

Title: Conference Talk

Speakers: Bianca Dittrich

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On the positivity of the LQG Hamiltonian

Bianca Dittrich,

Perimeter Institute

Lee's Fest 2025



Trying to answer Lee's question ... from 20 years ago

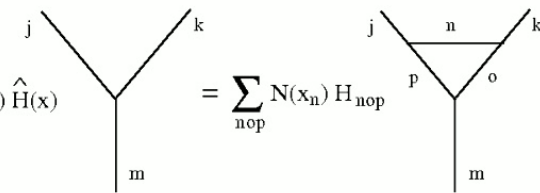
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LQG Hamiltonian

[Rovelli, Smolin; ..., Thiemann]

$$\int_{\Sigma} N(x) \hat{H}(x) = \sum_{nop} N(x_n) H_{nop}$$


Ultra-local?

And therefore not leading to propagating dof?

[Smolin]

Physical semi-classical states?

Asymptotic infinity? Or finite boundary?

This is too bold for me.

Leaving the canonical path

Covariant LQG dynamics: spin foams

Configuration space of spin foams larger than expected.

Need to impose shape matching constraints.

[BD, Speziale 2007]

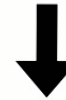
Shape matching constraints cannot and are not implemented strongly.

[BD, Ryan 2008+]

Reason: Discrete (asymptotic. equidistant) area spectrum

[Rovelli, Smolin]

[Asante, BD, Haggard 2020]



“Flatness problem” of spin foams.

[Bonzom; Hellmann, Kamiński;
Han, Engle, Dona, Speziale, ...]

Do spin foams give GR in semi-classical/ continuum limit?

Was working a lot on methods to obtain continuum/ refinement limit, but spin foam models were highly computational challenging, even on very coarse triangulations.

Whereas Lee emphasized that there should be simple(r) models, which capture gravity, by universal mechanisms in the continuum limit.



Effective spin foams

Replacing gauge with higher gauge theory leads to much simpler formulation.

[Girelli, Pfeiffer] [Baratin, Freidel]
[Asante, BD, Girelli, Riello, Tsimiklis]

Coupling symbol of higher gauge theory = exponential of Regge action.

[Baratin, Freidel]

Effective spin foams:

Exponential of Area Regge action + weak implementation of shape matching constraints.

[Asante, BD, Haggard]
[Asante, BD, Padua-Arguelles]

1) Allowed first explicit numerical computation of expectation values for small triangulations.

Showed that correct discrete EOM are implemented (in the tested examples).

[Asante, BD, Haggard]

Flatness problem can be avoided (for sufficient small Barbero-Immirzi parameter).

2) Allowed construction of (perturbative) continuum limit (on flat background).

[BD]

Surprise: In the continuum limit only 10 length metric degrees of freedom become massless/ acquire infinite correlation length. **Universal mechanism.**

Flatness problem not an issue in continuum limit.

Shape mismatch degrees of freedom **are massive** and therefore suppressed, without explicit implementation of the shape matching constraints.

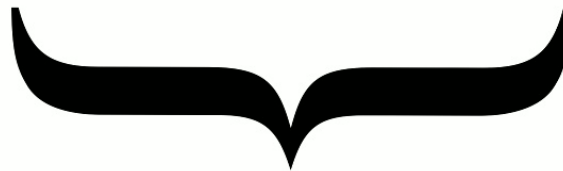
Continuum limit of Area Regge action

massless length metric degrees of freedom
(local action and linearly diffeo symmetry)

→ (linearized) general relativity to leading order in c -limit (or $1/k$)

massive non-shape matching dof
(via coupling to length metric dof)

→ Weyl squared correction to subleading order



area metric,
splits into length metric and non-length degrees of freedom

[BD, Kogios]

Conjecture: Area metric dynamics: can capture the effective continuum dynamics of spin foams.

Spin foam degrees of freedom are area metric degrees of freedom (also microscopically).

[BD, Padua-Arguelles]

Area metric actions in the continuum

Spin foams arise from quantization of Plebanski formalism: BF action + constraints.

[Reisenberger, Rovelli, Baez, Barrett, Crane, Livine, Speziale, Freidel, Krasnov, Baratin, Oriti, Bahr, Lewandowski, Kaminski, ...]

Modified Plebanski framework: constraints \rightarrow suppression by potential (or mass terms).

[Krasnov]

Chiral version: just a deformation of GR (no extra pole). Due to an extra symmetry in kinetic term.

[Freidel, Krasnov]

Non-chiral version (on which spin foams are based) : rather bi-metric gravity.

[Speziale]

Motivated by effective spin foam - **Modified Modified Plebanski framework**: Replace only part of constraints by mass terms.

Choose constraints such that one obtains a field theory of area metrics.

[Borissova, BD]

Can be done! Yields an action **consistent with continuum limit of effective spin foams**.

[Borissova, BD]

$${}^{(2)}\mathcal{L}(h) = {}^{(2)}\mathcal{L}_{EH}(h) - {}^{(1)}C_{\mu\nu\rho\sigma}(h) \frac{1}{p^2 + M^2} {}^{(1)}C^{\mu\nu\rho\sigma}(h)$$

Has no extra pole! Ghost-free despite being higher derivative.

Kirill Krasnov wanted to understand why this also holds in this case.

The space of area metric actions

[Borissova, BD, Krasnov]

Determined (linearized) general area metric actions with (linearized) diffeomorphism symmetry.

Canonical analysis of (linearized) area metric actions with shift symmetry in kinetic term (as in partially modified Plebanski).

Consider also Lorentzian signature.

Euclidean signature: Hamiltonian for non-length metric degrees of freedom positive definite.

(Wick rotation)

(similar for EM Lagrangian:

$$E^2 + B^2 \rightarrow E^2 - B^2)$$

Lorentzian signature: Hamiltonian for half of the non-length metric degrees of freedom negative definite!

Does generically lead to instabilities,

but in this case the **positive and negative definite parts decouple** and dynamics is thus stable (in the linearized theory).

Independent confirmation of indefiniteness of mass terms in spin foam actions:

analysis of Lorentzian Area Regge action on regular lattice.

[Asante, BD to appear]

Is the LQG Hamiltonian positive definite?

Maybe not (in Lorentzian signature).

If we accept that LQG leads to an extended configuration space of area metrics:

Covariant (Lorentzian) actions of area metrics lead to indefinite Hamiltonians.

Nevertheless linearized dynamics as defined from partially modified Plebanski formalism is stable.

Considered effective continuum theory. Is the 'fundamental' theory different?

Does it help us? Avoiding instabilities might nicely reduce the vast space of area metric actions at higher order.

[Borissova, BD, Eichhorn, Schiffer:
Renormalization flow of area metrics, to appear]

Area metric actions - rich phenomenology

Parity symmetry breaking (in EOM) - parametrized by Barbero-Immirzi parameter.

[Borissova, BD, Krasnov]

Effects on propagation of light and matter.

[Schuller et al]

Particles travelling backwards in time.

[Donoghue, Percacci]

Area metrics in string theory and holography?

