Title: Dark Matter as the leftover of historical violations of the Hamiltonian constraint

Speakers: Joao Magueijo

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

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Abstract:

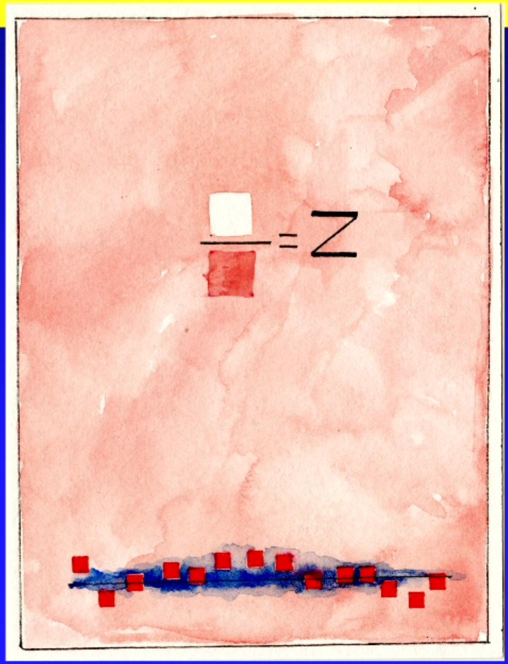
If the Hamiltonian constraint were ever violated (in the early Universe, at high energies, etc), its subsequent restoration could never erase a memory effect of the original violation. Depending on the technicalities of restoration, the memory effect may be as boring as something which mimics standard dark matter, or an anisotropic extension of dark matter... Or, as crazy as something that acts like a dust fluid capable of attracting other matter but being attracted to nothing. Attracting without being attracted: we discuss how it might be possible in a relativistic theory of gravity, its implications to the foundations of physics (specifically Dirac's algebra of constraints underpinning relativistic physics), and what its observational hallmarks might be.

Dark Matter: missing matter, or missing physics?

João Magueijo

2025

Imperial College, London



Moral of this conference: We all owe a lot to Lee…

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Lorentz Invariance with an Invariant Energy Scale

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We propose a modification of special relativity in which a physical energy, which may be the Planck energy, joins the speed of light as an invariant, in spite of a complete relativity of inertial frames and agreement with Einstein's theory at low energies. This is accomplished by a nonlinear modification of the action of the Lorentz group on momentum space, generated by adding a dilatation to each boost in such a way that the Planck energy remains invariant. The associated algebra has unmodified structure constants. We also discuss the resulting modifications of field theory and suggest a modification of the equivalence principle which determines how the new theory is embedded in general relativity.

DOI: 10.1103/PhysRevLett.88.190403 PACS numbers: 03.30.+p, 04.50.+h, 04.60.−m

A debt I owe to Lee, besides our science and friendship:

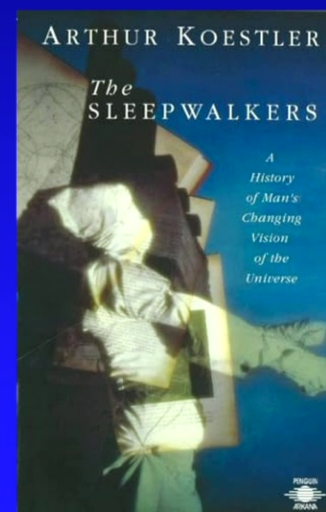
- In a ghost house, on the inaptly named "Euclid Avenue" nearby…

I was a philistine until I was exposed to Lee's remarkable library:

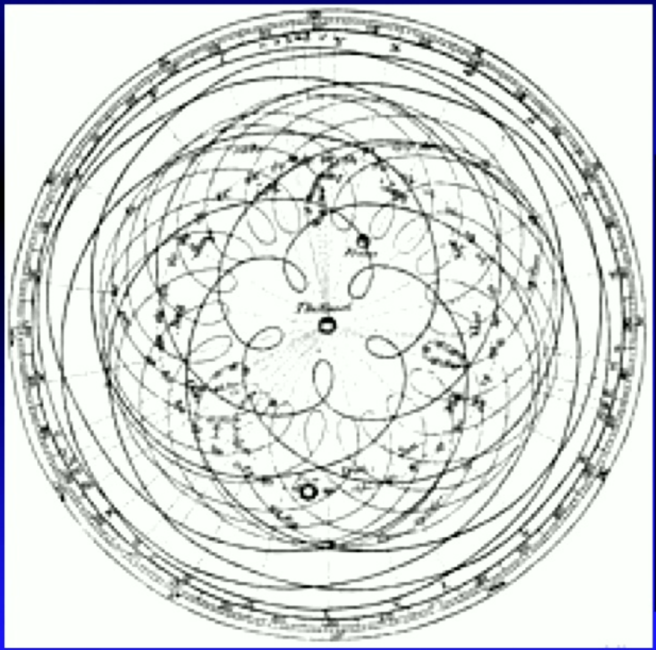
- Copernicus couldn't add up to save his life
- Kepler's mother was a witch
- Newton was a "mystic"…

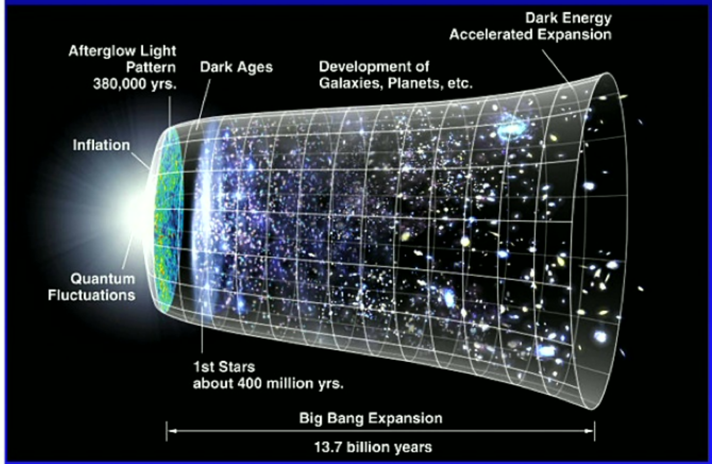
- Major discoveries are often fortuitous and misunderstood by the discoverer!

- Wrong science can be beautifully logical, whilst correct science can come out of nonsense and sleepwalking…



Could “cosmic concordance” have got it all spectacularly WRONG?





We should question everything. Could the pains of quantum gravity be self-inflicted?

- Canonical quantum gravity is particularly tied to diffeomorphism invariance (a throwback to Leibnitz)

$$x^\mu \to x^{\mu\prime} = x^{\mu\prime}(x^\mu)$$

- When we split 4=3+1 this translates into the famous result:

$$\mathcal{H}_E = N\mathcal{H} + N^i\mathcal{H}_i$$

lapse N, shift N^i,

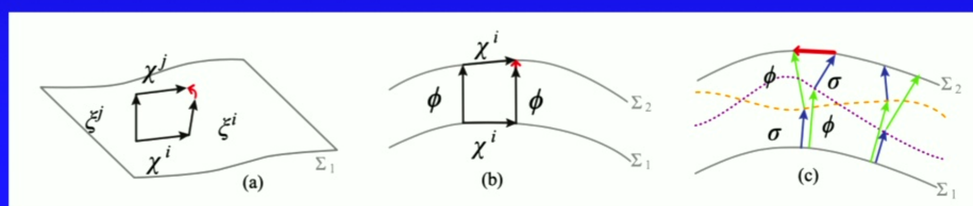
$$\mathcal{H} = 0 \qquad \mathcal{H}_i = 0$$

The algebra of constraints closes, and represents diffeomorphisms:

- This is the famous Dirac Hypersurface Deformation algebra:

$$\{H_i(f^i), H_j(g^j)\}_L = H_i([f,g]^i)$$
$$\{H_i(f^i), H(g)\}_L = H(f^i \partial_i g)$$
$$\{H(f), H(g)\}_L = H_i(h^{ij}(f\partial_j g - g\partial_j f))$$

$$H(f) = \int d^3y\, f(y)\mathcal{H}(y)$$
$$H_i(f^i) = \int d^3y\, f^i(y)\mathcal{H}_i(y)$$



(Small print: they are an "algebroid" or "business class", but that's OK…)

But what if we break this symmetry in extreme regimes? Can we still restore it?

- Examples include:
  - Evolution (time-dependence in the laws of physics) in the Early Universe $$\mathcal{H}(t,x) = \int_{C} \left(\frac{\partial \mathcal{H}}{\partial \alpha}\dot{\alpha} + \frac{\partial \mathcal{H}}{\partial T_\alpha}\dot{T}_\alpha \right)$$
  - A UV effect as in Horava-Lifshitz theory.
- Typically we are left with a non-zero leftover Hamiltonian even after the violating effects cease.
- No big deal if we reabsorb the leftover Hamiltonian into a vanishing Hamiltonian $$\mathcal{H} \approx \Delta \implies \bar{\mathcal{H}} = \mathcal{H} - \Delta \approx 0$$

…. AND…

…AND we evolve the residual Hamiltonian according to the Dirac HDA

- This amounts to:

$$\dot{\mathcal{H}}|_L = \{\mathcal{H}, \mathbf{H}\}_L = \partial_i(N^i\mathcal{H}) + \partial_i(\mathcal{H}^i N) + \mathcal{H}^i\partial_i N$$
$$\dot{\mathcal{H}}_i|_L = \{\mathcal{H}_i, \mathbf{H}\}_L = \mathcal{H}\,\partial_i N + \partial_j(N^j\mathcal{H}_i) + \mathcal{H}_j\partial_i N^j.$$

- And is equivalent to CDM if the foliation is geodesic…

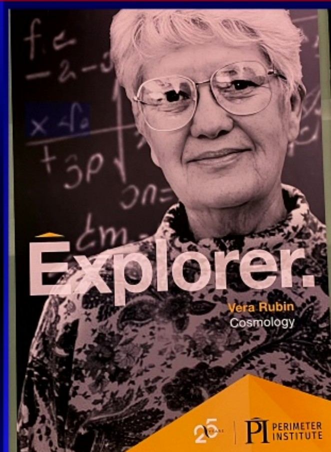
$$T^{\mu\nu}_\Delta = \frac{\Delta(x)}{\sqrt{h}} n^\mu n^\nu$$

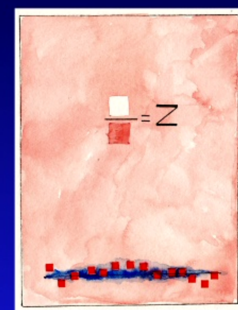
- But what if we insist on a frozen-in Hamiltonian on a non-geodesic foliation:

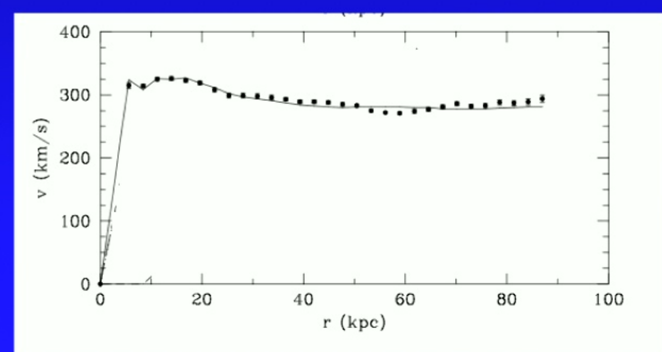
$$\Delta = \Delta(x) \qquad \text{i.e.} \qquad \dot{\Delta} = 0$$

- Then, we must have caused permanent damage to diffeormorphism invariance! SEE ABOVE.

Why would we want to do this? Let’s return to Lee’s talk wich I am plagiarizing

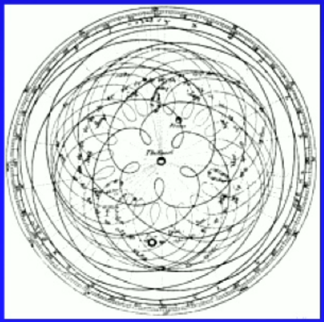






The trouble with dark matter:

- Either it works brilliantly, or it is a disaster.
- This suggests "dark matter" is actually a conflation of very disparate things.
- The disasters typically happen when dark matter does not stay where it should stay, under its own gravity and of other matter (something related to the issues of bias, etc).
- This happens with galaxy rotation curves: the dark matter wants to form a cusp instead of the required halo.
- Now…it can be fixed with epicycles (fuzzy DM, triaxial bars, etc…)



…Or else it can be taken face value and note that:

- A frozen-in Hamiltonian on a non-geodesic foliation has the property that in the "perverted Newtonian limit" (i.e. non-relativistic weak gravity), it is equivalent to matter that attracts without being attracted.
- (Obviously this violates everything we hold dear but so what?)
- It certainly mimics what early dark matters called: painted-on dark matter.

How might we do this?

- The Dirac HD algebra acquires a "central charge'' Δ(x). This is what dark matter is.

$$\{H_i(f^i), H_j(g^j)\} = H_i([\vec{f}, \vec{g}]^i)$$
$$\{H_i(f^i), H(f)\} = (H - \Delta)(f^i \partial_i f)$$
$$\{H(f), H(f)\} = H_i(h^{ij}(f\partial_j g - g\partial_j f)),$$

implying:

$$\{H_i(f^i), H_j(g^j)\} \approx 0$$
$$\{H_i(f^i), H(f)\} \approx -\int d^3x\, \Delta(x) f^i \partial_i f$$
$$\{H(f), H(g)\} \approx 0.$$

- Obviously the algebra then becomes second class, but this merely signals we are working with a phase space that is too large

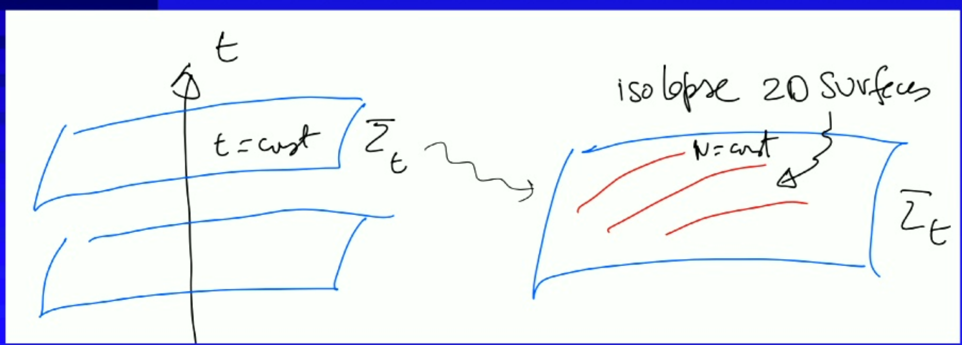
We need to clip phase space:

- Recall the equivalent thing in unimodular gravity:

$$P_{\mu\nu}{}^{\alpha\beta}[G_{\alpha\beta} - 8\pi(T^{\Delta}_{\alpha\beta} + T^{M}_{\alpha\beta})] = 0$$
$$\nabla_\mu T^{\mu\nu}_M = 0$$

$$P_{\mu\nu}{}^{\alpha\beta} = \delta^\alpha_\mu \delta^\beta_\nu - \frac{1}{4} g_{\mu\nu} g^{\alpha\beta}$$

- Here we could, for example, choose a spatial clipper orthogonal to the iso-lapse 2D surfaces on the foliation:



$$P_{\mu\nu}{}^{\alpha\beta} = \delta^\alpha_\mu \delta^\beta_\nu - \frac{1}{2}\, {}^{(2)}h_{\mu\nu}\, {}^{(2)}h^{\alpha\beta}$$

An example of spherically symmetric solutions (see Isichei and Magueijo 2505.04544 for details)

$$ds^2 = -e^{2\Phi(r)}\,dt^2 + e^{2\lambda(r)}\,dr^2 + r^2 d\Omega^2$$

$$\rho_\Delta = \frac{\Delta(r)}{r^2 e^\lambda}$$

$$\Delta = \Delta_0$$

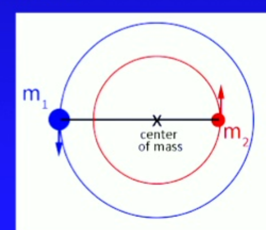
$$\Delta_0 = \frac{m_0}{4\pi\sqrt{1-2m_0}}$$

$$ds^2 = -\left(\frac{r}{r_c}\right)^{\frac{2m_0}{1-2m_0}} dt^2 + \frac{dr^2}{1-2m_0} + r^2 d\Omega^2.$$

$$\Phi = \frac{m_0}{1-2m_0} \ln \frac{r}{r_c}$$

Conclusions:
(Against territorial physics)

- This is not a replacement of dark matter; but the result of thinking that dark matter is probably more than once thing.
- On the minus side: since Δ is non-dynamical, we can always evade any observational constraint by setting it to zero where the constraint would come from:
- On the plus sign, this is so weird that, if Δ≠0, what comes out is pretty inimitable by anything else.
- A toy example:



Imagine a double star. The center of mass accelerates, and this is testable even if the dark companion is not seen.

More seriously:

- Lensing and gravitational waves will be different but only inside regions with $\Delta \neq 0$.
- Many of other signatures …. WORK IN PROGRESS!

- THANK YOU LEE AND IT IS YOUR FAULT IF I ONLY HAVE MAD IDEAS IN MY OLD AGE

(but then again you might think this is not mad enough)