Title: Bulk-Boundary in Spinfoams

Speakers: Etera Livine

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

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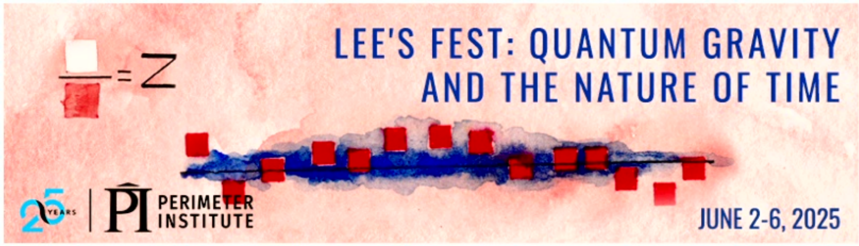
Abstract:

Spinfoams define a path integral for quantum gravity based on topological quantum field theory (TQFT). Following Lee’s early intuitions about the implementation of the holographic principle in the formalism, I will explain how the bulk dynamics arises as a network of boundary theories and draw lessons about the bulk-boundary relations. Diving into the example of 3d quantum gravity, this is illustrated by a beautiful holographic-like duality with the 2d inhomogeneous Ising model, with an underlying emergent supersymmetry, opening an intriguing interplay with the world of statistical physics with an elegant geometric formula for the zeroes of the Ising partition function.

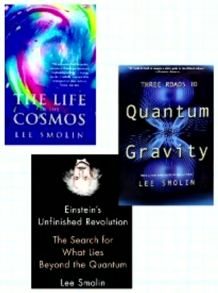


Bulk-Boundary in Spinfoam Gravity

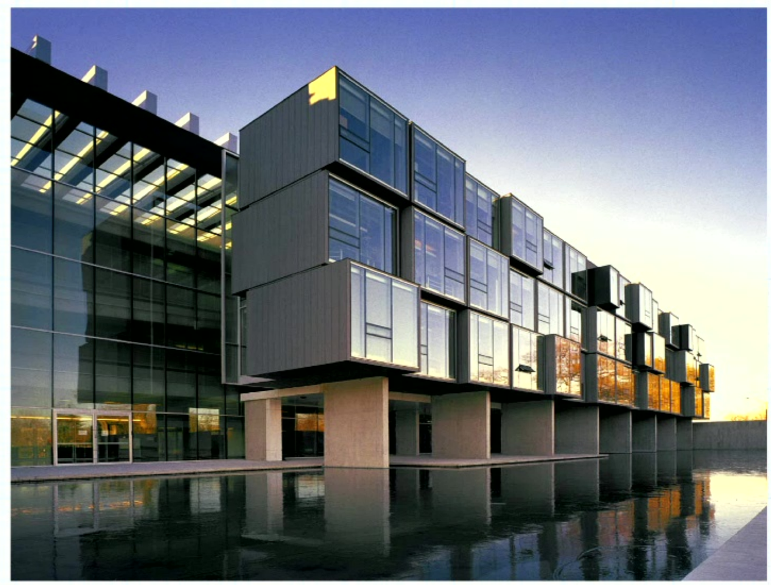
Etera Livine
Laboratoire de Physique LP ENSL & CNRS

Perimeter Institute - for Lee

Quantum Gravity Explorers



Imperial College
London



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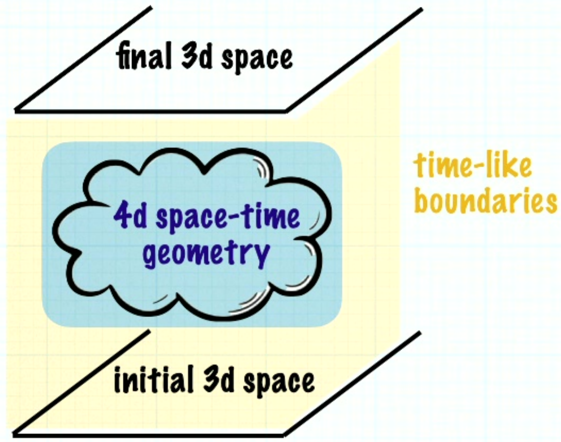
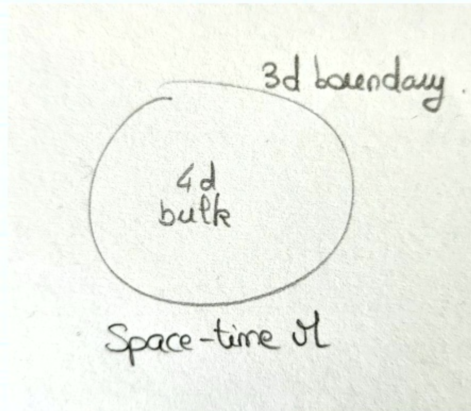
Path Integral for Quantum Geometry





$$Z_{\mathcal{M}}[\varphi_{boundary}] = \int \mathcal{D}\varphi_{bulk}\, \underbrace{\mathcal{A}_{\mathcal{M}}[\varphi_{bulk}]}_{\underset{\text{semi-classical regime}}{\sim}\; e^{i\frac{1}{\hbar}S_{\mathcal{M}}[\varphi_{bulk}]}}$$

Bulk geometry d.o.fs

Integrate over bulk fields given boundary conditions

boundary state(s)

Depends on boundary state + properties of 4d-M

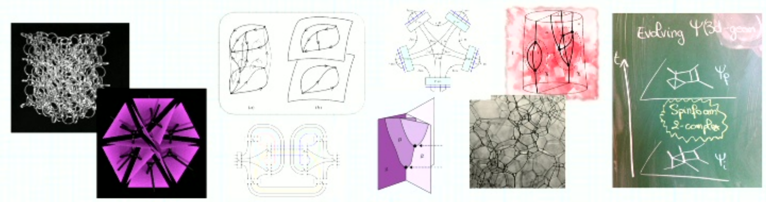
Bulk to Boundary

A simple example ?

Spinfoam Models from TQFTs

Spinfoams = a picture of 4d quantum space-time at Planck scale
+
local amplitudes for each possible quantum geometry

- Reformulate gravity as topological BF with extra stuff
- Discretize/Quantize topological BF exactly



- Re-introduce local bulk d.o.fs as sea of defects
- Check renormalization flow & large scale limit ?

Modify A_v [Ψ_{bulk}]

- Get boundary theory
- Identify suitable representation(s) of boundary symmetry algebra (bootstrap ?)

Adjust Z_v [$\Psi_{boundary}$].

Bulk to Boundary

scalar field

U(1) connect°

$$S[\phi,\omega] = \int_{\mathcal{M}_{2d}} \phi F[\omega] - \kappa \phi^2$$

curvature

mass-term

2d BF theory

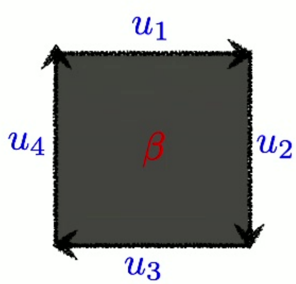
Bulk to Boundary

scalar field — U(1) connection

$$S[\phi,\omega]=\int_{M_{2d}} \phi F[\omega]-\kappa\phi^2 \qquad \text{2d BF theory}$$

curvature — mass-term

- **Boundary data:** U(1)-holonomy around 2d cell



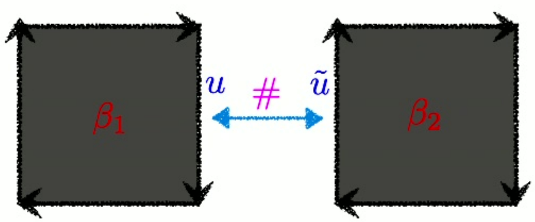
$$Z_\partial^{(\beta)}[\{u_i\}]=e^{i\beta\left(\sum_k u_k\right)^2}$$

with boundary coupling: $\beta^{-1}\propto\kappa\times\text{Area}$

Bulk to Boundary and

- **Boundary data:** U(1)-holonomy around 2d cell

- **Fusion rule:** To glue cells



$$Z_\partial^{(\beta_1)} \# Z_\partial^{(\beta_2)} = \int du\, d\tilde{u}\, \delta(u+\tilde{u})\, Z_\partial^{(\beta_1)} Z_\partial^{(\beta_2)} = Z_\partial^{(\beta)}$$

implies $\beta^{-1} = \beta_1^{-1} + \beta_2^{-1}$ extensive

boundary coupling = area = bulk observable

Bulk to Boundary and Boundary to Bulk

scalar field U(1) connection

$$S[\varphi,\omega] = \int_{\mathcal{M}_{2d}} \varphi F[\omega] - \epsilon\varphi^2$$

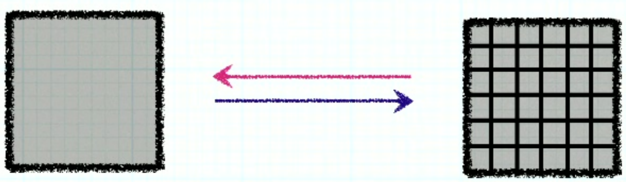
2d BF theory

curvature mass-term

- Boundary data: U(1)-holonomy around 2d cell

- Fusion rule: To glue cells

- Bulk as refined network of boundary theories

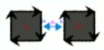


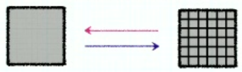
$\mathcal{Z} = Z_\partial$ $\mathcal{Z} = Z_\partial \# Z_\partial \# \ldots$

Bulk to Boundary and Boundary to Bulk - Recipe

$$\underbrace{S[\overset{\text{scalar field}}{\phi}, \overset{U(1)\text{-connect}^\circ}{\omega}] = \int_{M_{2d}} \phi \underbrace{F[\omega]}_{\text{curvature}} - \underbrace{\kappa \phi^2}_{\text{mass-term}}}$$

2d BF theory

- Boundary data: U(1)-holonomy around 2d cell  $Z_\partial^{(\beta)}[\{\omega_i\}] = e^{i\beta(\sum_i \omega_i)^2}$
- Fusion rule: To glue cells  $Z_\partial^{(\beta_1)} \# Z_\partial^{(\beta_2)} = Z_\partial^{(\beta)}$ $\quad \beta^{-1} = \beta_1^{-1} + \beta_2^{-1}$
- Bulk as refined network of boundary theories



$\mathcal{Z} = Z_\partial \qquad \mathcal{Z} = Z_\partial \# Z_\partial \# \ldots$

Bulk observables ⇄ Boundary couplings

Bulk fields ⇄ Network of boundary modes

Bulk coarse-graining ⇄ Boundary renormalisation

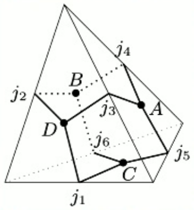
3d QG <-> 2d Ising duality

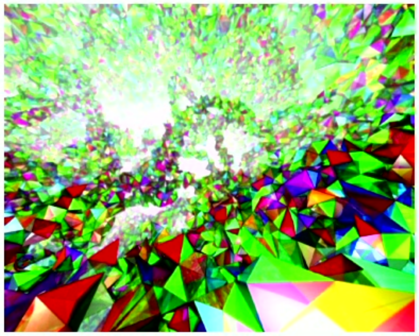
Let's go beyond 2d !

3d Quantum Gravity path integral given by Ponzano-Regge model

- 3d LQG fits with RSt, Rt, TV, RT, CS, cq
- Discretised space-time made from glued 3d cells

Quantum geometry defined by
a quantum state on each edge vector





$$\mathcal{Z}_\Delta = \sum_{\{j_\ell\}} \prod_\ell (-1)^{2j_\ell}(2j_\ell+1) \prod_t (-1)^{\sum_{\ell\in t} j_\ell} \prod_T \{6j\}_T$$

3d QG <-> 2d Ising duality

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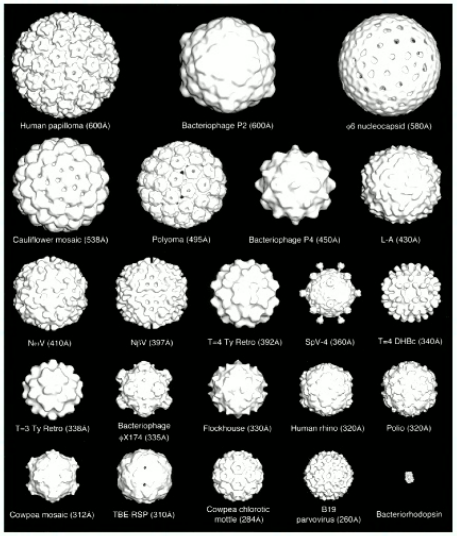

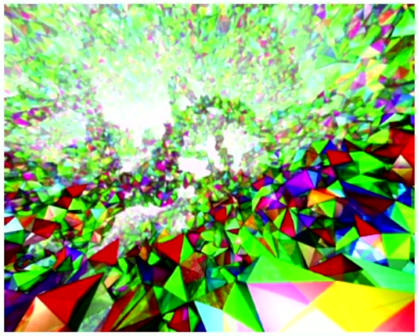

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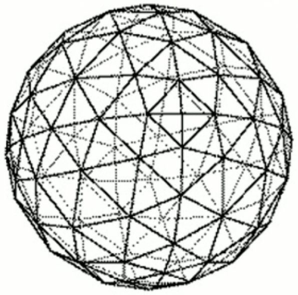
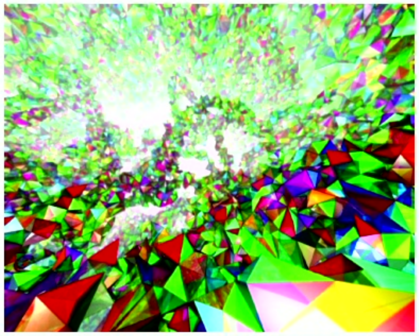
- Discretised space-time made from glued 3d cells
- Topological state-sum



Bulk Amplitude
=
Boundary State Evaluation
=
Spin Network Evaluation



$$s^{\Gamma}(\{j_e\}) = \psi^{\Gamma}_{\{j_e\}}(\mathbb{1}) = \sum_{\{m_e\}} \prod_e (-1)^{j_e - m_e} \prod_v \begin{pmatrix} j_{e_1^v} & j_{e_2^v} & j_{e_3^v} \\ \epsilon^v_{e_1} m_{e_1^v} & \epsilon^v_{e_2} m_{e_2^v} & \epsilon^v_{e_3} m_{e_3^v} \end{pmatrix}$$

3d QG <-> 2d Ising duality

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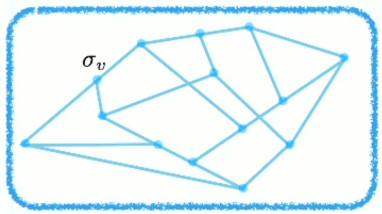
- Boundary theory given by inhomogeneous 2d Ising model

Bulk Amplitude
=
Boundary State Evaluation

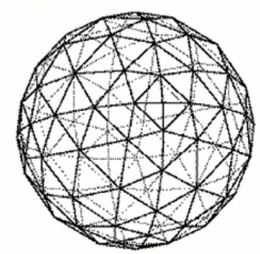
$$Z_\Gamma^{PR}[\{Y_e\}] \propto \frac{1}{Z^{Ising}[\{y_e\}]^2}$$

$$Y_e = \tanh y_e$$



$$\sigma_v = \pm 1 \in \mathbb{Z}_2$$

$$Z_\Gamma^{Ising}(\{y_e\}) = \sum_\sigma \exp\left(\sum_e y_e \sigma_{s(c)} \sigma_{t(c)}\right)$$



3d QG <-> 2d Ising duality

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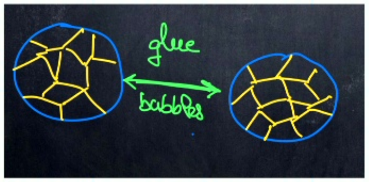
- Boundary theory given by inhomogeneous 2d Ising model
- Underlying supersymmetry !
- 3d QG = network of 2d Ising models !

Bulk Amplitude
=
Boundary State Evaluation



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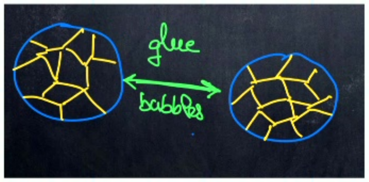
- Boundary theory given by inhomogeneous 2d Ising model
- Underlying supersymmetry !
- 3d QG = network of 2d Ising models !
- Coarse-graining 3d QG = renormalising 2d Ising model ?

Bulk Amplitude
=
Boundary State Evaluation



Side-product: Critical Ising couplings from QG

$$Z_\Gamma^{PR}[\{Y_e\}] \propto \frac{1}{Z^{Ising}[\{y_e\}]^2}$$

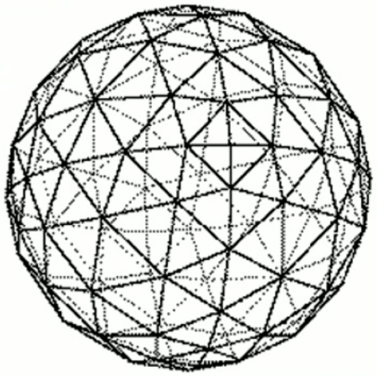
$$Y_e = \tanh y_e$$

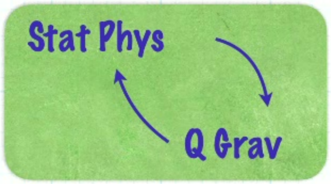
Ising zeroes

QG Poles

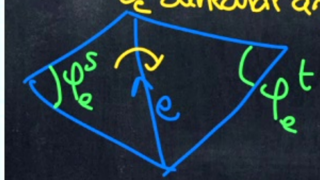
Semiclassical geometry
(Large spins)

Side-product: Critical Ising couplings from QG





θ_e dihedral angle



$$y_e^{crit} = \underbrace{e^{\frac{i}{2}\epsilon\theta_e}}_{\text{from 3j asymptotics}} \underbrace{\sqrt{\tan\frac{\varphi_e^s}{2}\tan\frac{\varphi_e^t}{2}}}_{\text{from factorial weights}}$$


Soudainement surpris
D'une idée me renversant,
D'une question me retournant,
Déclic, merci Lee
Merci!
Lee
Le ciel bleu sur nous peut s'effondrer
Et la Terre peut bien s'écrouler
Peu m'importe si tu t'souviens de moi
Je me fous de l'arXiv journalier
Tant qu'tes concepts innonderont mes matins
Tant qu'm'pensées frémiront de tes idées
Peu m'importe ...