Title: Building and (hints of) seeing gravitational statistical mechanics

Speakers: Seth Major

Collection/Series: Lee's Fest: Quantum Gravity and the Nature of Time

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Abstract:

A statistical mechanics of geometry based on the quasi-local energy of constantly accelerating observers outside black holes is proposed. By assuming particle statistics for the geometric quanta, the geometry condenses in the large area limit, providing a quantum gravity cutoff for the entropy of the thermal atmosphere of geometric excitations. These excitations model the fluctuating ``shape of the black hole" and form an effectively two dimensional quantum thermal geometric atmosphere. The entropy from these ``Debye" excitations is proportional to the area. Prospects of for the observation of the fluctuations are briefly discussed.

Building and (hints of) seeing gravitational statistical mechanics

Seth Major

3 June 2025

Lee's Fest @ PI

Thank you Lee!

Hamilton

• What is the gravitational statistical mechanics for observers maneuvering to ensure that their proper acceleration is constant?

- For high acceleration observers around black holes

there is simple form of the quasi-local energy for such observers

$$\boxed{E_g \simeq \frac{A}{8\pi\ell} \simeq \frac{g\,A}{8\pi}}$$

with ℓ proper distance to horizon
and g the proper acceleration

Frodden, Ghosh, Perez, Phys. Rev. D 87 (2013) 121503(R)

See also "Black Holes: The Membrane Paradigm"

S. Major Phys Rev D 105 (2022) 104050

Statistical Mechanics Model

- Two-stage statistical model in high g, large area limit in Schwarzschild spacetime

I - Use quasi-local energy for constantly accelerating observers

- Assume quantization of area (and thus energy)
- Assume **quantum statistics** for the atoms of geometry
- Model gas of indistinguishable, non-interacting atoms of geometry

$\rightarrow$ Condenses in large area limit to the $j = 1$ state

$\rightarrow$ Entropy of condensed gas is low

II (Sketch of work in progress) `quasi-normal' phonons - Debye model

- The phonon frequency is cutoff by the condensed state area

$\rightarrow$ Entropy of phonons is proportional to area

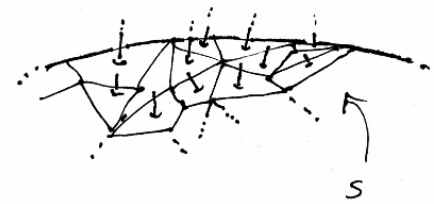
- Thoughts on possible observations

Statistical Model - I

Schwarzschild, large g,

- With spin network states observers surface has area

$$\hat{A}_S \mid \vec{j}\,\rangle = 8\pi l_{eff}^2 \sum_{i \in S \cap G} \sqrt{j_i\left(j_i+1\right)} \mid \vec{j}\,\rangle$$



where $l_{eff} = \sqrt{\hbar\gamma}$ * $\qquad (G = c = 1)$

$$\hat{H}_g \mid \vec{j}\,\rangle = \frac{g\hat{A}_S}{8\pi} \mid \vec{j}\,\rangle = g l_{eff}^2 \sum_{i \in S \cap G} \sqrt{j_i\left(j_i+1\right)} \mid \vec{j}\,\rangle$$

- A gas of indistinguishable, non-interacting quantum geometric particles

Each geometric particle has degeneracy

$$d_j = 2j + 1$$

(**No** `holographic' degeneracy $exp(\lambda A/l_P^2)$)

Asin et. al., Ghosh et. al.

Partition function with non-vanishing chemical potentials $\mu_\pm$

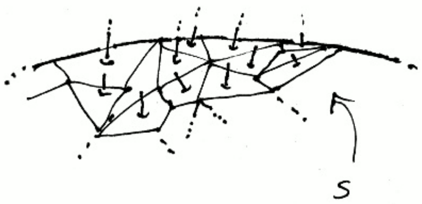
$$Z_\pm = \prod_j^\infty \left(1 \pm e^{-\beta(\epsilon\sqrt{j(j+1)} - \mu_\pm)}\right)^{\pm(2j+1)} \quad \text{with} \quad \epsilon = g l_{eff}^2$$

Statistical Model - I

Schwarzschild, large g, large A limit

- Observers are at the Unruh temperature

$$\beta \simeq \left(\frac{2\pi}{g\,\hbar}\right)$$



At Unruh temperature the Boltzmann weight scales with

$$\beta\epsilon = 2\pi\gamma$$

The Barbero-Immirizi parameter acts like an inverse temperature

- Bosonic area

(Fermionic excitations sub-dominant.)

$$\frac{\langle A\rangle_b}{8\pi l_{eff}^2} = \sum_{j=1}^{\infty} \frac{(2j+1)\sqrt{j(j+1)}}{e^{2\pi\gamma(\sqrt{j(j+1)}-\tilde{\mu}_b)} - 1)}$$

Positivity requires maximum chemical potential with

$$\tilde{\mu}_{max} = \frac{\mu_b}{\epsilon} = \sqrt{2} - \delta \qquad \langle A\rangle_b \simeq \frac{12\sqrt{2}l_P^2}{\delta}$$

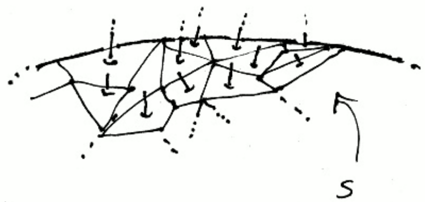
⇨ Large area requires condensation $\delta \to 0$ (or at least suitably small)

Statistical Model - I

Schwarzschild, large g, large A limit

The free energy is $\Omega = -\frac{1}{\beta}\ln Z$

Entropy: $S = \beta^2 \partial_\beta \Omega \quad = 3\ln\left(\frac{\langle A\rangle_b}{24\sqrt{2}\pi\, l_{eff}^2}\right)$



In condensed state the entropy is low, sub-dominant

Story so far: a low entropy geometric `condensate' with area quanta a_1 and

$$a_1 = 8\pi\sqrt{2}l_{eff}^2$$

$$A \simeq N a_1$$

See also Asin et. al.

See also Major and Setter

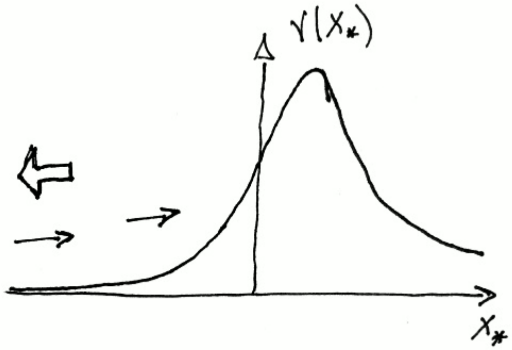
Dreyer PRL 2003

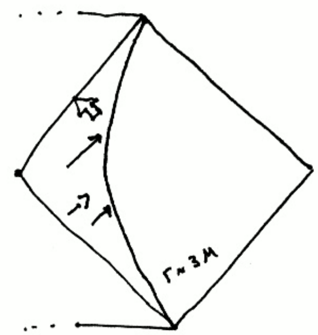
Statistical Model - II

Schwarzschild, large g, large A limit

• Thermal or quantum atmosphere via Hawking process

York, 't Hooft, Frolov and Novikov, Jacobson and Parentani, …





• These `phonon modes' contribute entropy in near horizon geometry.

• Asymptotic QNM spectrum is $\omega_n \simeq \frac{1}{8\pi M}\left(\ln 3 + 2\pi i\,(n+\frac{1}{2})\right)$ Motl, Neitzke

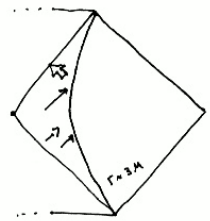
Statistical Model - II

Schwarzschild, large g, large A limit

- Energy of quasi-normal-like modes at energy $\Delta E = \hbar \frac{\ln 3}{8\pi M} \rightarrow \Delta E_{local} = \hbar \frac{g \ln 3}{2\pi}$ Hod PRL 1998

  Fixes upper bound on phonon modes

  2D Debye model has a Debye (max) frequency of $\omega_D = \frac{2}{L}\sqrt{\frac{\pi A}{a_1}} = \frac{g \ln 3}{2\pi}$



Geometric `phonon' partition function $$Z_P = \prod_{n=0}^{n_{max}} \frac{1}{e^{\beta\hbar\omega_n} - 1}$$

Helmholtz free energy $$F = -\frac{1}{\beta}\ln Z_P$$

- The phonon entropy is

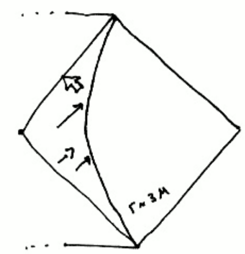
$$S_P = \beta^2 \partial_\beta F \simeq \left(\frac{4A}{a_1}\right)\left(\frac{1}{x_D^2}\right) I(x_D)$$ with $x_D = \beta\hbar\omega_D = \ln 3$

and $$I(x_D) = \int_0^{x_D} dx \left[\frac{x^2}{e^x - 1} - x \ln\left(1 - e^{-x}\right)\right]$$

Statistical Model - **II**

Schwarzschild, large g, large A limit

Observers experience thermal bath at Unruh temperature thus



$$x_D = \beta_U \hbar\omega_D = \ln 3 \text{ and } I(x_D) \simeq 0.863$$

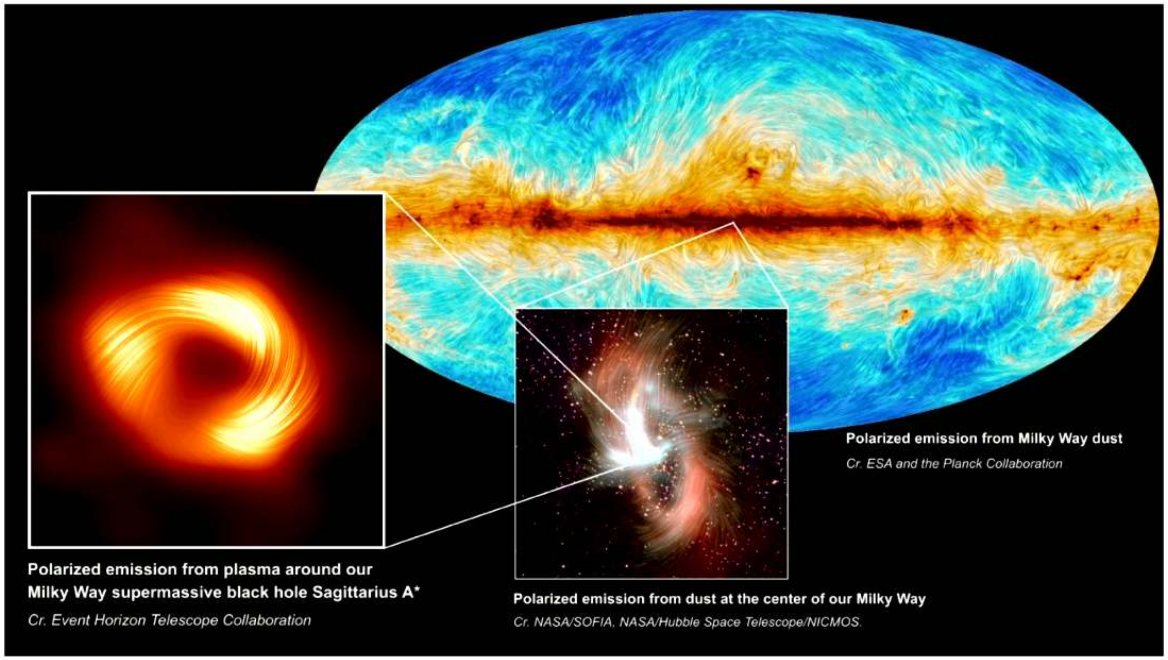
$$\implies S_P \simeq \frac{4 \cdot 0.863\, A}{(\ln 3)^3 a_1} \simeq 0.294 \frac{A}{l_P^2} \; *$$

Not surprising from earlier studies by Frolov and Novikov, 't Hooft, and others but still nice :-)

These fluctuations in geometry are **outside** the black hole. They may be observable.

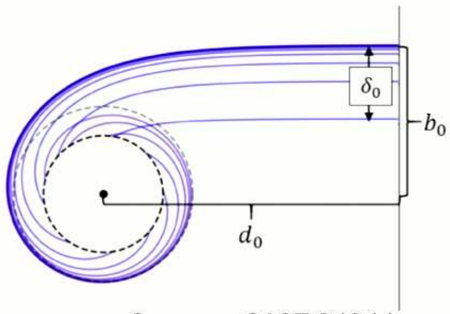
`Seeing' black hole entropy

- Event Horizon Telescope images (and movies) of Sgr A*



`Seeing' black hole entropy

- Flare from accreting matter around black holes e.g. Sgr A*



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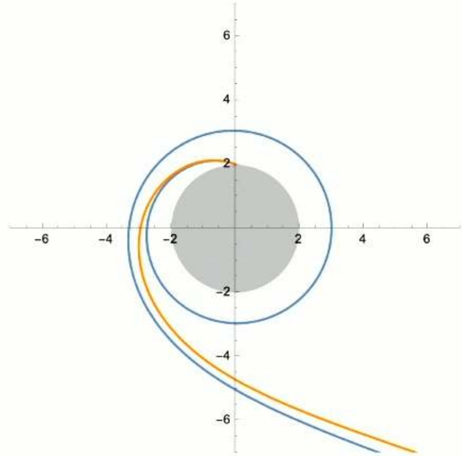

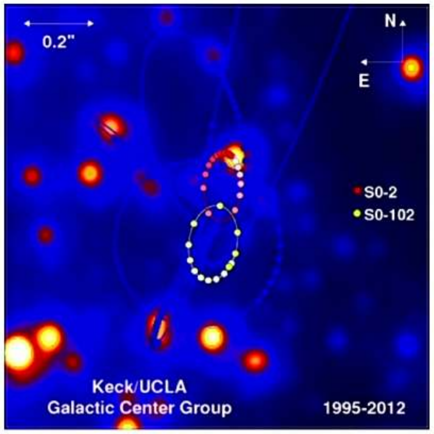

Characteristic fluctuation $\omega_\infty \sim \frac{1}{M}$

- Photon ring sub-structure via very long baseline interferometry

Johnson et al Sci. Adv. 2020

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Statistical Model - II

Schwarzschild, large g, large A limit

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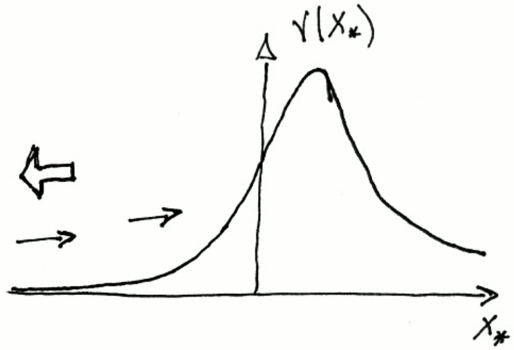

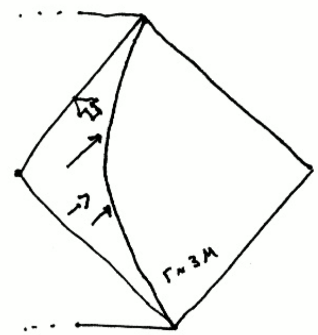

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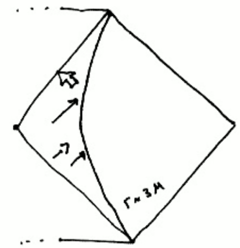

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