Title: Quasi-topological principle and its degrees of freedom

Speakers: Simone Speziale

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Abstract:

I will briefly review an idea spurring from Lee, Stephon and Joao for an action with a varying cosmological constant and at most only three new free parameters, based on a `quasi-topological principle', and show how one can determine its number of degrees of freedom at the non-perturbative level, and some of the interesting features that arise in the analysis.

The quasi-topological principle and its degrees of freedom

Simone Speziale

Lee's Fest, June 4 '25

CPT Centre de Physique Théorique

cnrs

It is a great honor and a great joy for me to participate to this event celebrating Lee

His passion, his curiosity, and never-ending push to support creative and bold ideas has been very inspirational on what science is about

One of Lee's teachings that I hold dearest, is that one can
be ambitious without being arrogant,
be driven without being narrow minded.
Especially in the current world of high energy theoretical physics,
where often opinions count as experiments,
these are really important and deep pieces of advice.

Another one, is the importance of a having fun in what we do!

Another important piece of advice from Lee was ***not be afraid of being wrong.***

This advice came in pretty handy in our second paper together,
where we predicted a Higgs mass that was wrong by 105 orders of magnitude!

Unification of gravity, gauge fields, and Higgs bosons

G. Lisi, L. Smolin, S '10

based on a previous idea by Lee "The Plebanski action extended to a unification of gravity and Yang-Mills theory," 2007, we considered

$$S(\omega, B, \Phi) = \int B \wedge F + \Phi B \wedge B + \Phi^2 B \wedge B$$

with gauge group spin(4+N) to be spontaneously broken to spin(4)x spin(N)
of gravitational and YM sectors, with a Higgs field as bonus

The model is elegant and has only one free parameter, and it turns out that $m_H \sim \Lambda$
So taking the measured value $\Lambda \sim 10^{-122}$ in Planck units, our Higgs mass was wrong by 105 orders of magnitude

The quasi-topological principle

S. Alexander, J. Magueijo, L. Smolin '18

S. Alexander, M.Cortes, A. Liddle, J. Magueijo, R. Sims, L. Smolin '19

What makes theoretical physics beautiful is the possibility of understanding the world around us using a few simple principles

Simple and possibly elegant maths, **as few free** parameters as possible such as Λ-CDM.

But if DESI/Euclid e.g. determine that Λ has varied over the epochs,
then the genie is out of the bottle, with a large number of models on the table,
each typically with a large number of free parameters to tweak and make it work

To tame the genie, they came up with a beautiful idea that allows a varying Λ without allowing infinitely many free parameters as with most other models:

the quasi-topological principle
allow new terms in Λ only if they are topological when Λ is constant

The quasi-topological principle

S. Alexander, J. Magueijo, L. Smolin '18

S. Alexander, M.Cortes, A. Liddle, J. Magueijo, R. Sims, L. Smolin '19

The intuition behind this idea is that when gravity is described using differential forms, there is only a finite number of terms that can be written down

$$S(e,\omega,\Lambda) = \frac{1}{2}\int\left(\frac{1}{2}\epsilon^{IJ}{}_{KL} + \frac{1}{\gamma}\delta^{IJ}_{KL}\right) e_I \wedge e_J \wedge \left(F^{KL}(\omega) - \frac{\Lambda}{6}\, e^K \wedge e^L\right)$$
$$+ \frac{1}{2}\int\left(\frac{\alpha}{2}\epsilon^{IJ}{}_{KL} + \beta\,\delta^{IJ}_{KL}\right)\frac{1}{\Lambda} F_{IJ}(\omega) \wedge F^{KL}(\omega), \qquad F_{IJ}(\omega) \wedge F^{KL}(\omega),$$

γ Barbero-Immirzi parameter

α coupling of Euler term

β coupling of Pontryagin term

Consider now coupling to Λ, and promote it to a dynamical field

Only 2 free parameters!

This changes dramatically the field equations

- torsion is allowed
- Λ acquires a spacetime dependence determined by F
- there are new degrees of freedom on top of the graviton

An appealing feature is that there is a varying-Λ de Sitter solution, which can be used for cosmological models, but only for $\alpha = -\frac{3}{2}, \qquad \beta = -\frac{3}{2\gamma}.$

⇨ **zero** new free parameters

Canonical analysis

S. Alexandrov, S, T. Zlosnik '21

How many new degrees of freedom are there?

$$S(e,\omega,\cancel{X}) = \frac{1}{2}\int \left(\frac{1}{2}\epsilon^{IJ}{}_{KL} + \frac{1}{\gamma}\delta^{IJ}_{KL}\right) e_I \wedge e_J \wedge \left(F^{KL}(\omega) - \frac{\Lambda}{6}\, e^K \wedge e^L\right)$$

Without knowing the general solution to the field equations, it is possible to count the degrees of freedom using the canonical analysis and Dirac's method of stabilizing constraints

Einstein-Cartan-Holst action (same solutions as vacuum GR) :

- 4 Diffeos, 6 Gauss → 10 first class constraints
- 6 simplicity, 6 secondary → 12 second class constraints

Counting : 18+18 - 20 - 12 = 4 ⇨ 2 dofs

Canonical analysis

S. Alexandrov, S, T. Zlosnik '21

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Full action :

- 4 Diffeos, 6 Gauss → 10 first class constraints
- 6 simplicity, 2 secondary → 12 second class constraints

Counting : 18+18+2 - 20 - 8 = 10 ⇨ 5 dofs

Canonical analysis

S. Alexandrov, S, T. Zlosnik '21

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Without knowing the general solution to the field equations, it is possible to count the degrees of freedom using the canonical analysis and Dirac's method of stabilizing constraints

Full action :

- 4 Diffeos, 6 Gauss → 10 first class constraints
- 6 simplicity, 2 secondary → 12 second class constraints

Counting : 18+18+2 - 20 - 8 = 10 ⇨ 5 dofs

Exception: $\left(1 - \frac{1}{\gamma^2}\right)\beta = \frac{2\alpha}{\gamma}$ → 6 secondary ⇨ 3 dofs

These results confirm and generalize what found in cosmological perturbations

S. Alexander, M.Cortes, A. Liddle, J. Magueijo, R. Sims, L. Smolin '19

J. Magueijo, T. Zlosnik '19

de Sitter degeneracy:

linearized theory around de Sitter solution has additional conformal symmetry ⇨ 4 dofs

(similar to the partial masslessness at $3m^2=2\Lambda$ in massive gravity)

Canonical analysis

S. Alexandrov, S, T. Zlosnik '21

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$$+\frac{1}{2}\int\left(\frac{\alpha}{2}\epsilon^{IJ}{}_{KL} + \beta\,\delta^{IJ}_{KL}\right)\frac{1}{\Lambda}F_{IJ}(\omega)\wedge F^{KL}(\omega),$$

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What is the geometric interpretation of the extra dofs?

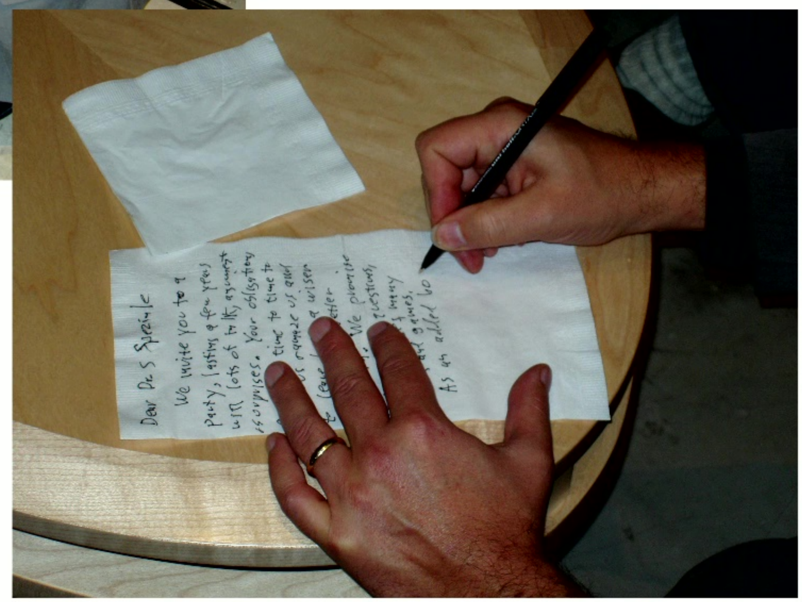
de Sitter degeneracy:

linearized theory around de Sitter solution has additional conformal symmetry ⇨ 4 dofs

(similar to the partial masslessness at $3m^2=2\Lambda$ in massive gravity)

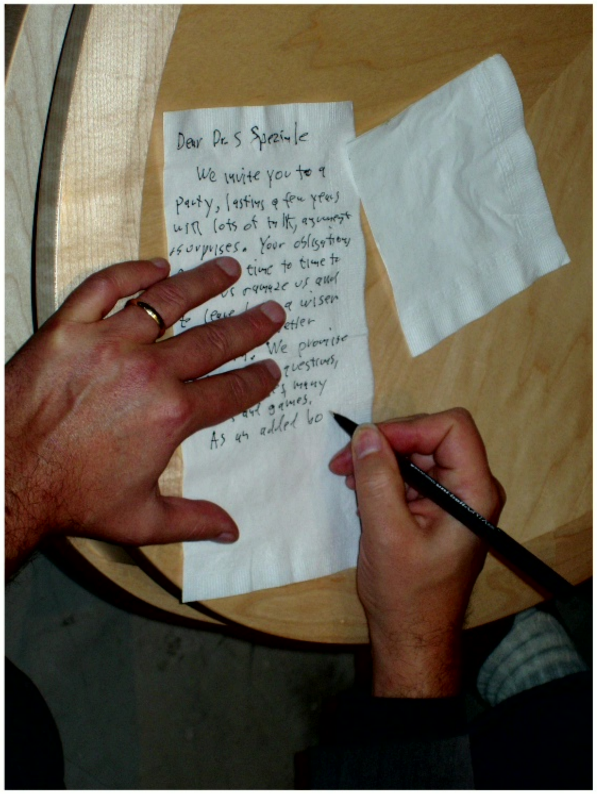
Now, coming back to that party…


PLEASE HELP US
KEEP OUR MALL
CLEAN
THROW YOUR
GARBAGE IN THE
CONTAINERS
Thank You



We invite you to a party
with lots of talk,
discussions and surprises…

Thank you Lee,
for this ever-going party full of talk,
discussions and surprises!



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Thank you Lee,
for all you have done for the community,
for always pushing for creativity
and boldness in science!

Thank you Lee,
for infusing your vision and energies
in the Perimeter Institute!

Thank you Lee,
for showing us that even the hardest of times
can be faced with honesty, strength and courage!