Title: No time machine on the cheap: why semiclassical wormholes won’t do

Speakers: Daniel Terno

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

Date: June 06, 2025 - 10:25 AM

URL: https://pirsa.org/25060080

Abstract:

If you can control a wormhole, you can time-travel. The issue is whether they can exist at all. Wormhole solutions in general relativity have spectacular local and global features. Invariantly, a wormhole throat is an outer marginally trapped surface satisfying additional constraints. Some of its properties — like violation of the energy conditions — it shares with black holes that are required to trap light in finite time for a distant observer. This condition may be contentious for black and white holes, but it is the essential part of what “traversable” means.

Standard traversable wormholes, such as those described by the Ellis-Morris-Thorne or Simpson-Visser metrics, are static and spherically symmetric. We show that no dynamical solution of the semiclassical Einstein equations can asymptote to these geometries. Conversely, dynamical solutions that do exist either fail to yield a traversable static limit, or breach quantum energy inequalities that bound violations of the null energy condition, or lead to divergent tidal forces. These conclusions hold independently of the choice of quantum fields. Such symmetric wormholes, therefore, are ruled out in semiclassical gravity — making time travel a costlier proposition.

No time machines on the cheap: why semiclassical wormholes won't do

2025.06.06

Daniel Terno

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MACQUARIE University

Astrophysics and Space Technologies Research Centre

RQi Relativistic Quantum Information COST Action 23115

JSF Julian Schwinger Foundation

LEE'S FEST: QUANTUM GRAVITY AND THE NATURE OF TIME

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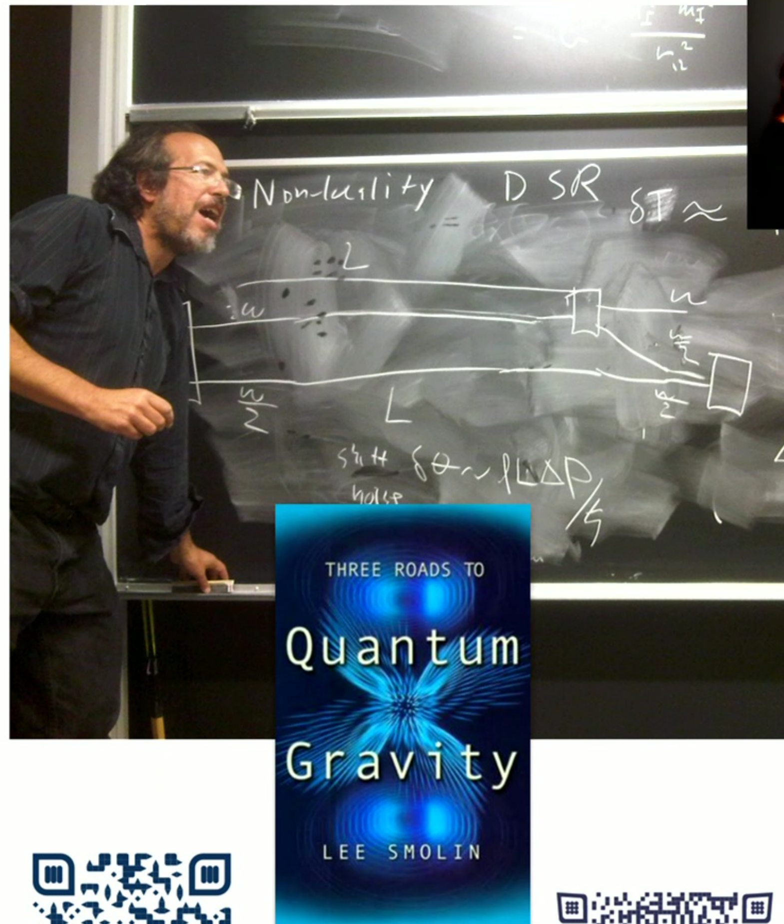
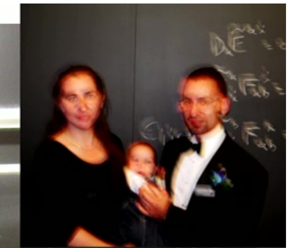




wormholes

sci-fi & astrophysics

Design brief

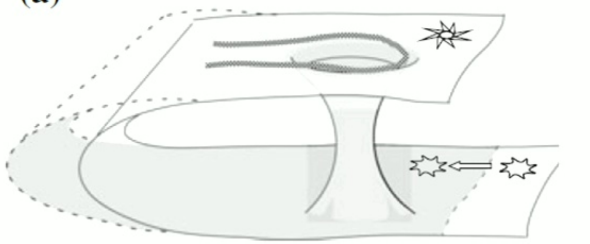
(1) *Spherically symmetric (& static)*
(2) Satisfies the Einstein equations
(3) Connects asymptotically flat regions
(4) No horizon
(5) Bearably small tidal forces
(6) Finite & reasonable travel time
(7) Reasonable EMT
(8) Perturbatively stable
(9) Sub-Universe-scale resources

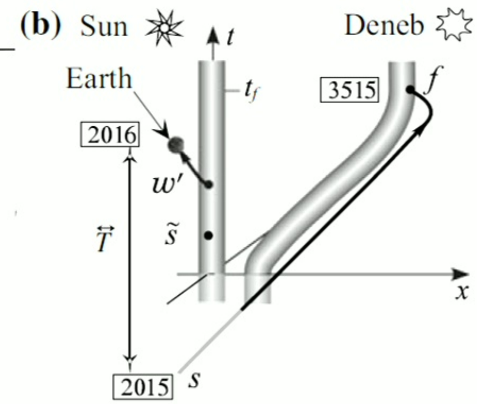
▲ Morris and Thorne, Am J Phys **56**, 395 (1988) ▼

Ellis, J Math Phys (N.Y.) **14**, 104 (1973) ▼

$$ds^2 = -dt^2 + \left(1 - \frac{a^2}{r^2}\right) dr^2 + r^2 d\Omega_2$$

(a)





$$ds^2 = -\left(1 - \frac{2m}{\sqrt{\eta^2 + a^2}}\right) dt^2 + \frac{d\eta^2}{1 - \frac{2m}{\sqrt{\eta^2+a^2}}} + (\eta^2 + a^2) d\Omega_2,$$

◀ Simpson and Visser, J Cosm Astropart Phys 02 (2019) 042

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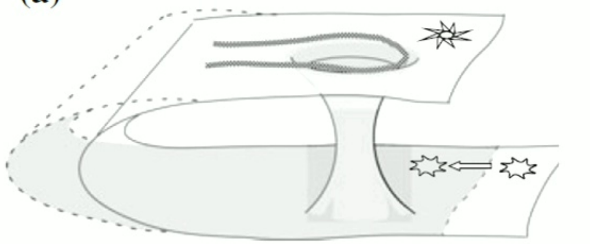
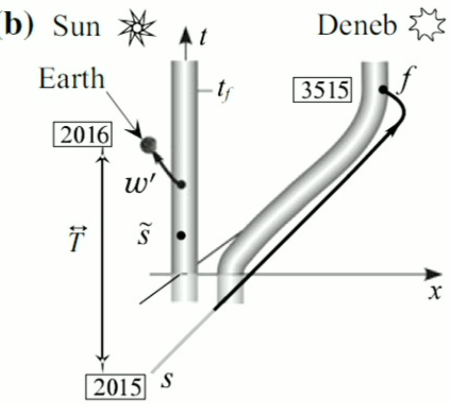


(b)



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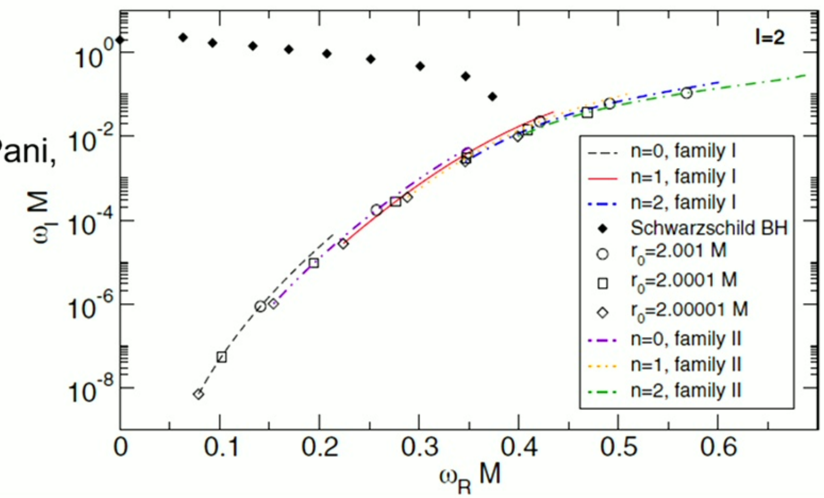
◀ Simpson and Visser, J Cosm Astropart Phys 02 (2019) 042

Cardoso, Franzin, and Pani, Phys Rev Lett **116**, 171101 (2016) ▶



wormholes

design math

Design brief

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- A wormhole throat: a marginal outer trapped surface + flare-out condition

Hochberg and Visser, Phys Rev D **58**, 044021 (1998)
McNutt et al, Phys Rev D **103**, 124024 (2021)

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

- circumference: $2\pi r$
- physical time at infinity: t

$$f = 1 - 2M(t,r)/r$$

$$2M(t,r) \equiv C(t,r)$$

MSH invariant mass

(1)

(2) $$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi \left\langle \hat{T}_{\mu\nu} \right\rangle_{\omega} + E_{\mu\nu}$$

$$ds^2 = -e^{\Psi} dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

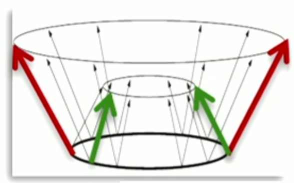
$$\Psi \equiv h + \tfrac{1}{2} \ln f$$

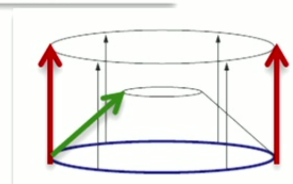
throat

$$r_0 = C(t, r_0)$$

(4)

(6) $$\int^{r_0} \frac{dr}{e^h f} < \infty$$





$$\theta^{\text{out}} = 0$$

$$l^{\mu}_{\text{out}} = \left(e^{-h}, f, 0, 0\right)$$

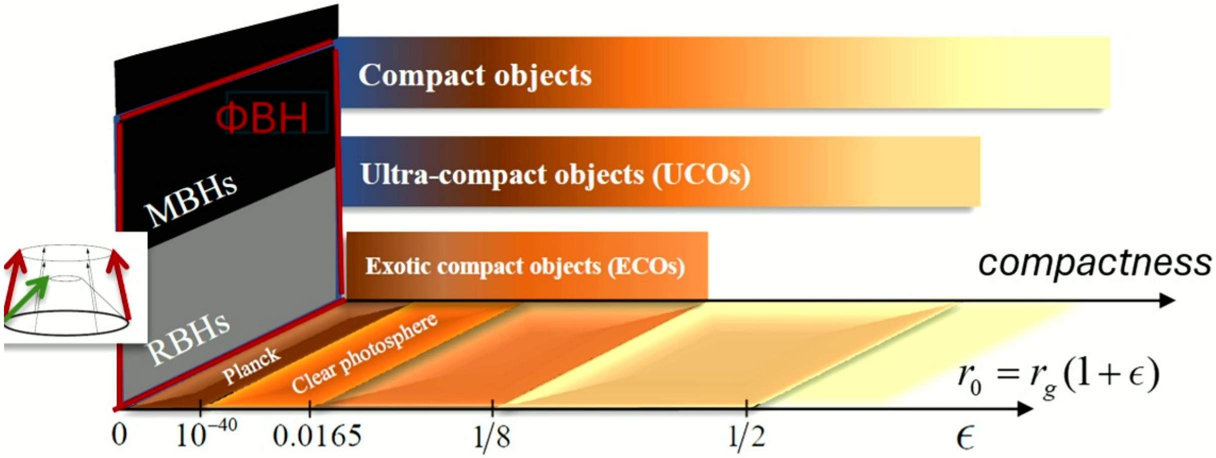
$$l^{\mu}_{\text{out}} \theta^{\text{out}}_{,\mu} = \frac{d\theta^{\text{out}}}{d\lambda} > 0$$

$$\theta^{\text{out}} = l^{\mu}_{\text{out};\mu} - \kappa = \frac{2f}{r}$$

ultra-compact objects

V. P. Frolov, arXiv:1411.6981 (2014)

the zoo of models & physical black holes



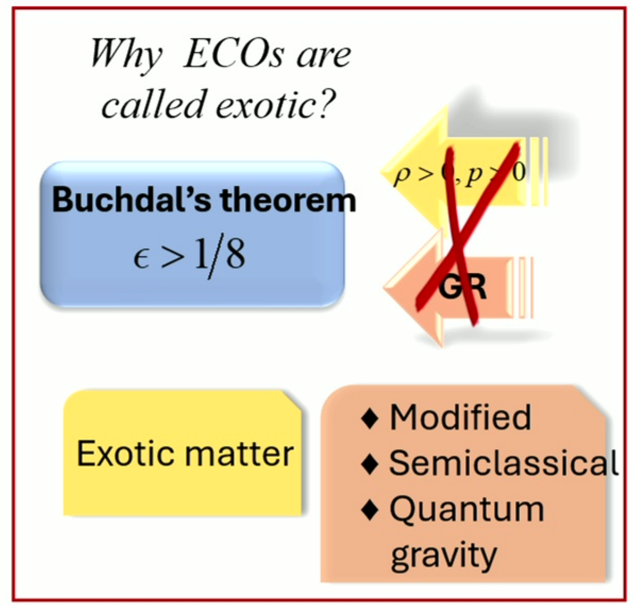
UCO: has a photosphere

BH: has a horizon

MBH: has an event horizon

ΦBH: has a trapped region

ECO: non-BH UCO



Cardoso and Pani, Nat. Astron. **1**, 586 (2017)

[1] Perpetual ongoing collapse, with an asymptotic horizon $\epsilon \to 0 \; @ t \to \infty$

[2] Formation of a transient or an asymptotic object, where the compactness reaches a minimum at some finite asymptotic [=distant observer] time $\epsilon \to \epsilon_{\min}$

[3] Formation of a ΦBH with the apparent horizon in finite distant observer's time $\epsilon(t_f) = 0$

Mann, Murk, DRT, Int J Mod Phys D **31**, 2230015 (2022)

physical black holes

Mann, Murk, DRT, Int J Mod Phys D **31**, 2230015 (2022)

assumptions

(2) $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \left\langle \hat{T}_{\mu\nu} \right\rangle_{\omega} + E_{\mu\nu}$

not assumed: type of the metric theory, global structure, singularity, types of fields, quantum state, presence of Hawking radiation

(*) Collapsing matter + excitations are the total EMT

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

(1)

$$l^{\mu}_{\text{out}} = \left(e^{-h}, f, 0, 0\right)$$

$$\theta^{(l)} = l^{\mu}_{\text{out};\mu} - \kappa = \frac{2f}{r}$$

$$r_g(t) = 2M\left(t, r_g(t)\right)$$

Apparent horizon at the Schwarzschild radius: the largest* root of $f=0$

ΦBH

i. light-trapping region forms at a **finite** time of a distant observer (Bob) **(9)**

ii. curvature scalars [contractions of the Riemann tensor] are **finite** on the boundary of the trapped region **(5)**

Logic

- use "bad"coordinates to express (i)
- require "self-renormalisation" to have (ii)

Curvature scalars

$$\mathrm{T} := T^{\mu}{}_{\mu}$$

$$\mathfrak{T} = T^{\mu\nu} T_{\mu\nu}$$

useful

$$\tau_t := e^{-2h} T_{tt}$$

$$\tau_t{}^r := e^{-h} T_t{}^r$$

$$\tau^r := T^{rr}$$

ΦBH structure

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

- circumference: $2\pi r$
- physical time at infinity: t

$$f = 1 - 2M(t,r)/r$$

$$2M(t,r) \equiv C(t,r)$$

MS invariant mass

Curvature scalars

$$\mathrm{T} := T^{\mu}{}_{\mu}$$

$$\mathfrak{T} = T^{\mu\nu} T_{\mu\nu}$$

$$\mathfrak{T} := \left((\tau^r)^2 + (\tau_t)^2 - 2(\tau_t{}^r)^2\right)/f^2$$

useful

$$\tau_t := e^{-2h} T_{tt}$$

$$\tau_t{}^r := e^{-h} T_t{}^r$$

$$\tau^r := T^{rr}$$

$$\mathrm{T} := (\tau^r - \tau_t)/f$$

+regular terms*

All three components go to zero or diverge in the same way

Schwarzschild radius

$$\max r_g = C(t, r_g)$$

Einstein equations

$$\partial_r C = 8\pi r^2 \tau_t / f,$$

$$\partial_t C = 8\pi r^2 e^h \tau_t{}^r,$$

$$\partial_r h = 4\pi r \left(\tau_t + \tau^r\right)/f^2$$

$$\lim_{r \to r_g} \tau_a \sim \begin{cases} \pm \Upsilon^2 f^0 \\ \tau_a(t) f^k \end{cases}$$

k=0,1* [who]

$$C = r_g(t) + W(t,r)$$ ▶

$$\lim_{r \to r_g} \tau_t = \lim_{r \to r_g} \tau^r = -\Upsilon^2 \qquad k = 0$$

spherical symmetry

ΦBH metrics, mostly *k*=0

1. The limiting form (close apparent horizon) of dynamical metrics is almost uniquely defined (both *k*=0 and *k*=1).

$x := r - r_g$

$$C = r_g - 4\sqrt{\pi r_g^3}\Upsilon\sqrt{x} + \ldots \quad h = -\frac{1}{2}\ln\frac{x}{\xi} + \ldots$$

$$f \propto \sqrt{x}$$

2. Parameters are related via evaporation rate

$$\frac{dr_g}{dt} = \mp 4\sqrt{\pi r_g \xi}\Upsilon$$

3. Most convenient coordinates are retarded (u_-, r) for *white holes* and advanced (v_+, r) for *black holes*

$r_g' < 0 \blacktriangleright (v,r)$ $\theta_{\text{in}} < 0, \theta_{\text{out}} < 0$ BH solutions	$r_g' > 0 \blacktriangleright (u,r)$ $\theta_{\text{in}} > 0, \theta_{\text{out}} > 0$ WH solutions

- ❑ No static *k*=0 solutions (the *k*=1 limit exists)
- ❑ Vaidya metrics are *k*=0 solutions
- ❑ Reissner-Nordström, STU & static RBHs are *k*=1 solutions
- ❑ Popular dynamic RBH models are *k*=0 solutions
- ❑ ϒ & ξ are identified using BH evaporation law*

If* ΦBH/WH exist, *then

- ❑ Finite infall time (according to a distant Bob)
- ❑ Outer apparent horizon is **timelike** **(6)**
- ❑ Null energy condition is violated (but QEI are satisfied) near outer horizon (7)

wormholes

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

popular models in our formalism

- ❑ Ellis-Morris-Thorne (original) $ds^2 = -dt^2 + dl^2 + (l^2 + a^2) d\Omega_2$

transformation & identification $r^2 = a^2 + l^2$ $\qquad \Psi = 0,\ C = a^2/r$

expansion

$$C = a - x + \frac{x^2}{a} + \mathcal{O}(x^2) \qquad h = -\frac{1}{2}\ln\frac{2x}{a} + \mathcal{O}(x)$$

- ❑ Simpson-Visser

$$ds^2 = -\left(1 - \frac{2m}{\sqrt{\eta^2 + a^2}}\right)dt^2 + \frac{d\eta^2}{1 - \frac{2m}{\sqrt{\eta^2+a^2}}} + (\eta^2 + a^2)d\Omega_2,$$

transformation $r^2 = a^2 + \eta^2$

expansion $a > 2m$

$$C = a + \frac{4m - a}{a}x + \mathcal{O}(x^2),$$

$$h = -\frac{1}{2}\ln\frac{2(a-2m)}{a}x + \mathcal{O}(x)$$

$a = 2m$

$$C = r + \frac{2}{a}x^2 + \mathcal{O}(x^3)$$

$$h = -\ln\frac{\sqrt{2}x}{a} + \mathcal{O}(x)$$

$$ds^2 = -e^{\Psi} dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

$$\Psi \equiv h + \tfrac{1}{2}\ln f$$

throat
$r_0 = C(t, r_0)$

(4)

$f = 1 - C/r \qquad x := r - r_g$

Classification:

All belong to the class k=1 (two versions)

Problems:

No static solutions in class k=0
Mismatch of the ln perfectors for k=1
Infinities† for k>1

DRT, Phys Rev D **106**, 044035 (2022)

wormholes

exoticity bounds

Kontou and Olum, Phys. Rev. D **91**, 104005 (2015).
Kontou and Sanders, Class. Quant. Grav. **37**, 193001 (2020).

(7) Reasonable matter satisfies quantum energy inequalities

$$\int_\gamma \mathfrak{f}^2(\tau)\rho d\tau \geq -B(R,\mathfrak{f},\gamma)$$

$B>0$
smearing function $\mathfrak{f}$
timelike geodesic γ

- Ingoing test particles (aka Alice falling into a BH, $r_g'(t)<0$)

Alice's 4-velocity: $u_A=(\dot{T},\dot{R},0,0)$ $\quad \dot{T}=\dfrac{\sqrt{F+\dot{R}^2}}{e^H F} \sim \dfrac{|\dot{R}|}{|r_g'|}$

Quantity to track: $X \coloneqq R(\tau)-r_g\big(T(\tau)\big)$

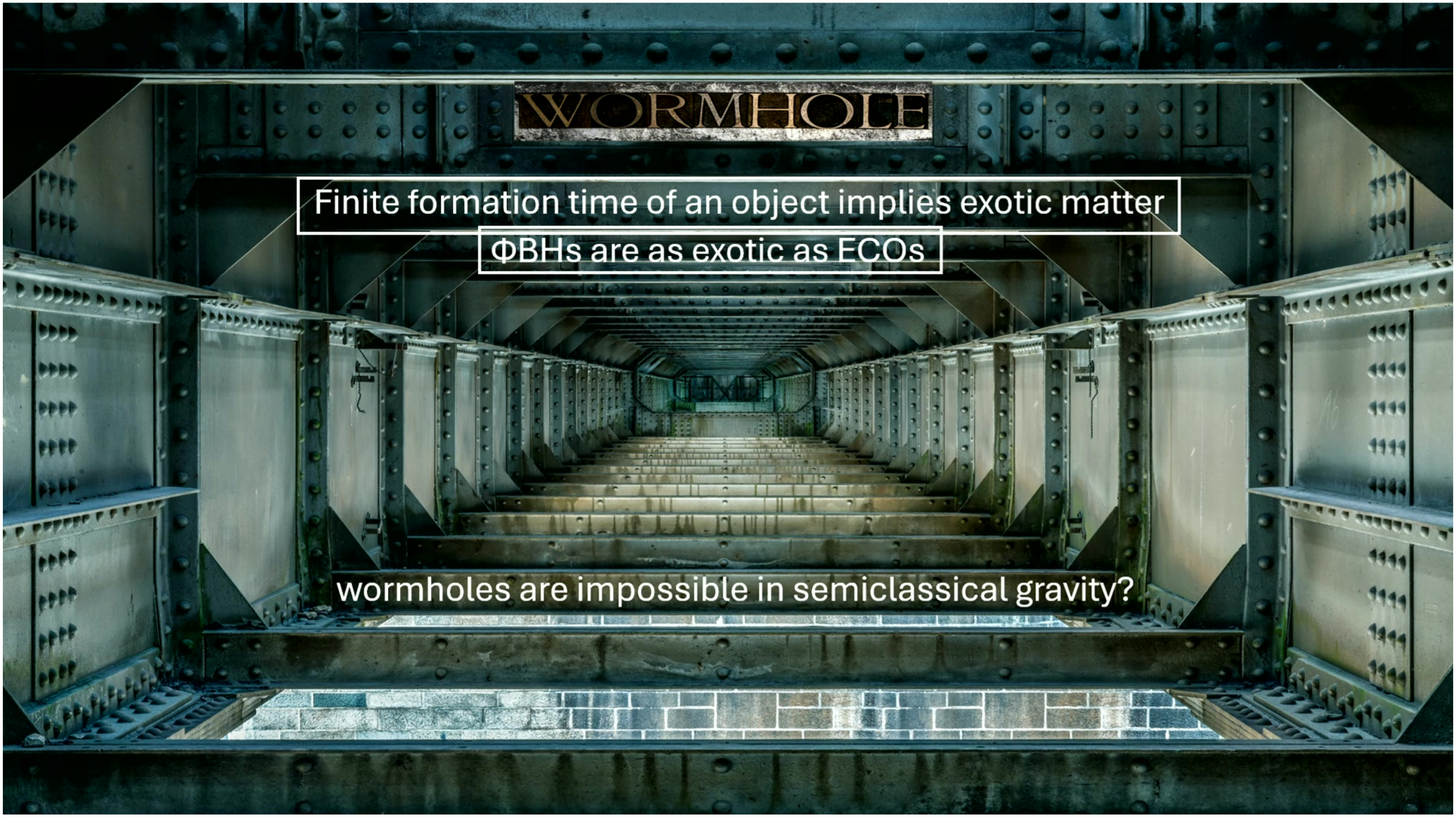
Alice's energy density:

$\dot{R}<0 \quad \rho_{\rm in}=-\dfrac{\Upsilon^2}{4\pi\dot{R}^2}$

$\dot{R}>0 \quad \rho_{\rm out}=-\dfrac{\dot{R}^2}{4\pi r_g X}$

Integrate energy density on exit diverges.
Impossible for BHs (no exit with $\dot{R}>0$)
Allowed for wormholes

QIEs are violated: energy (intermediate) singularity + breaking down of QFT


WORMHOLE
Finite formation time of an object implies exotic matter
ΦBHs are as exotic as ECOs
wormholes are impossible in semiclassical gravity?