Title: Quantum Foundations

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Quantum Foundations at Lee Fest

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The measurement problem

According to unitary evolution we can have

$$\psi = \psi_{\text{big thing at } A} + \psi_{\text{big thing at } B}$$

If we believe this, we are on the horns of a dilemma. Either

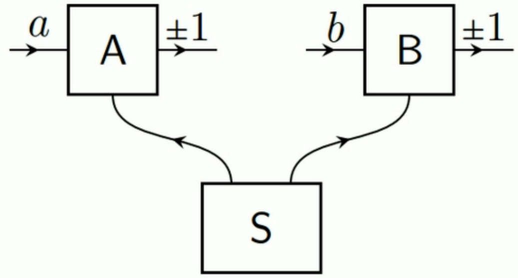
Reality problem: There is a mismatch between the quantum description and way the world appears to be (as big objects appear to be in definite places), or

Incompleteness: The wavefunction is not a complete description of reality

The measurement problem is the elephant in the room.

Bell nonlocality

Consider making measurements entangled particles



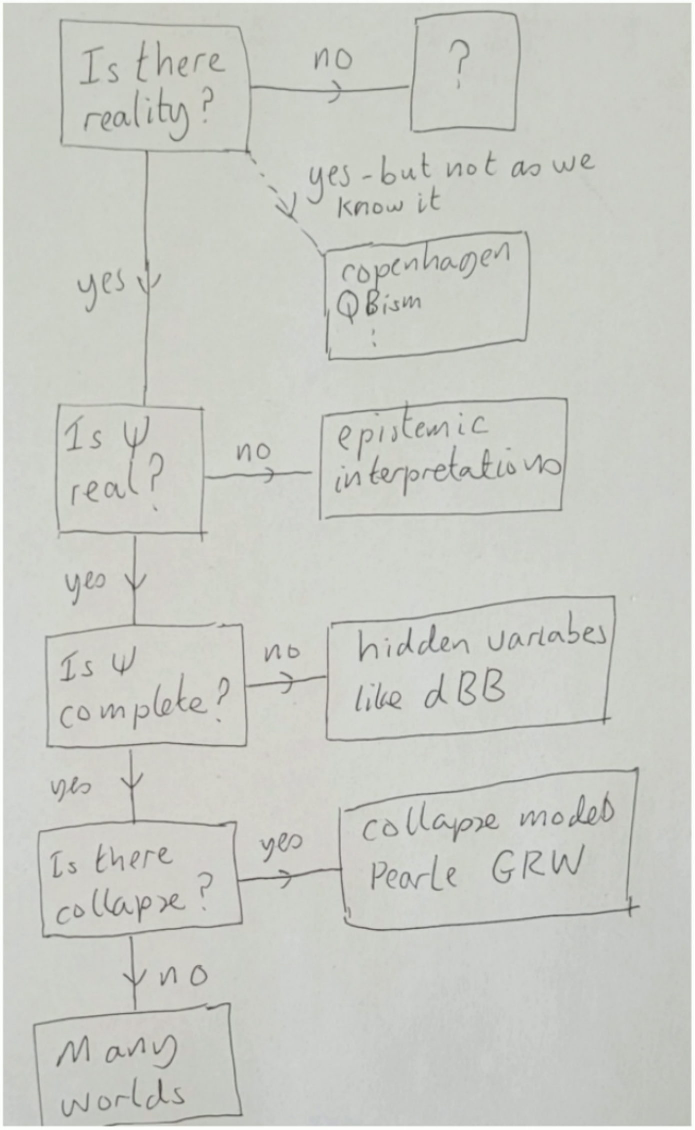
We can measure a correlation function

$$E(a,b)$$

(equal to the average value of the product of the outcomes).
According to *local hidden variable theories* must have

$$-2\sqrt{2} \leq E(a,b) + E(a,b') + E(a',b) - E(a',b') \leq 2\sqrt{2}$$

Quantum predictions for $E(a,b)$ violate these inequalities.


Is there reality?
no
?
yes - but not as we know it
copenhagen
QBism
yes
Is ψ real?
no
epistemic interpretations
yes
Is ψ complete?
no
hidden varlabes like dBB
yes
Is there collapse?
yes
collapse models Pearle GRW
no
Many worlds

Questions for an interpretation

1. What are the beables?
2. What are the dynamics for the beables?
3. How is the measurement problem solved?
4. Does this interpretation say anything about Bell nonlocality?
5. Does the interpretation serve as a good departure point for making progress in physics? (In particular, finding a theory of Quantum Gravity).

Lee, unusually, in the field of QF takes on all these questions - head on, and does more.

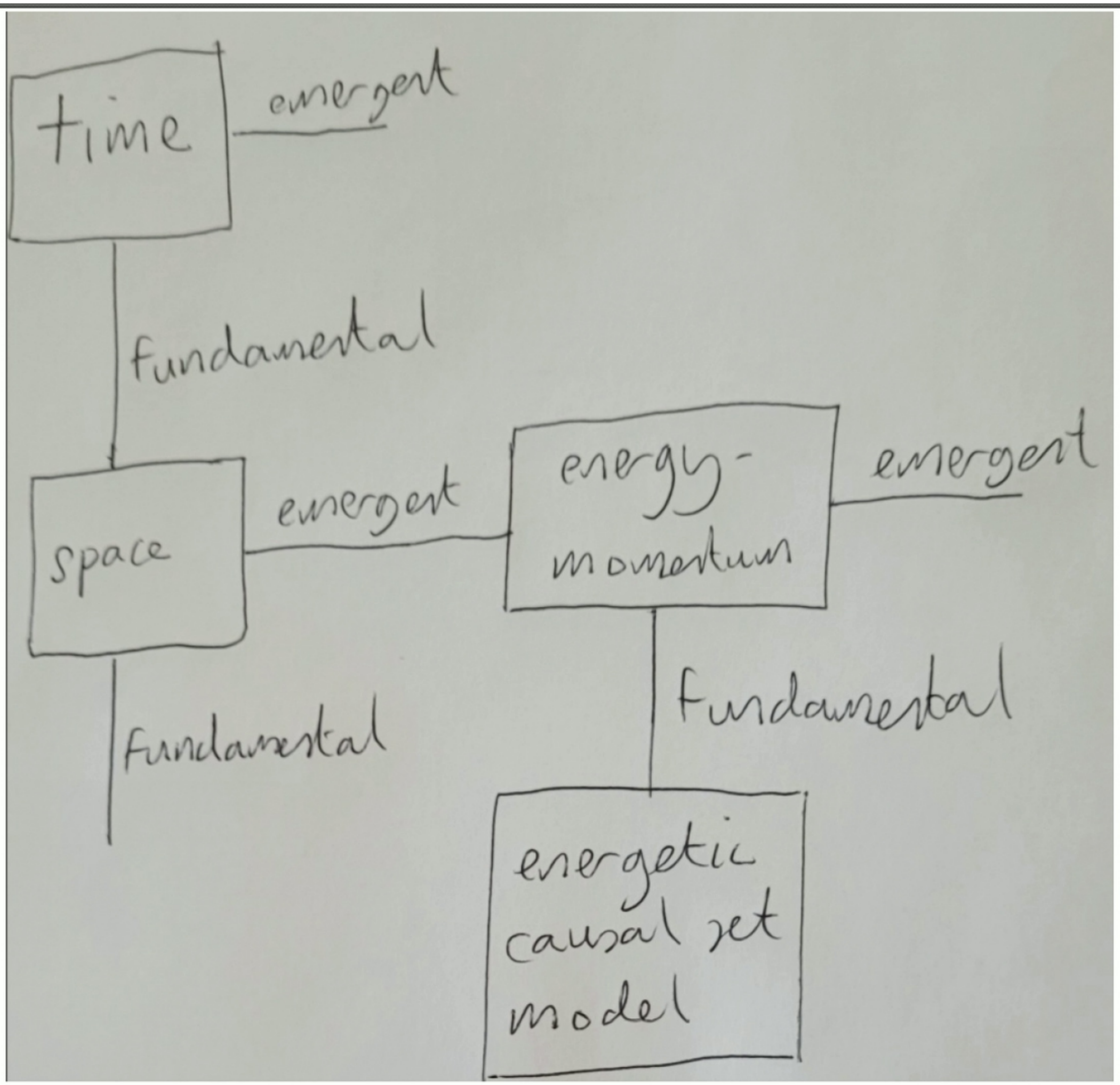
Lee's mission statement?

Lee's mission statement (perhaps):

I would like to introduce a research program aimed at solving the foundational issues in quantum mechanics in a way that also addresses the problem of quantum gravity.

His research program is motivated by deep questions coming from philosophy and Quantum Gravity.

1. He distinguishes between what is fundamental and what is emergent.
2. Time is fundamental.
3. Space is emergent.
4. He states that QT is incomplete though, also, the wavefunction is emergent
5. He attempts to explain Bell nonlocality through deeper notions of locality (in the pre-space theory).
6. Time asymmetric.
7. The viewpoint he has developed has led to a number of results.


time
emergent
fundamental
space
emergent
energy-momentum
emergent
fundamental
fundamental
energetic causal set model

In collaboration with the philosopher, Roberto Mangabeira Unger, Lee developed a point of view which, to some extent, forms the basis for much of the physics work he has done since. A short very readable paper by Lee outlining some of these ideas is the following.

A naturalist account of the limited, and hence reasonable, effectiveness of mathematics in physics

Lee Smolin

My aim in this essay is to propose a conception of mathematics that is fully consonant with *naturalism*. By that I mean the hypothesis that *everything that exists is part of the natural world, which makes up a unitary whole.* This is in contradiction with the Platonic view of mathematics held by many physicists and mathematicians according to which, *mathematical truths are facts about mathematical objects which exist in a separate, timeless realm of reality, which exists apart from and in addition to physical reality.*

Temporal naturalism

Mangabeira Unger and I hypothesize two principles which we take to define temporal naturalism.

1. **The singular universe:** All that exists is part of a single, causally connected universe. The universe and its history have no copies, and are not part of any ensemble. There is no other mode of existence, in particular neither a Platonic realm of mathematical objects nor an ensemble of possible worlds exist apart from the single universe.

2. **The inclusive reality of time:** All that is real or true is such within a moment, which is one of a succession of moments. The activity of time is a process by which novel events are generated out of a presently existing, thick set of present events. There are no eternal laws; laws are subsidiary to time and to a fundamental activity of causation and may evolve. There is an objective distinction between past, present and future.

Against Platonism

The choice between whether mathematics is discovered or invented is a false choice. Discovered implies something already exists and it also implies we have no choice about what we find. Invented means did not exist before AND we have choice about what we invent.

So these are not opposites. These are two out of four possibilities on a square whose dimensions are whether an entity allows choice or not and whether it already exists or not.

I would like to propose that there is a class of facts about the world, which concerns structures and objects which come to exist at specific moments, which, nevertheless, have rigid properties once they exist.

Let us call this possibility *evoked*. I would then propose that mathematics consists of the study of certain of these evoked structures. The four possibilities are indicated in the following diagram:

Has rigid properties? \ Existed prior?	Yes	No
Yes	*Discovered*	*Evoked*
No.	*Fantasized*	*Invented*

There are many things that did not exist before we bring them into existence but about which we have no choice, or only highly constrained choices, once it does exist. So the notion of evocation applies to many things besides mathematics.

In closing, I would like to mention two properties enjoyed by the physical universe which are not isomorphic to any property of a mathematical object.

1. In the real universe it is always some present moment, which is one of a succession of moments. Properties off mathematical objects, once evoked, are true independent of time.
2. The universe exists apart from being evoked by the human imagination, while mathematical objects do not exist before and apart from being evoked by human imagination.

A paper that summarizes much of Lee's research program is the following.

Temporal relationalism

Lee Smolin*

Abstract

Because of the non-locality of quantum entanglement, realist approaches to completing quantum mechanics have implications for our conception of space. Quantum gravity also is expected to predict phenomena in which the locality of classical spacetime is modified or disordered. It is then possible that the right quantum theory of gravity will also be a completion of quantum mechanics in which the foundational puzzles in both are addressed together. I review here the results of a program, developed with Roberto Mangabeira Unger, Marina Cortes and other collaborators, which aims to do just that. The results so far include energetic causal set models, time asymmetric extensions of general relativity and relational hidden variables theories, including real ensemble approaches to quantum mechanics. These models share two assumptions: that physics is relational and that time and causality are fundamental.

The paper opens as follows

I would like to introduce a research program aimed at solving the foundational issues in quantum mechanics in a way that also addresses the problem of quantum gravity.

There are, roughly speaking, two kinds of approaches to the measurement problem and the other issues in quantum foundations. The first take it that the principles of quantum mechanics are largely correct and therefore aim to make our thinking more compatible with the practice of quantum physics. The second kind of approach begins with the hypothesis that the foundational puzzles are consequences of an incompleteness of our understanding of nature, and seeks to resolve those puzzles by means of novel hypotheses about physics. Consequently these approaches are realist, whereas the first kind of approach are mostly anti-realist or operational. The aim is a deeper theory, inequivalent to quantum mechanics, which completes the partial description the standard theory now reveals. As it is a distinct theory, we may hope this completion will be testable. The two main examples of such completions of quantum mechanics which have been studied so far, pilot wave theory[1] and dynamical collapse models[2], do make testable predictions which differentiate them from quantum mechanics.

Two comments:

- I think one can pursue an operational methodology whilst still being committed to realism.
- Lee's approach is deeper than dBB and collapse models (and, indeed, many worlds) because ψ is emergent.
- Lee says that what distinguishes his approach from the other realist models is an emphasis on relationalism.

Space emergent

Once one has decided that time is going to be fundamental or emergent, one must ask the same question with regard to space. I will explain why it is unlikely that, within a realist framework, time and space can both be fundamental. Since I choose time to be fundamental, I must choose space to be emergent. It is, by the way, the fact that in this construction time is fundamental, but space is not, that resolves the apparent conflict of having an objective present with special relativity. The relativity of inertial frames, and the consequent lorentz invariance, is an emergent symmetry, which comes into effect only when space emerges, and is not hence a symmetry of the fundamental laws, which govern a domain of events with causal relations, but no space.

Energy and momentum

The third choice one has to make concerns energy and momentum: are they fundamental or emergent? The more I reflect on the structure of our physical theories, the more I realize that momentum and energy are at the heart of the foundations of physics, and that this distinguishes physics, in ways that are sometimes under-appreciated, from other fields of science which describe systems that change in time. Only physics has the canonical structure indicated by $\delta\Gamma = p_a \dot{x}^a$, and the related principles of inertia and of the relativity of inertial frames. These lead to the most fundamental structure in quantum mechanics, the Heisenberg algebra, $[\hat{x}, \hat{p}] = \imath\hbar$, which cannot be expressed in a system constructed with a finite number of qbits.

This is one reason I find the claims to analogize nature to a computer, classical or quantum, to be inadequate: such claims neglect the fundamental roles energy and momentum each play in the structure of our physical theories, indeed, I would suggest, in nature. For reasons I describe below, I think it is interesting to develop the idea that energy and momentum are both fundamental.

Energetic causal set models

Thus, the class of theories I will develop treat time, energy and momentum as fundamental. Space–and hence spacetime and its symmetries–are to be recovered as emergent. Towards the end I will describe a class of theories we constructed and studied with Marina Cortes, which realize these ideas, called *energetic causal set models [3]-[6]*.

Temporal relationalism is a part of a larger program of *temporal naturalism* which Roberto Mangabeira Unger and I presented in [9, 10, 11]. The adjective temporal is again meant to emphasize the centrality of the hypothesis that time is fundamental and irreducible.

These temporal approaches are to be distinguished from timeless, or atemporal, forms of relationalism and naturalism, which hold that the most fundamental levels of description of the world are formulated without time, as time, in the sense of its flow or passage, is held not to exist fundamentally. According to this view, time is real, but only in the sense that it is emergent, in the same sense that pressure and temperature are real. The block universe is an aspect of timeless naturalism, so is the notion that the laws of nature are fixed and unchanging.

Principle of increasingly sufficient reason

- *Seek to progress by making discoveries and inventing hypotheses and theories that lesson the arbitrary elements of our theories.*

I call this the principle of *increasingly sufficient reason* and it will be the form I employ here.

Here are some examples of steps in the history of physics which exemplified this advice.

- Eliminate references to unobservable absolute positions and motions and replace them by measurable relative positions and motions.
- Seek a theory of light in which its speed is not arbitrary but is computable in terms of known constants.

The principle of increasingly sufficient reason has a number of consequences which are each worthy principles in their own light. In each case they are to be read as advice, i.e. of the kinds of theories we ought to be aiming for, if we hope to progress fundamental physics[1].

1. The principle of increased background independence.
2. The principle that properties that comprise or give rise to space, time and motion are relational.
3. The principle of causal completeness.
4. The principle of reciprocity.
5. The principle of the identity of the indiscernible.

Each of these requires some elaboration.

Background independence

In many cases the observables of a background dependent theory describe how some quantity changes, or some body moves, with respect to those fixed external structures. This is the role of reference frames, they implicitly reflec a division of the universe into two parts-a dynamical system we aim to study, and a part whose dynamics we neglect and fictionalize as fixed-for the purpose of pretending that relational quantities like relative position and relative motion, have an absolute meaning.

It follows that any theory with fixed external structures can be improved if the external elements can be unfrozen, made dynamical, and brought inside the circle of mutually interacting physical degrees of freedom. This was the strategy that led Einstein to general relativity. The geometry of space and time is frozen in Newtonian physics and it is also frozen in special relativity. In these theories the spacetime geometry provides an absolute and fixed background against which measurements are defined. These are reflected in the role played by reference frames. Mach pointed out the fiction involved by identifying inertial frames with the "fixed stars." General relativity unfreezes geometry, making it dynamical. This freed the local inertial frames from an absolute and fixed dependence on the global mass distribution, and made their relation dynamical, subject to solutions of the equations of motion.

Background independence

It follows that the only complete theory of physics must be a cosmological theory, for the universe is the only system which has nothing outside of it. A theory of the whole universe must then be very different from theories of parts of the universe. It must have no fixed, frozen, timeless elements, as these refer to things outside the system described by the theory, but there is nothing outside the universe. A complete cosmological theory must be fully background independent.

It follows that quantum mechanics cannot be a theory of the whole universe because it has fixed elements. These include the algebra of observables and the geometry of Hilbert space, including the inner product.

This implies that there is no wave-function of the universe, because there is no observer outside the universe who could measure it. The quantum state is, and must remain, a description of part of the universe. Relational quantum mechanics develops this idea.

Completing quantum theory

We then seek to complete quantum theory by eliminating background structures. We do this by exposing and then unfreezing the background and giving it dynamics. In other words, rather than quantizing gravity we seek to gravitize the quantum. We mean by that identifying and unfreezing those aspects of quantum theory which are arbitrary and fixed, making them subject to dynamical laws. Turning this around, we hope to understand the challenging features of quantum physics as consequences of separating the universe into two parts: the system we observe, and the rest containing the observer and their measuring instruments. Closely related to background independence is another key idea: that the observables of physical theories should describe relationships.

Relational hidden variable theories

Relational hidden variable theories.

One way to complete quantum mechanics would be by the introduction of additional degrees of freedom, whose statistical fluctuations are responsible for the fluctuations and uncertainties in quantum physics. These are called hidden variables, because it was presumed that they are not measured by the procedures which measure quantum observables. Given the ubiquity of non-locality in quantum systems, due to entanglement, it is natural to hypothesize that the hidden variables are relational, in the sense that they describe shared properties of pairs of quantum systems.

We then might expect to have a hidden degree of freedom for every pair of ordinary degrees of freedom. These could be arranged as a matrix or a graph.

Several such relational hidden variables have been constructed[20, 21, 22]. These were formulated by constructing an explicit stochastic process that, in a certain large N limit, the assumptions Nelson[23] imposes on a stochastic process to yield a solution to the Schrodinger equation would be realized. This required a fine tuning to keep the dynamics time reversible, as discussed in [24].

Relational Hidden variable theory two

From "Nonlocal beables"

The beables of the theory are d, $N \times N$ real symmetric matrices $X^{\,j}_{ai}$, with $a = 1, ..., d$ and $i, j = 1, ..., N$. The classical, local observables are taken to be the eigenvalues of these matrices, λ^a_i. These can be imagined to give the positions of N particles in d dimensional space. Relative to these, the matrix elements are non-local, as a shift in the value of any one matrix element perturbs all the eigenvalues. Our aim is to give a dynamics to the matrices such that quantum dynamics emerges for their eigenvalues.

The dynamics of these matrices is given by an action[1],

$$S = \mu \int dt Tr \left[\dot{X}^2_a - \omega^2 [X_a, X_b][X^a, X^b] \right] \quad (1)$$

We choose the matrices X^a to be dimensionless. ω is a frequency and μ has dimensions of mass $\cdot$ length^2. We do not assume $\hbar = 1$, in fact, as we aim to derive quantum mechanics from a more fundamental theory, $\hbar$ is not yet meaningful. We will introduce $\hbar$ as a function of the parameters of the theory when we derive the Schroedinger equation as an approximate evolution law. We may note that the parameters of the theory define an energy $\epsilon = \mu\omega^2$.

We then define the wave functional

$$\Psi(\lambda, t) = \sqrt{\rho(\lambda, t)} e^{\imath S(\lambda,t)/\hbar} \quad (12)$$

...

The model I have sketched shows that quantum mechanics can be recovered from an explicit hidden variables model whose beables are non-local. This is in accord with the reasons I stressed that the beables of quantum theory should be taken as non-local. I would thus propose that the ultimate legacy of Bell's fundamental work will be the discovery that quantum theory is a description of an a-local world, which we happen to see in a phase where space has emerged. When we try to describe the physics of local subsystems of the universe, delineated by the emergent and approximate concept of locality, we are forced to neglect interactions which are really there between the subsystem's microscopic degrees of freedom and other degrees of freedom now emerged in distant parts of the universe. These non-local interactions are mediated by relational degrees of freedom that are non-local, in the sense that they are shared between subsystems that are distant from each other in the emergent concept of locality.

Because of the neglect of these non-local degrees of freedom, the quantum physics of local subsystems is stochastic and subject to a persistent and universal Brownian motion, which is the cheshire cat smile of the fundamental a-locality of the world. In this sense $\hbar$ is a measure of the resistance of the world to a local description.

The dynamics of difference

In *Temporal Relationalism* Lee gives a simplified account of the "dynamics of difference" model.

1. He posits a network of relationships in the form of a graph
2. Have the metric $g_{J,K}$ = number of steps between J and K.
3. Neighbourhood of J is $\mathcal{N}_J^n$ (all K s.t. $g_{J,K} \leq$
4. A view is represented by $\mathcal{V}_J = \{\mathcal{N}_J^0, \mathcal{N}_J^1, \mathcal{N}_J^3, \dots\}$
5. Distinctiveness of views

$$\mathcal{D}_{J,K} = \frac{1}{n_{JK}^p}$$

where n is a fixed power and n_{JK} is the smallest n such that $\mathcal{N}_J^n$ is not isomorphic to $\mathcal{N}_K^n$ (under maps that preserve the origin).

6. Locality is a proxy for similarity of views.

We then define the causal version of variety

$$\mathcal{V} = \frac{1}{N^2} \sum_{A \neq B} \mathcal{D}_{AB} \tag{38}$$

We can then consider theories based on an energetic causal set, in which the variety defined in this way acts as a potential energy. The dynamics of such a theory aims to maximize the variety of the system. This acts preferentially on pairs with small distinctiveness, and changes them so as to increase the overall variety. In other words, the more similar two events are, the more likely they are to interact.

Under appropriate conditions, this leads also to a derivation of Schrodinger quantum mechanics, from a theory whose dynamics involves extremizing the variety. I leave the details to the original paper[44].

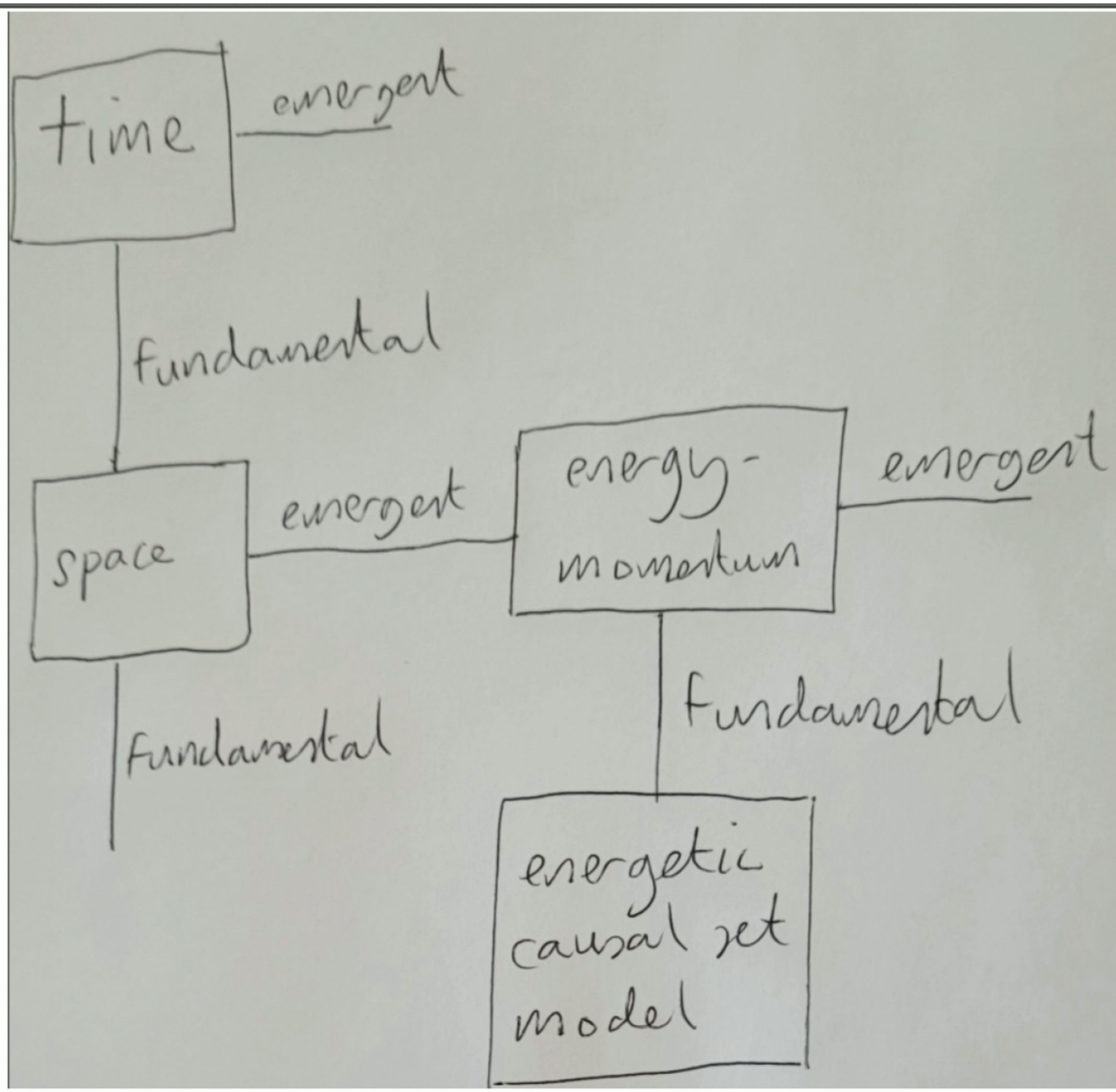
Wrap up

Results stemming from the world view Lee has developed with his colleagues

1. Relative locality - studying a limit of GR, find an extension of SR whereby whether two events coincide or not depends on their energies and distance from the observer
2. Energetic causal set models. Views and varieties.
3. Irreversible extensions of GR.
4. The real ensemble formulation of quantum mechanics.
5. Evolving laws.
6. Causal theory of views.
7. Emergence of Einstein's equations from thermodynamics of an energetic causal set.
8. ...

Lee's views (as I understand them)

1. Temporal naturalism. Time is fundamental. Novelty is evoked.
2. Space is emergent.
3. He aims to complete QT through shifting the relational curtain within the viewpoint of temporal relationalism.
4. It is hypothesised that Bell nonlocality is due to residual local effects living across emergent space.
5. Energy and momentum are taken to be fundamental also.
6. He has developed hidden variable models along the lines of this philosophy. In particular, the energetic causal set model.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