Title: Spin Networks

Speakers: Ted Jacobson

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

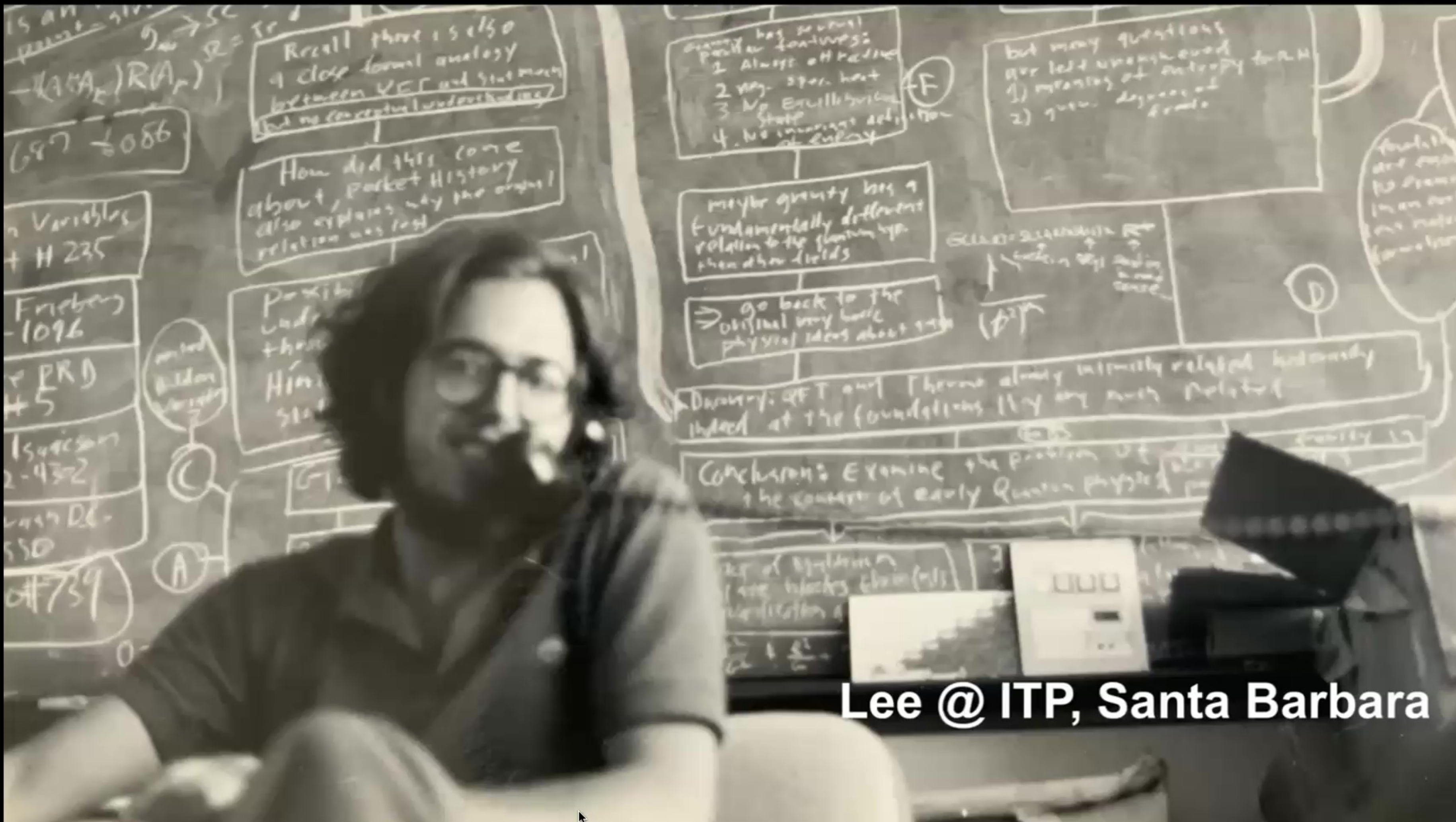
Date: June 05, 2025 - 2:10 PM

URL: https://pirsa.org/25060043

Spin Networks

the allure of singlets

Ted Jacobson, University of Maryland, Lee's Fest, Perimeter Institute, June 5, 2025


Lee @ ITP, Santa Barbara


Ted Jacobson
Lee Smolin
John Dell

A Theory of Quantized Directions

Abstract: — ? —

by Roger Penrose
c/o Battelle Memorial Institute, 4000 N.E. Street, Seattle Wash. 98105

1. Introduction The concept of the continuum (or real number system) is one of the most fruitful mathematical notion employed in physical science. ~~Any~~ All (~~[illegible]~~ successful) and reasonably comprehensive physical theories that ~~[illegible]~~ have so far been proposed have rested heavily on this notion — if only because space and time according to our present ideas, form a continuum and are therefore ~~to~~ be represented by continuous coordinates. And quantum mechanics (as a well ...

PHYSICAL REVIEW D VOLUME 52, NUMBER 10 15 NOVEMBER 1995

Spin networks and quantum gravity

Carlo Rovelli*
Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

Lee Smolin†
Center for Gravitational Physics and Geometry, Department of Physics, Pennsylvania State University, University Park, Pennsylvania 16802-6360
and School of Natural Science, Institute for Advanced Study, Princeton, New Jersey 08540

(Received 5 May 1995)

We introduce a new basis on the state space of nonperturbative quantum gravity. The states of this basis are linearly independent, are well defined in both the loop representation and the connection representation, and are labeled by a generalization of Penrose's spin networks. The new basis fully reduces the spinor identities [SU(2) Mandelstam identities] and simplifies calculations in nonperturbative quantum gravity. In particular, it allows a simple expression for the exact solutions of the Hamiltonian constraint (Wheeler-DeWitt equation) that have been discovered in the loop representation. The states in this basis diagonalize operators that represent the three-geometry of space, such as the area and the volume of arbitrary surfaces and regions, and therefore provide a discrete picture of quantum geometry at the Planck scale.

PACS number(s): 04.60.Ds, 75.25.+z

ISO a universe that is

- discrete
- background-independent
- quantum mechanical
- relativistic

The simplest quantum system is a single qubit …
… but what are the two possibilities to which it refers?

A background-independent theory should be purely relational.

The simplest relational quantum system is
a maximally entangled state of two qubits,
and the most symmetric of these is the singlet:

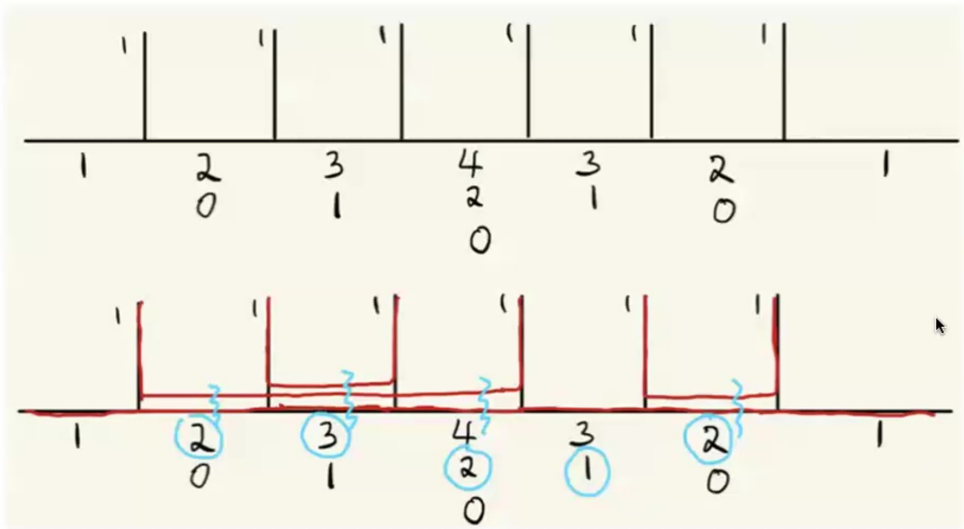
$$\epsilon^{AB}$$

It is invariant under all SL(2,C) transformations:

$$L^A{}_M L^B{}_N \, \epsilon^{MN} = (\det L) \, \epsilon^{AB} = \epsilon^{AB}$$

SL(2,C) is the double-cover of the Lorentz group SO(3,1),
so the singlet is Lorentz invariant.

Spin network combs provide a basis for these higher rank singlets:

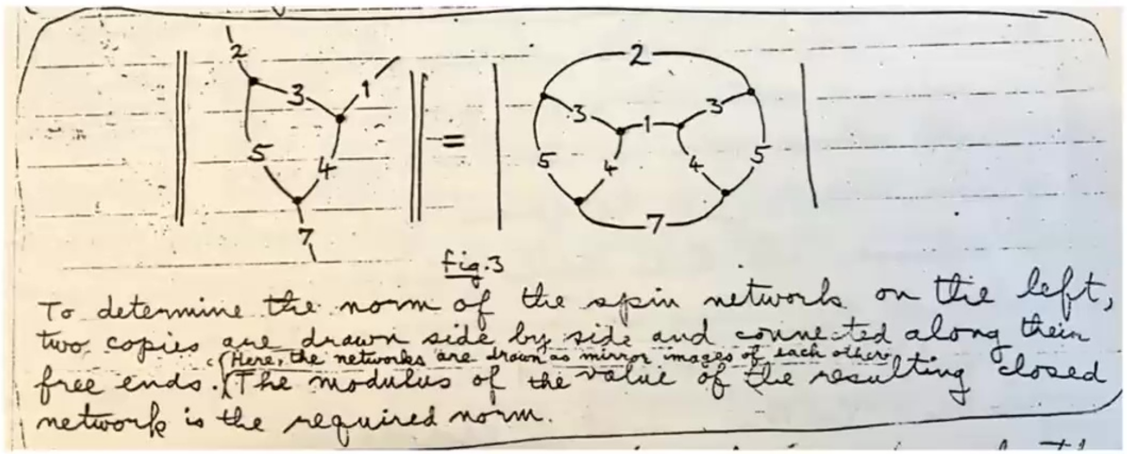


The dimension of the space built with N two-qubit singlets is the Nth Catalan number, exponentially the same as the $2N$ qubit space:

$$\frac{(2N)!}{N!(N+1)!} \approx \frac{1}{\sqrt{\pi N^3}} 2^{2N}$$

Penrose spin networks form a subset of the space of singlet tensors.
But distinct spin networks can represent the same tensor.
I will focus on the vector space of singlet tensors.

To define quantum probabilities we need an inner product.
Penrose defined a norm diagrammatically:



This amounts to using the inverse singlet tensor ϵ_{AB} to define an inner product. But is that Hermitian??

A positive, Hermitian ("real") tensor $t_{AA'} = \bar{t}_{A'A}$ defines an Hermitian inner product on the single qubit state space:

$$(\lambda, \mu) := t_{AA'}\,\bar{\lambda}^{A'}\mu^{A}$$

which defines an Hermitian inner product on the multi-qubit space:

$$(\psi, \phi) := t_{AA'} \cdots t_{BB'}\,\bar{\psi}^{A' \ldots B'}\phi^{A \ldots B}$$

On <u>singlets</u> this is independent of the choice of $t_{AA'}$ apart from the overall scale: $L_A{}^M \bar{L}_{A'}{}^{M'} t_{MM'}$ defines the <u>same</u> inner product, so the inner product is Lorentz invariant, even though $t_{AA'}$ is not.

Remarkably, the same inner product is defined by ϵ_{AB}

$$t_{AA'}t_{BB'}\,\epsilon^{AB} = \frac{1}{2}\,t^2\,\epsilon_{A'B'}\,, \qquad t^2 \equiv t_{MM'}t_{NN'}\,\epsilon^{MN}\epsilon^{M'N'}$$

$$\epsilon_{AA'}\epsilon_{BB'}\,\epsilon^{AB} = \epsilon_{A'B'}$$

(identifying the spin space with its conjugate)

This justifies Penrose's diagrammatic definition of the norm.

Penrose spin geometry theorem and $\sqrt{-1}$

For any sufficiently entangled spin network,
the collection of intrinsically defined angles
between spin directions with small dispersion
is consistent with embedding of these spin directions
into a three dimensional Euclidean space.

This raises a puzzle: Complex superpositions are required to span irreducible unitary representations of the rotation group, yet spin networks involve only rational numbers.

Does $\sqrt{-1}$ "emerge" from the structure of a spin network?

$$\epsilon_{abc} J^a J^b J^c = i \hbar J^2$$

$$J^x J^y J^z + J^y J^z J^x + J^z J^x J^y + J^y J^x J^z + J^x J^z J^y + J^z J^y J^x = i\hbar J^2$$

If $\{W, X, Y, Z\}$ represent four spin units with well-defined relative angles, the latter three mutually orthogonal, and we define (intrinsically to the network)

$$J^x := \frac{\vec{J}_X \cdot \vec{J}_W}{\sqrt{j_X(j_X+1)}}$$

and similarly for J^y and J^z, then

- ✳ Does the triple product define an operator that squares to $-I$?
- ✳ If so, is it the <u>same</u> operator for any choice of the W, X, Y, Z ?
- ✳ If so, is there a more symmetric, global way to define this $\sqrt{-1}$ operator?


Thank you Lee
for the inspiration
and your passion!