Title: Spinfoams and the discreteness of time

Speakers: Francesca Vidotto

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

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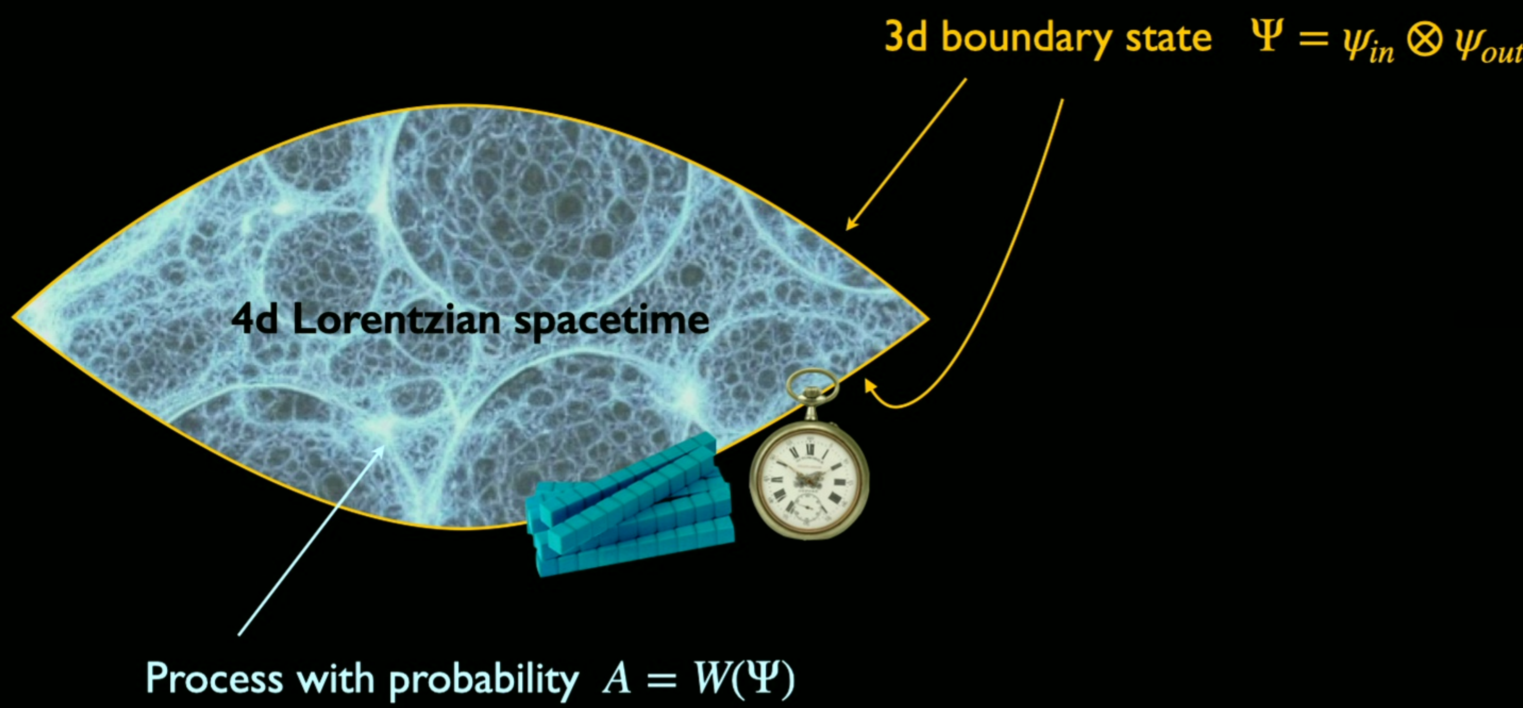
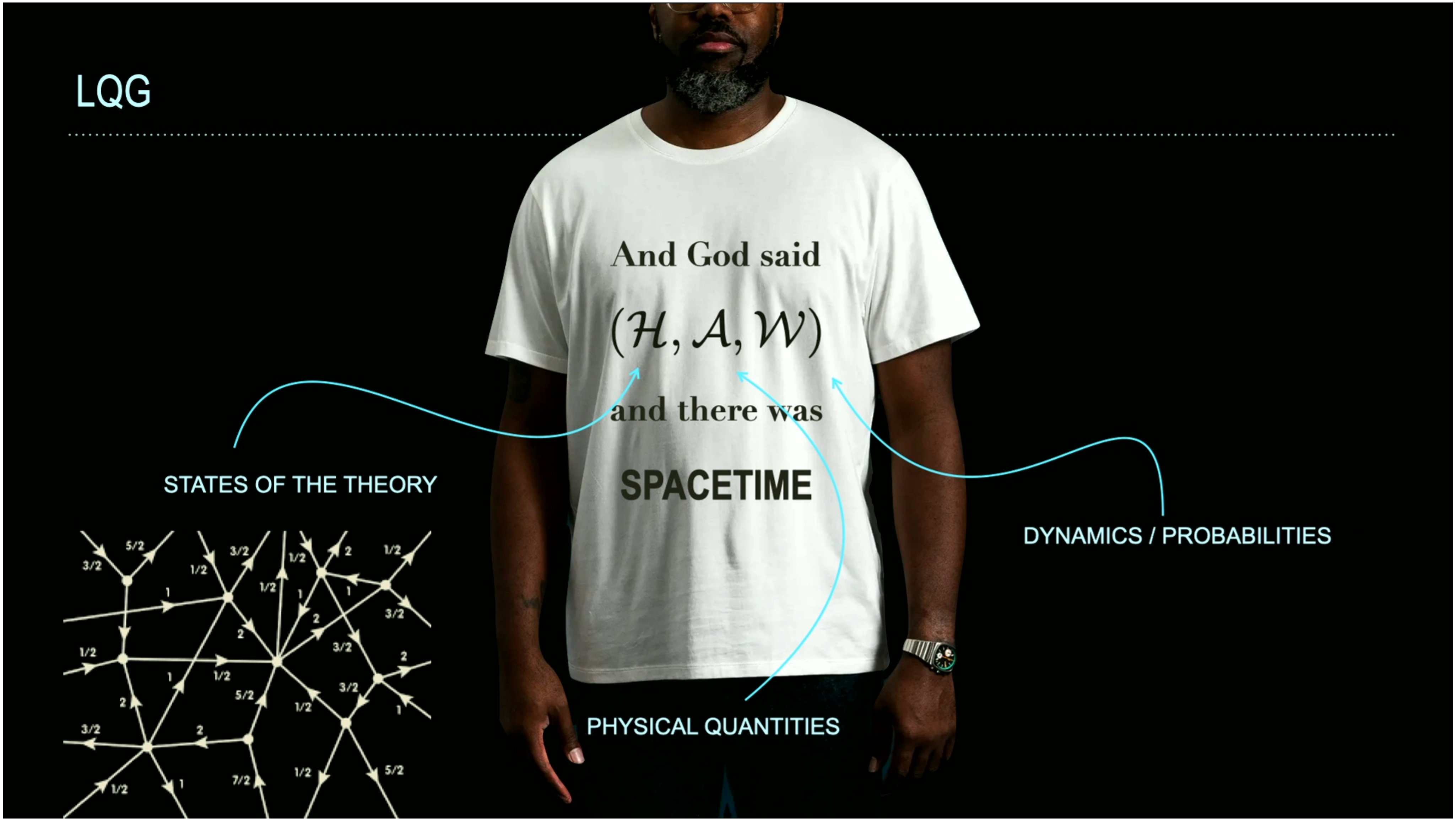
Abstract:

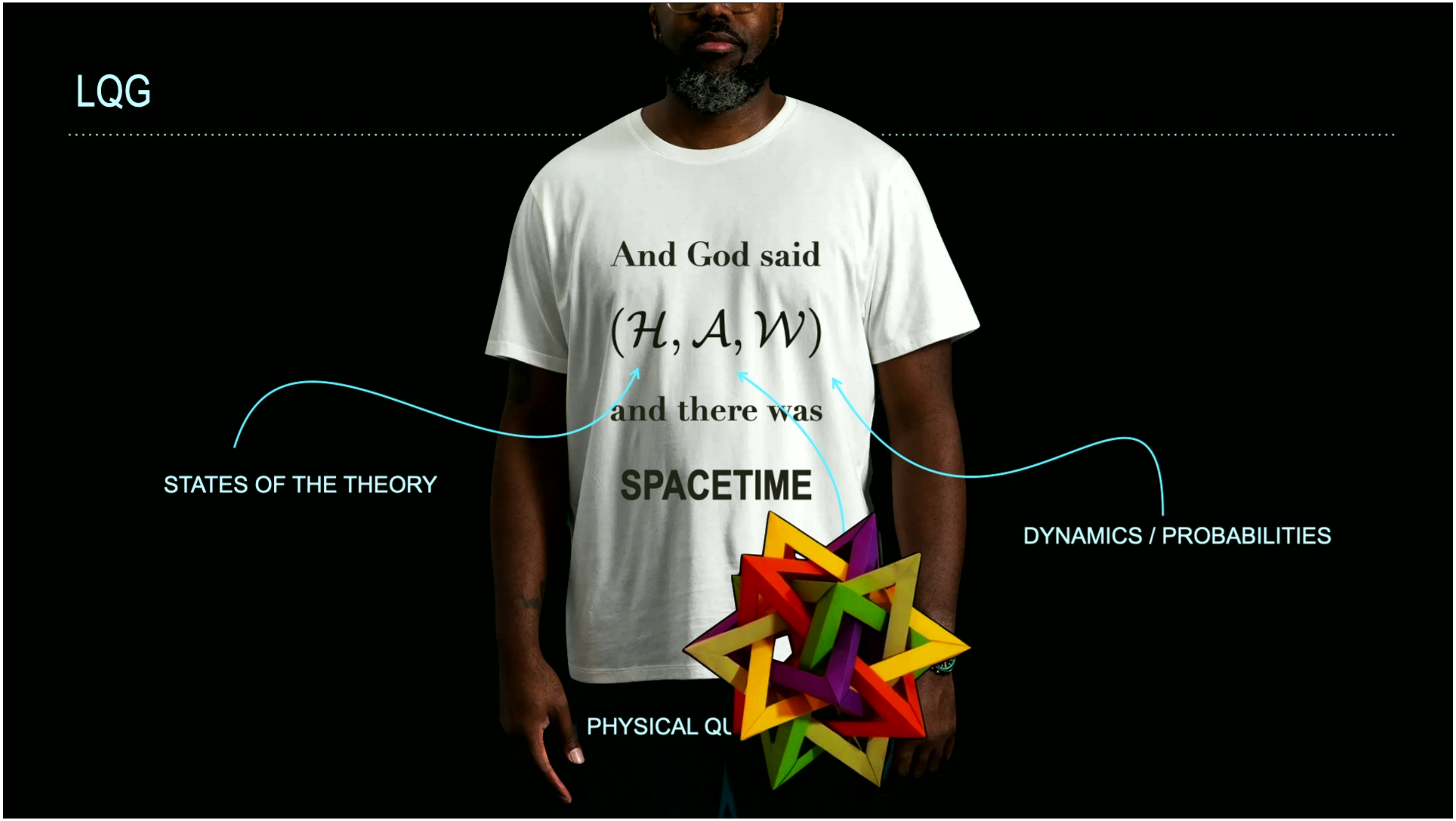
The covariant dynamics of Loop Quantum Gravity answers the question of what we can in principle predict and observe in quantum gravity. It features a characteristic discrete quantum evolution, as proposed in a seminal paper by Smolin and Markoupoulou in 1997. Spinfoam transition amplitudes are local exactly thanks to this form of discreteness. Another property of the amplitudes is to be exactly finite when their formulation includes a cosmological constant: this adds further to the fundamental discreteness of spacetime quanta, and again it is a feature rooted in Lee's Smolin seminal works.

What is a measurement in quantum gravity?

SPACETIME IS A PROCESS




LQG
And God said
$(\mathcal{H}, \mathcal{A}, \mathcal{W})$
and there was
SPACETIME
STATES OF THE THEORY
PHYSICAL QUANTITIES
DYNAMICS / PROBABILITIES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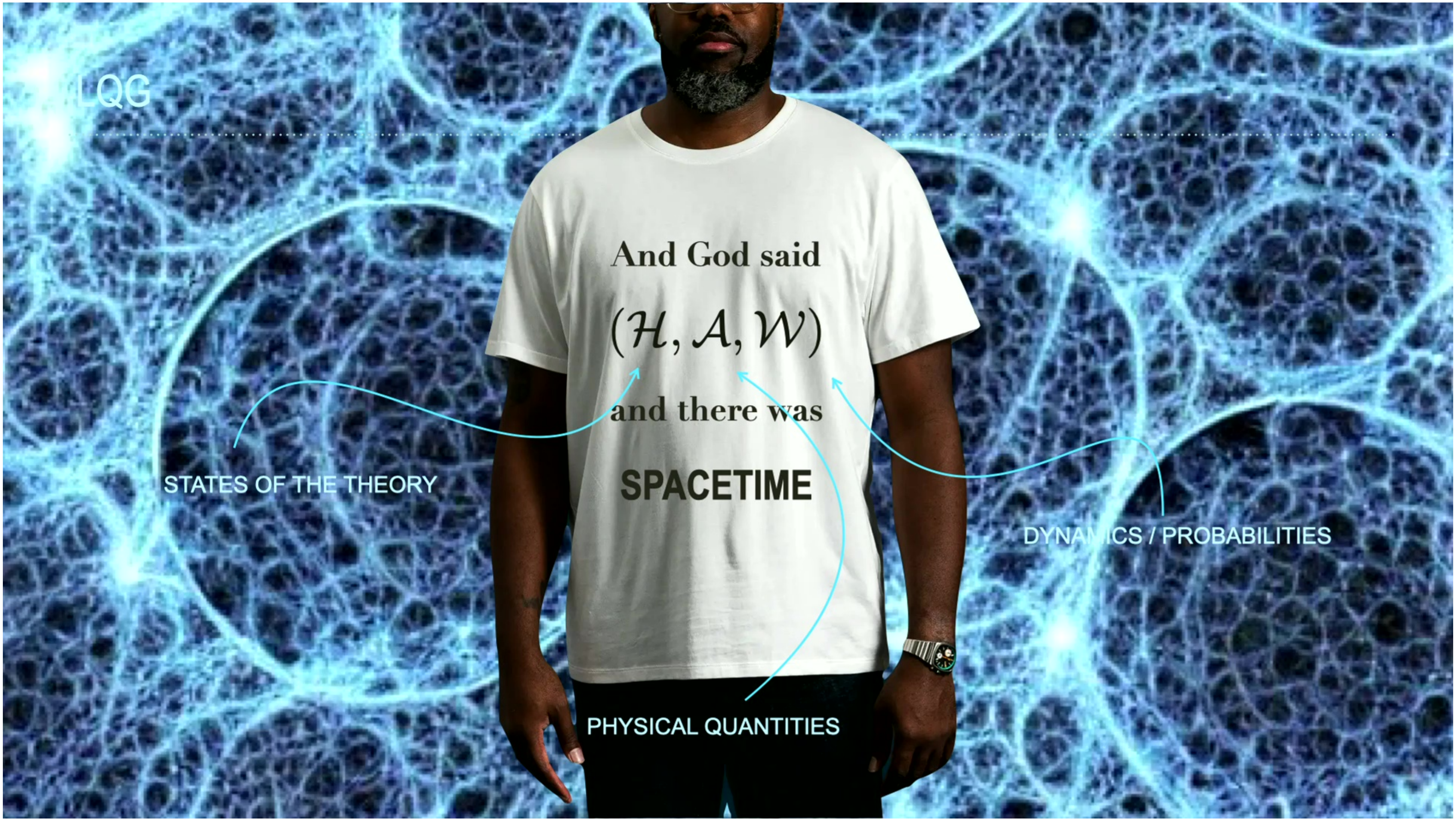
LQG
And God said
(ℋ, 𝒜, 𝒲)
and there was
SPACETIME
STATES OF THE THEORY
PHYSICAL QUANTITIES
DYNAMICS / PROBABILITIES

"Sum over Surfaces" form of Loop Quantum Gravity

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Erwin Schrödinger Institute, A-1090 Vienna, Europe.

We derive a *spacetime* formulation of quantum general relativity from (hamiltonian) loop quantum gravity. In particular, we study the quantum propagator that evolves the 3-geometry in proper time. We show that the perturbation expansion of this operator is finite and computable order by order. By giving a graphical representation *à la* Feynman of this expansion, we find that the theory can be expressed as a sum over topologically inequivalent (branched, colored) 2d surfaces in 4d. The contribution of one surface to the sum is given by the product of one factor per branching point of the surface. Therefore branching points play the role of elementary vertices of the theory. Their value is determined by the matrix elements of the hamiltonian constraint, which are known.

The formulation we obtain can be viewed as a continuum version of Reisenberger's simplicial quantum gravity. Also, it has the same structure as the Ooguri-Crane-Yetter 4d topological field theory, with a few key differences that illuminate the relation between quantum gravity and TQFT. Finally, we suggests that certain new terms should be added to the hamiltonian constraint in order to implement a "crossing" symmetry related to 4d diffeomorphism invariance.

gr-qc/9612035 May 1997

I. INTRODUCTION

An old dream in quantum gravity [1] is to define a manifestly spacetime-covariant Feynman-style "sum over trajectories" [2], sufficiently well defined to yield finite results order by order in some expansion. The hamiltonian theory has obtained encouraging successes in recent years, but it suffers for the well-known lack of transparency of the frozen time formalism, for the difficulty of writing physical observables and for operator ordering ambiguities. These problems are related to the lack of manifest 4d covariance. Here, we derive a covariant spacetime formalism *from* the hamiltonian theory. This is of course the path followed by Feynman to introduce his sum over trajectories in the first place [3]. What we obtain is surprising: we obtain a formulation of quantum gravity as a sum over surfaces in spacetime. The surfaces capture the gravitational degrees of freedom. The formulation is "topological" in the sense that one must sum over topologically inequivalent surfaces only, and the contribution of each surface depends on its topology only[1]. This contribution is given by the product of elementary "vertices", namely points where the surface branches. The sum turns out to be finite and explicitly computable order by order. The main result of this paper is the construction of this finite "sum over surfaces" formulation of quantum gravity.

Let us sketch here the lines of the construction. Given gravitational data on a spacelike hyper-surface Σ_i, the three-geometry on a surface Σ_f at a proper time T in the future of Σ_i (as measured along geodesics initially at rest on Σ_i), is uniquely determined in the classical theory. It is then natural to study the corresponding evolution operator $U(T)$, that propagates states from Σ_i to Σ_f in the quantum theory. This operator, first considered by Teitelboim [4], codes the dynamics of the quantum gravitational field, and is analogous to the Feynman-Nambu proper time propagator [5] for a relativistic particle. Here, we construct the operator $U(T)$ in quantum GR, and we expand it in powers of T. We obtain a remarkable result: the expansion is finite order by order. This is our first result.

Next, we construct a graphical representation of the expansion. This is obtained by observing that topologically inequivalent colored 2d surfaces σ in spacetime provide a natural bookkeeping device for the terms of the expansion. We obtain an expression for $U(T)$ as a sum of terms labeled by surfaces σ bounded by initial and final states. A

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[1] More precisely: on the diff-invariant properties of the surface.

Causal evolution of spin networks

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University Park, PA, USA 16802

February 2, 1997

ABSTRACT

A new approach to quantum gravity is described which joins the loop representation formulation of the canonical theory to the causal set formulation of the path integral. The theory assigns quantum amplitudes to special classes of causal sets, which consist of spin networks representing quantum states of the gravitational field joined together by labeled null edges. The theory exists in 3+1, 2+1 and 1+1 dimensional versions, and may also be interepreted as a theory of labeled timelike surfaces. The dynamics is specified by a choice of functions of the labelings of $d+1$ dimensional simplices,which represent elementary future light cones of events in these discrete spacetimes. The quantum dynamics thus respects the discrete causal structure of the causal sets. In the $1+1$ dimensional case the theory is closely related to directed percolation models. In this case, at least, the theory may have critical behavior associated with percolation, leading to the existence of a classical limit.

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DISCRETE EVOLUTION

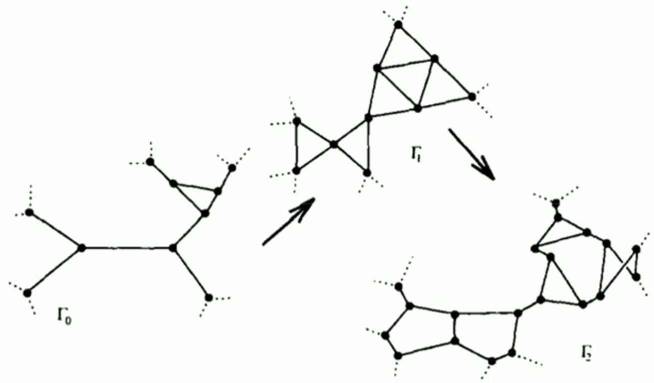


Fig. 2. Two steps in the evolution of a trivalent spin network, following first Rule 1, then Rule 2.

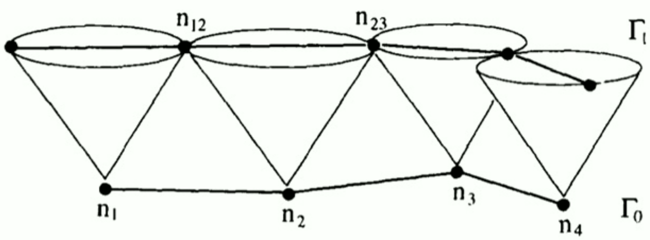


Fig. 3. The new nodes represent events defined by meeting of causal processes.

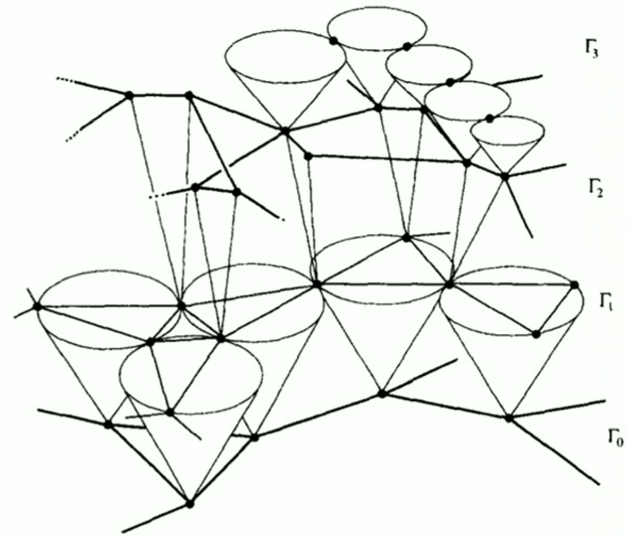


Fig. 12. A piece of a space-time network with the fourth level under construction. The light cones symbolize the null edges formed by Rule 1.

SPINFOAM AMPLITUDES

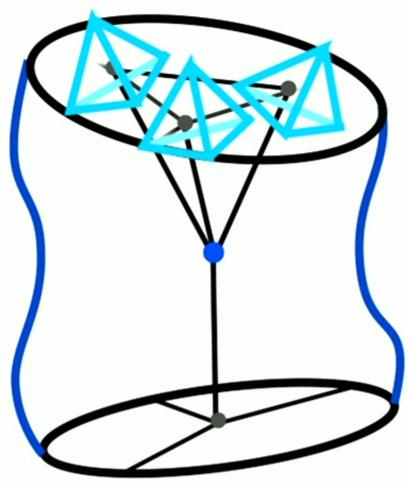
Probability amplitude $P(\psi) = |\langle W|\psi\rangle|^2$

for a state ψ associated to the boundary of a 4d region

$$W(q'_{ij}, q_{ij}) \sim \int_{\partial g = q', q} Dq\, e^{i\,S}$$

- Superposition principle $\langle W|\psi\rangle = \sum_\sigma W(\sigma)$
- Locality: vertex amplitude $W(\sigma) \sim \prod_v W_v.$
- Lorentz covariance $W_v = (P_{SL(2,\mathbb{C})} \circ Y_\gamma\, \psi_v)(\mathbf{1})$
- UV and IR finite (with Λ)
- Classical limit: GR (with Λ)
  (via Regge discretization)

Barrett et al. '09



www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf

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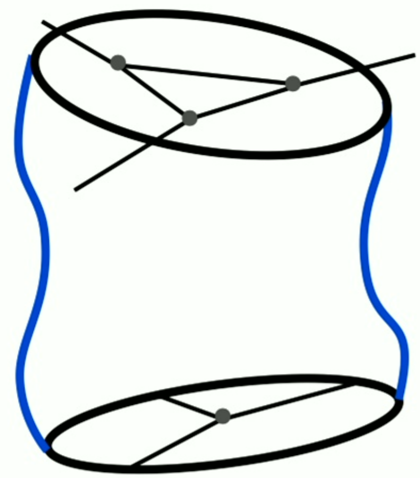


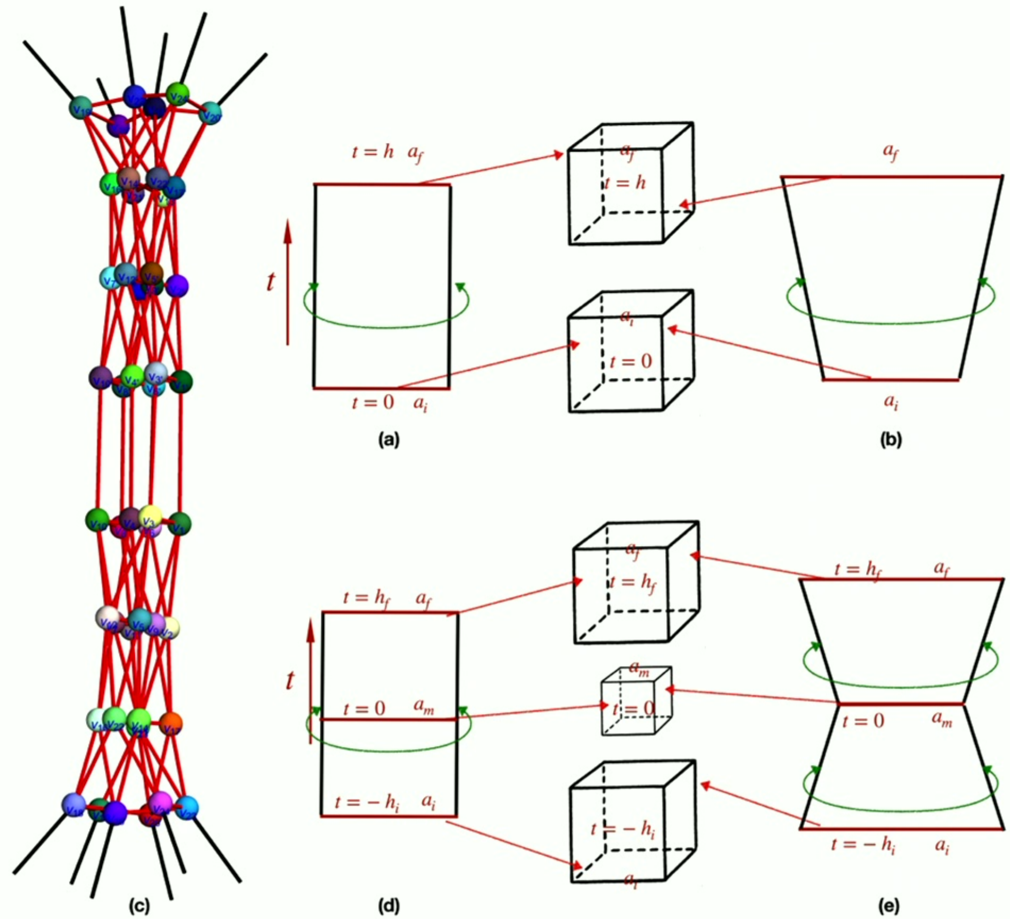
www.cpt.univ-mrs.fr/~rovelli/IntroductionLQG.pdf

COSMIC BOUNCE FROM SPINFOAM

[Han, Liu, Qu, Vidotto, Zhang '24]

- Hypercube ~ torus
- Coupling with a scalar field
- Same initial and final state
  but for a flip
  in the extrinsic curvature
- Tunneling given by suppressed but non-vanishing contributions

  [Donà, Haggard, Rovelli, Vidotto '24]
- In the semi-classical limit: action with extra higher-derivative terms



SPINFOAM AMPLITUDES

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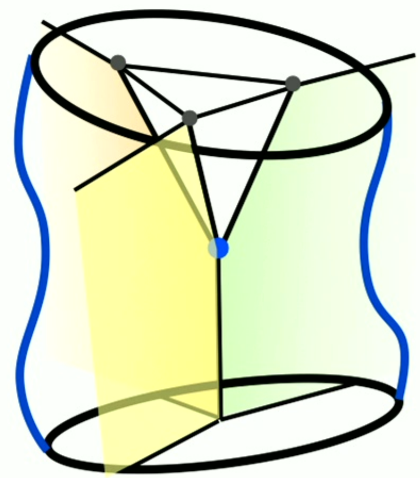
Barrett et al. '09



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ELSEVIER

Nuclear Physics B 473 (1996) 267–290

NUCLEAR PHYSICS B

Quantum deformation of quantum gravity

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Received 13 December 1995; accepted 8 May 1996

Abstract

We describe a deformation of the observable algebra of quantum gravity in which the loop algebra is extended to framed loops. This allows an alternative nonperturbative quantization which is suitable for describing a phase of quantum gravity characterized by states which are normalizable in the measure of Chern–Simons theory. The spinor identities are extended to a set of relations which are governed by the Kauffman bracket so that the spin network basis is deformed to a basis of $SU(2)_q$ spin networks. This deformation parameter, q, is $e^{i\hbar^2 G^2 \Lambda/6}$, where Λ is the cosmological constant. Corrections to the actions of operators in nonperturbative quantum gravity may be readily computed using recoupling theory; the example of the area observable is treated here. Finally, eigenstates of the q-deformed Wilson loops are constructed, which may make possible the construction of a q-deformed connection representation through an inverse transform.

1. Introduction

In the past five years a number of striking consequences of diffeomorphism invariance have emerged in the nonperturbative approach to quantum gravity based on the loop representation ([1–21], for reviews see [3,4]). One of these provides a *basis* of spatially diffeomorphism invariant states labeled by diffeomorphism equivalence classes of embeddings of spin networks [5,6] [3]. In this context, a spin network is a graph with edges labeled by representations of $SU(2)$ and vertices labeled by the ways that the edge representations may be combined into a singlet. This concept of a spin network was

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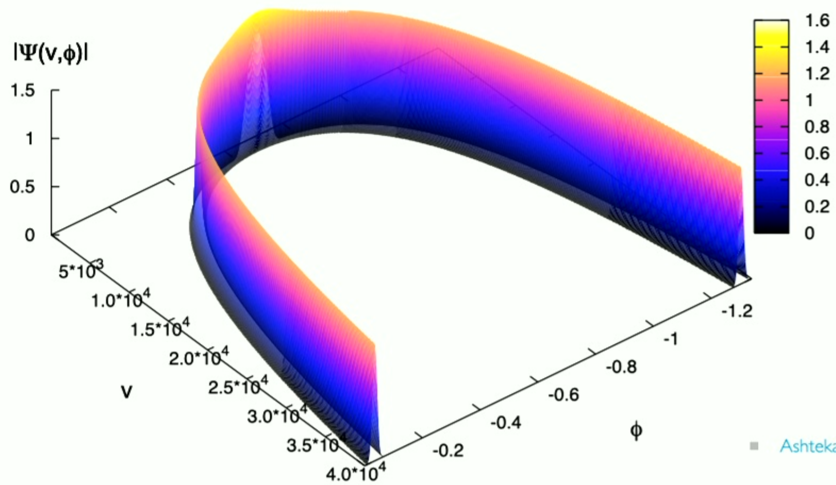
SINGULARITY RESOLUTION

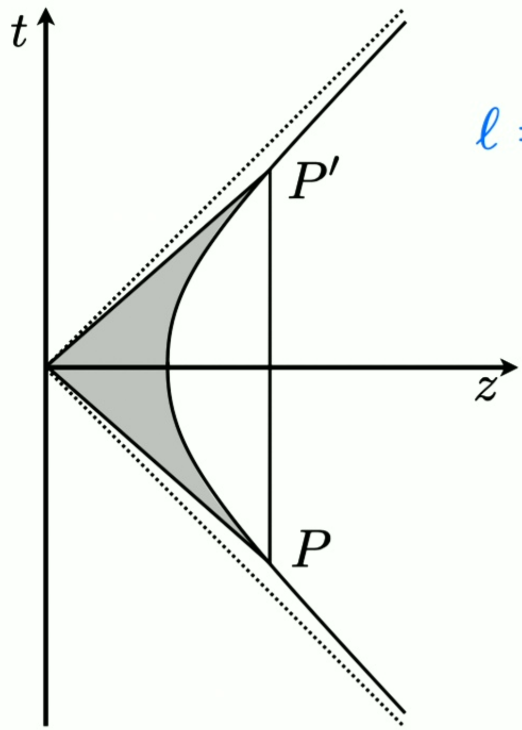


$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$

$$v'' - \left(1 - 2\frac{\rho}{\rho_c}\right)\nabla^2 v - \frac{z''}{z}v = 0$$

- Ashtekar, Pawlowski, Singh '06

MAXIMAL ACCELERATION



$\ell = 1/a$

[Rovelli,Vidotto '13]

$$\vec{K} + \gamma \vec{L} = 0$$

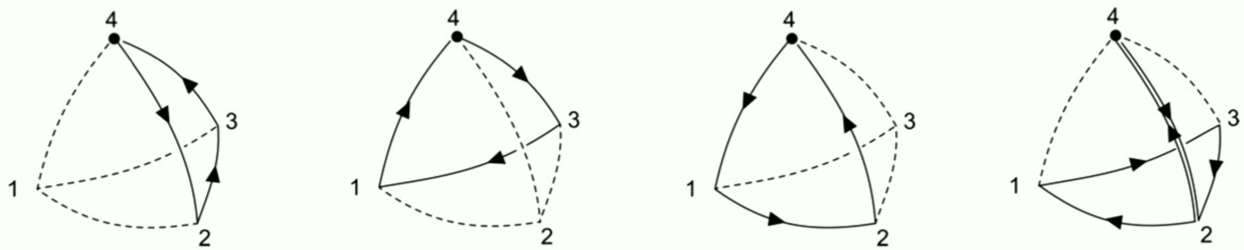
Boost generator — Rotation generator

$$W(\eta, j) = \langle j, j | Y^\dagger e^{i\eta K_z} Y | j, j \rangle$$

- motion of an accelerated observer in spacetime
- evolution of spacetime seen by an observer

COMPACT PHASE SPACE & DISCRETE TIME

$$(k = e^J, h) \in SU(2) \times SU(2)$$

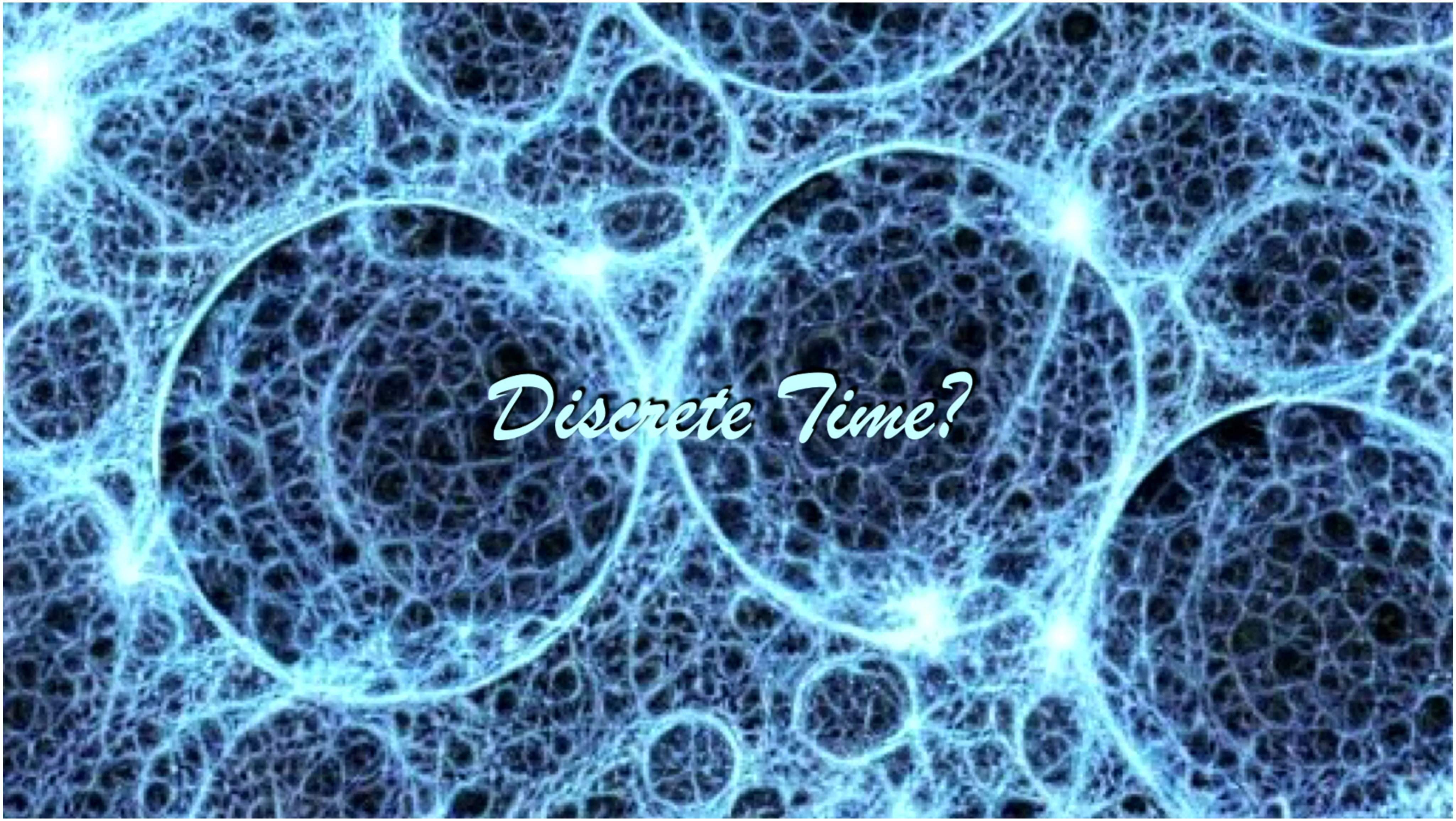


- Associate an SU(2) element for each face

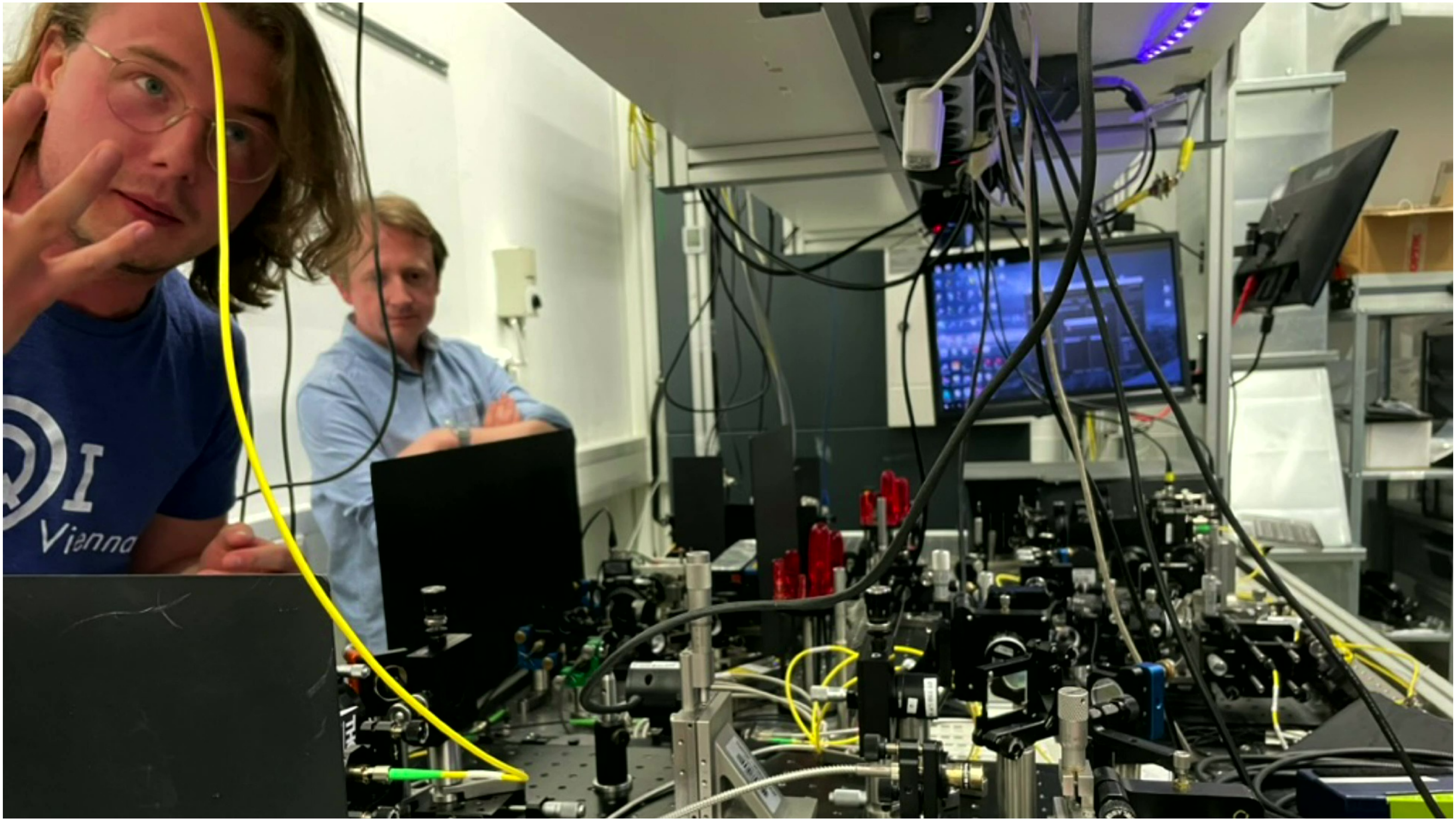
[Haggard-Han-Kaminski-Riello '14]
[Charles-Livine '15]

- **Compactness:** discretization of the intrinsic and extrinsic geometry [Rovelli-Vidotto '15]

- **Time discreteness:** $K_{ab} \sim dq_{ab}/dt$ where $q_{ab}(\Delta t) \sim q_{ab}(0) + dq_{ab}/dt \ \Delta t$
minimum proper time Planckian, full discrete spectrum depends on cosmological constant


Discrete Time?

Can we measure time discreteness?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Lee Smolin
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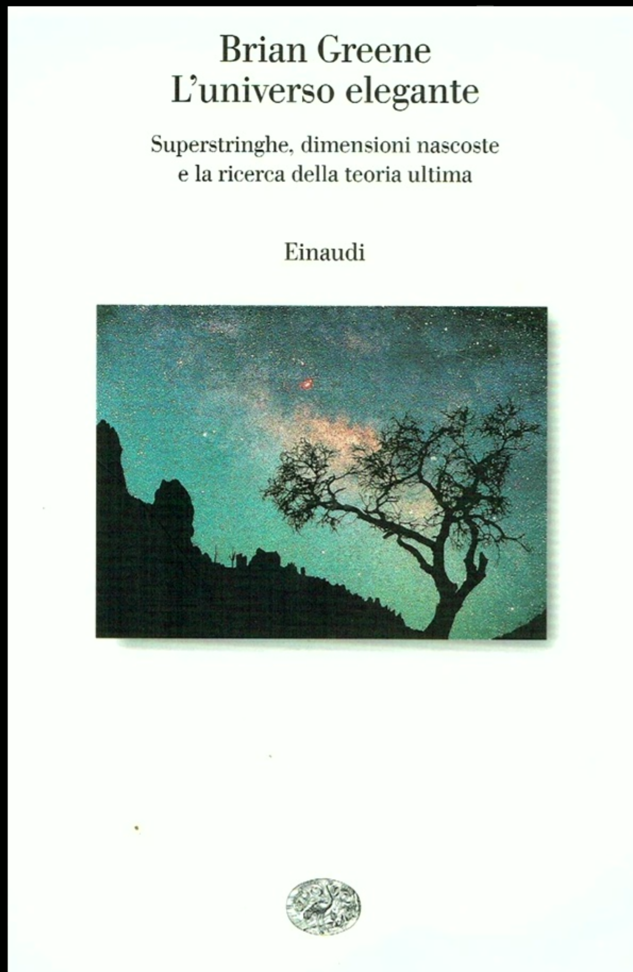
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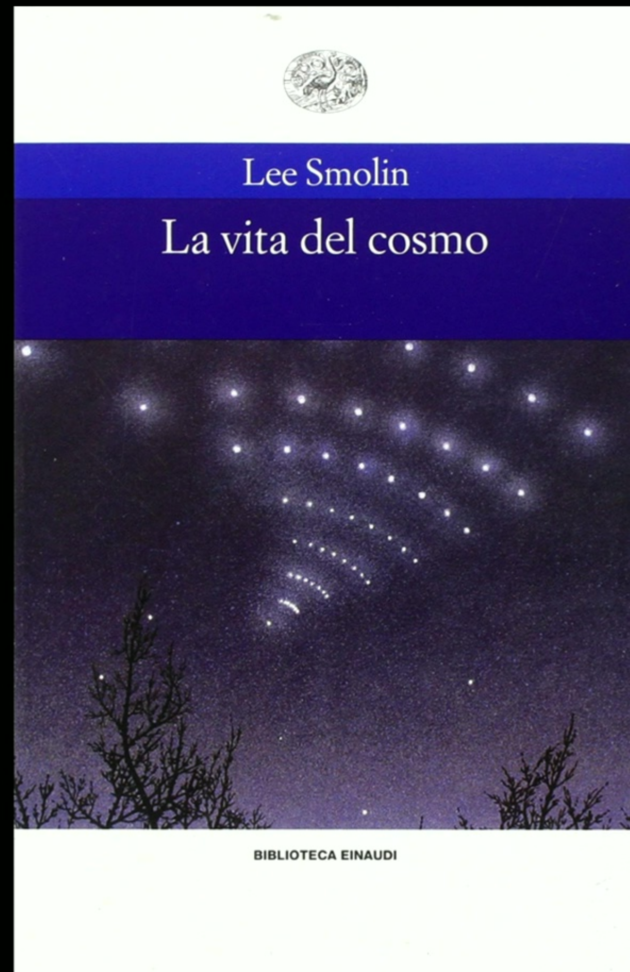
e la ricerca della teoria ultima
Einaudi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SLIDING DOORS


Thank you Lee!