Title: Reassessing the i.i.d. Assumption in Probability Assignments in Quantum Gravity

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Abstract:

I will examine a foundational assumption in quantum probability assignments, that experimental data arise from an identically and independently distributed (i.i.d.) ensemble. However, this assumption becomes problematic in the regime of quantum gravity. I will outline a proposal to resolve this issue by leveraging the tool of quantum reference frames in the measurement context.

Reassessing the i.i.d. assumption in quantum probability assignments

Based on a project with C. Brukner and R. Simmons

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Lee's Fest, June 2025

Thank you Lee to be our Yoda!
Happy birthday!

- As a PhD supervisor, Lee's kindness, passion, courage, originality, profound thinking, and open-mindedness will inspire and influence me for a lifetime!

- Lee continually inspires me to ask:
What are the fundamental principles underlying a theory of quantum gravity? Do we need to revise any of the core assumptions of quantum physics or general relativity to achieve such a theory?

- A consequence of Lee's view is that there is no such thing as the quantum state of the universe, or indeed of any macroscopic object. Quantum mechanics only applies to systems simple enough to be describable by identical quantum states.

— S. Weinstein's talk yesterday

- **How about simple systems but very heavy?**

The relative frequency operator

- The validity of probability assignment can be viewed through the relative frequency operator.

  [D. Finkelstein, 1963, "Logic of Quantum physics", Transactions of the NY Academy of Sciences, 25: 621- 637
  J. Hartle "Quantum Mechanics of Individual Systems", Am. Jour. Phys., 36, 704-712, (1967).]

- Although an individual measurement outcome is inherently unpredictable, the relative frequencies of an outcome become definite in the limit of an infinite ensemble of i.i.d systems:

$$|S\rangle^{\otimes N} := |S\rangle_1 \otimes |S\rangle_2 \otimes .... \otimes |S\rangle_N$$

- Supposingly we study observable A with eigenstates $\{|i\rangle\}$

$$\hat{F}^k_N(\hat{A}) := \sum_{i_1, i_2 .... i_N}^{d} \sum_{\alpha=1}^{N} \frac{\delta_{k,i_\alpha}}{N} \otimes_\alpha |i_\alpha\rangle\langle i_\alpha|$$

  The probability of observing outcome a_k becomes the eigenvalue of the joint state:

$$\lim_{N\to\infty} \hat{F}^k_N(\hat{A})|S\rangle^{\otimes N} = \lim_{N\to\infty} |\langle k|S\rangle_\alpha|^2 |S\rangle^{\otimes N}.$$

- In particular, for non-commuting observables $[\hat{A}, \hat{B}] \neq 0$, $\lim_{N\to\infty} [\hat{F}_N(\hat{A}), \hat{F}_N(\hat{B})]|S\rangle^{\otimes N} = 0$

  this ensures that the probabilities for all observables are simultaneously well-defined.

The i.i.d assumption in quantum gravity

The first level — the entanglement from gauge constraints

- For N copies of identical static quantum sources $S^{(N)}$, $|S\rangle = \int \mathcal{D}(E)\phi(E)|E\rangle$
in perturbative regime, we have the total wave function:

[LQC, F. Giacomini, C. Rovelli, 2023]
[LQC, F. Giacomini, 2024]

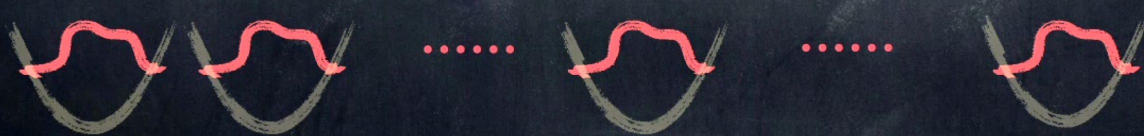
$$|\Psi\rangle_{S^{(N)},G} = \eta \int \mathcal{D}[h_{ij}] \prod_{\alpha}^{N} \mathcal{D}(E_\alpha)\phi_\alpha(E_\alpha)\delta\left(h^T - \mathrm{h}^T_{\sum_\alpha^N E_\alpha}\right) \Psi^G_{vac}[h_{ij}]|E\rangle_\alpha |h_{ij}\rangle_G$$

- The i.i.d assumption does not hold for this ensemble. One can check that for quantum sources:

$$[\hat{T}_{00}, \hat{x}_S] \neq 0 \qquad \lim_{N\to\infty} [\hat{F}_N(\hat{T}_{00}), \hat{F}_N(\hat{x}_S)]|\Psi\rangle_{S^{(N)},G} \neq 0,$$

- Practical resolution one may propose: put sources far enough from each other.

Or:



That's how i.i.d ensemble of charged particles are prepared, despite of Gauss constraints.

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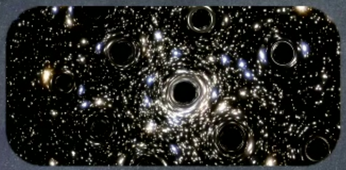
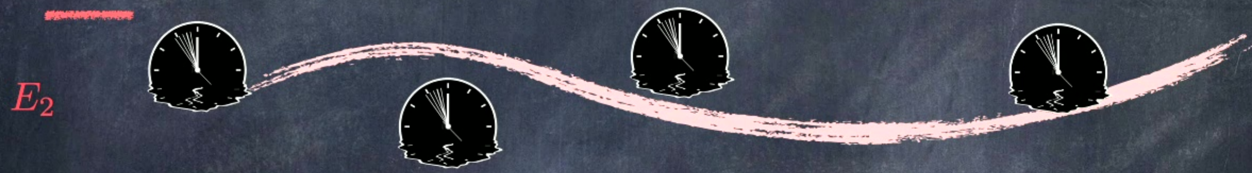
The i.i.d assumption in quantum gravity

- However, this seems to be a fundamental issue, rather than a merely practical concern, and is intrinsic to quantum gravity.



- Imagine if we try to define state tomography for N ensemble of quantum clocks far away from each other, in the asymptotic flat spacetime. The more precise we want the clocks to be, the larger energy gap it needs (between energy levels)!



The ADM energy for asymptotic flat spacetime $E = \dfrac{1}{16\pi G}\int_{S^\infty}\left(\partial^i h_{ij} - \partial_j h_{ii}\right) d\sigma^j$
tells us that even at far away, there is always imprint of clock's energy in the metric!

- If we try to define tomography, the measurement of any single clock energy would influence the quantum states and ticking rates of other clocks.

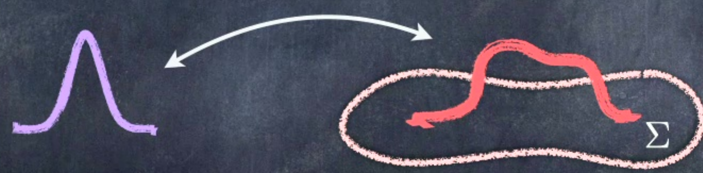
- The ensemble preparation can be identical, but not independent, in the first place!

The i.i.d assumption in quantum gravity

In summary, this second level of i.i.d problem is unique for QG

— the lack of split property and back-reaction:

- The universal coupling of gravity ensures that any operation and measurement all back-react to the next in the sequence, it is fundamentally non-independent.
- The unique feature in quantum gravity: one cannot assign quantum states independently in a subregion and its complement - the lack of split property.



- Related issues have also been discussed as QG violates assumptions of Cencov's theorem — the Fisher information metric is the unique metric invariant under sufficient statistics. The authors resolution is to make the Born rule/information metric dynamical!

  [Information Metrics and Possible Limitations of Local Information Objectivity in Quantum Gravity, P. Berglund, A. Geraci T. Hubsch, D. Mattingly, D. Minic arXiv:2501.19269v1]

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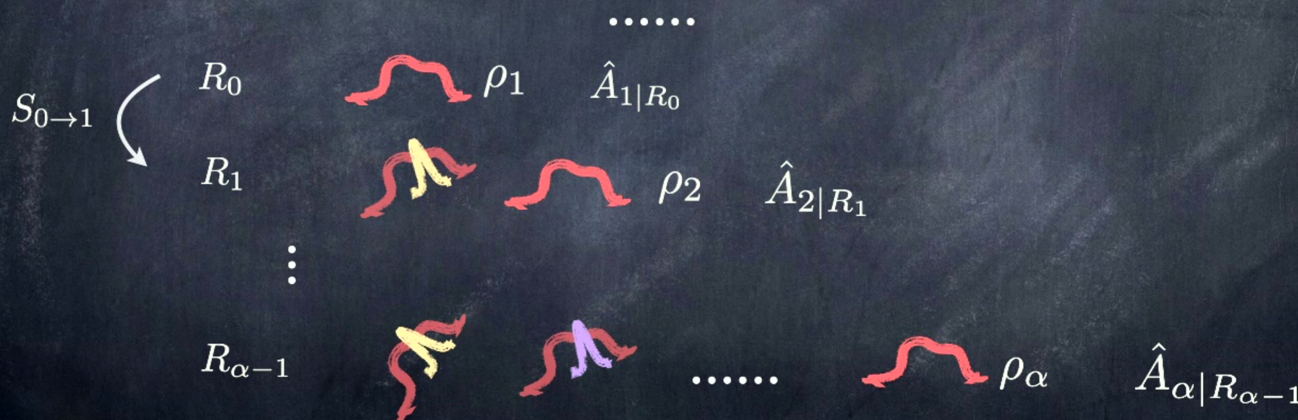
The outline of one proposal of resolution

- **Idea: Reset the reference frame!**
- We start with a local intertial frame R_0
- After preparing the first massive quantum state ρ_1 and performing measurement (relative to R_0), we preform quantum reference frame transformation $S_{0\to 1}$ into local inertial frame R_1



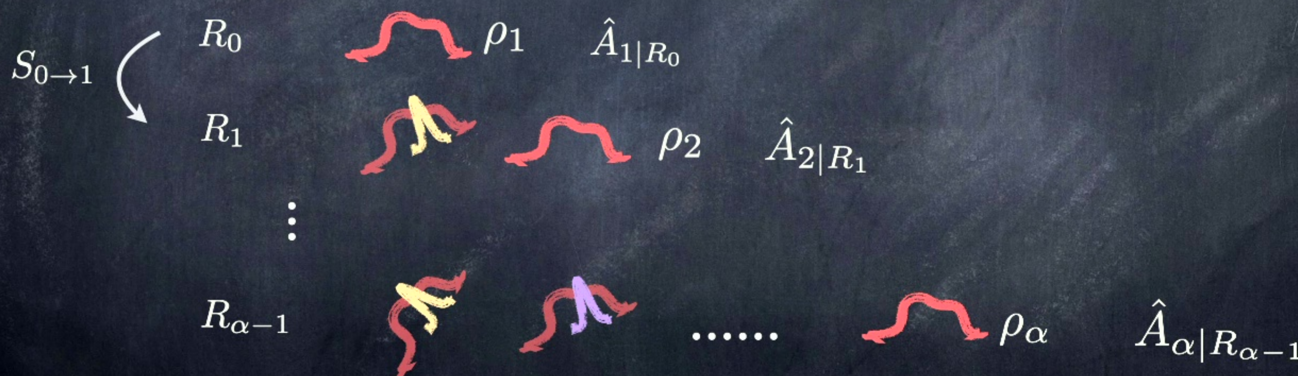
- After preparing the α th massive quantum state ρ_α and performing measurement (relative to $R_{\alpha-1}$), we preform quantum reference frame transformation into local inertial frame R_α

The outline of one proposal of resolution

- The ensemble becomes $\rho^{(N)} := \rho_{1|R_0} \vec{\otimes} \rho_{2|R_1} \vec{\otimes} \dots \vec{\otimes} \rho_{N|R_{N-1}}$

  The relative frequency operator: $\hat{F}_N^k := \sum_{i_1, i_2 \dots i_N}^{d} \sum_{\alpha=1}^{N} \frac{\delta_{k,i_\alpha}}{N} \vec{\otimes}_{\alpha|R_{\alpha-1}} \hat{\Pi}^i_{\alpha|R_{\alpha-1}}$

  in which $\hat{\Pi}^i_{\alpha|R_{\alpha-1}}$ is the measurement projector into the subspace of observable $\hat{A}_{\alpha|R_{\alpha-1}}$



- The correlation due to diffeo, back-reaction and lack of the split property shifts into the correlations among quantum reference frames; with assuming the quantum equivalence principle, the existence of local inertial frame is powerful enough assures that we can reset to i.i.d!

Thank you!

Wish Lee endless journey full of love, joy, and new ideas!

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