_i\right)}{m}\right]^{-1/2} \exp\left[\frac{im\left(x_f - x_i\right)^2}{2\left(T_f - T_i\right)}\right]$$

- **Violation of the Constraints**: $\hat{\mathcal{P}}_{Q_0}$, $\hat{\mathcal{P}}_{T_0}$ do not commute with $H \Rightarrow$ leaving constraint surface after acting

Remedy of Page-Wootters: ECM

- It turns out using evolving constants of motion (ECM) will fix all these issues [Gambini, Porto, 2001; Gambini, Porto, Pullin, Torterolo, 2009]
- ECM (parametrized Dirac observables)

$$O(t) \Rightarrow \{O(t), H\} \approx 0$$

The Montevideo Interpretation of QM

- Choose $\hat{Q}$, $\hat{T}$ to be **quantum evolving constants of motion**, instead of values of fields which in a totally constrained systems are not physically observable [Gambini, Porto, 2001; Gambini, Porto, Pullin, Torterolo, 2009]
- $\hat{T}$ **and** $\hat{Q}$ **interact (do not commute)**: (more) **realistic** model [Gambini, Pullin, Rastgoo, Roberts, *to appear soon*]
- The probability becomes

$$P\left(Q = Q_0 \middle| T = T_0\right) = \frac{\int_{-\infty}^{\infty} dt \mathrm{Tr}\left[\hat{\mathcal{P}}_{T_0}(t)\hat{\rho}\hat{\mathcal{P}}_{Q_0}(t)\hat{\mathcal{P}}_{T_0}(t)\right]}{\int_{-\infty}^{\infty} dt \mathrm{Tr}\left[\hat{\mathcal{P}}_{T_0}(t)\hat{\rho}\right]}$$

- "Time" is quantum! Can be discrete

The Model [Gambini, Pullin, Rastgoo, Roberts, *to appear soon*]

FLRW cosmology

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right)$$

with two scalar fields ϕ_i, $i = 1, 2$ with Hamiltonian constraint

$$\mathcal{H} = -\frac{6}{\gamma^2}c^2\sqrt{|p|} + \frac{8\pi G}{|p|^{\frac{3}{2}}}\sum_{i=1}^{2} p_{\phi_i}^2$$

where

$$c = \gamma\dot{a}, \qquad |p| = a^2$$

Dirac Observables

From two of EoM

$$\dot{p}_{\phi_i} = \{p_{\phi_i}, N\mathcal{H}\} = 0, \qquad i = 1, 2.$$

immediately see two Dirac Observables $O_1,\ O_2$

$$O_i = p_{\phi_i} \qquad i = 1, 2$$

Defining

$$\Pi_1 = -\ \phi_1, \qquad\qquad \Pi_2 = -\ \phi_2$$

leads to a 4D phase space

$$\{O_i, \Pi_j\} = \delta_{ij}, \qquad i, j = 1, 2$$

Evolving Constants of Motion (ECM)

Identify

$$t = \frac{\phi_1}{p_{\phi_1}}$$

Out of the phase space variables $O_1,\ O_2,\ \Pi_1,\ \Pi_2$, construct a clock ECM

$$T(t) := p_{\phi_1}\phi_2 = O_2\Pi_1 - O_1\Pi_2 + O_1O_2t$$

and another ECM (the one measured against the clock)

$$Q(t) := p_{\phi_1}p_{\phi_2}\ln\left(|p|\right) = \alpha\sqrt{O_1^2 + O_2^2}\,(O_2\Pi_1 + O_1O_2t)$$

where $\alpha = 4\mathrm{sgn}(c)\mathrm{sgn}(p)\sqrt{\frac{\pi G}{3}}$

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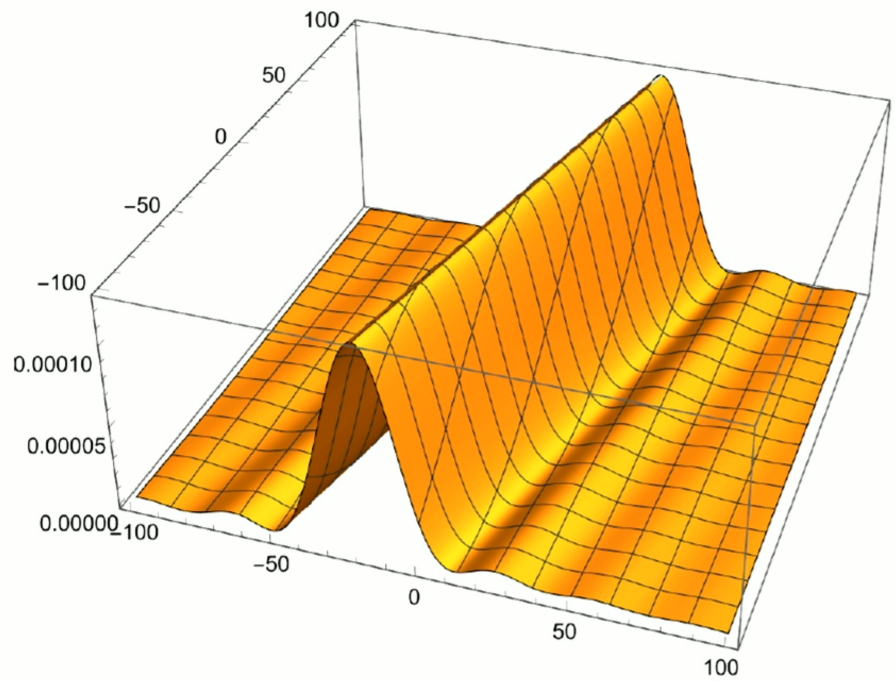
Schrödinger representation:

$$T \to \hat{T}, \qquad\qquad Q \to \hat{Q}$$

The Probability

A **very preliminary** result:



Summary

- A **Relational solution to the problem of time** based on Page-Wootters formalism: Montevideo interpretation
- Use of evolving constant of motion: **free of issues** of Page-Wootters formalism
- Clock and "other quantity" are both **quantum, observable, and can interact** with each other
- Complete results to follow soon

Thank You Lee!

Thank you Lee for all your great contribution to science and the scientific community!

and, **Happy Birthday!**