Title: Baby Universes and Holography: the Case of the Vanishing Universe

Speakers: Netta Engelhardt

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Abstract:

A number of recent arguments have suggested that baby universes in quantum gravity have a one-dimensional Hilbert space. I will provide a new argument, based on work with E. Gesteau, in favor of this conclusion. The argument makes use of a standard asymptotically AdS spacetime dual to a thermal state below the Hawking-Page transition. In particular, there is a low-energy, low-complexity causal wedge operator whose expectation value in the dual CFT is only consistent with a one-dimensional Hilbert space for baby universes.

Baby Universes and Holography: the Case of the Vanishing Universe

Netta Engelhardt

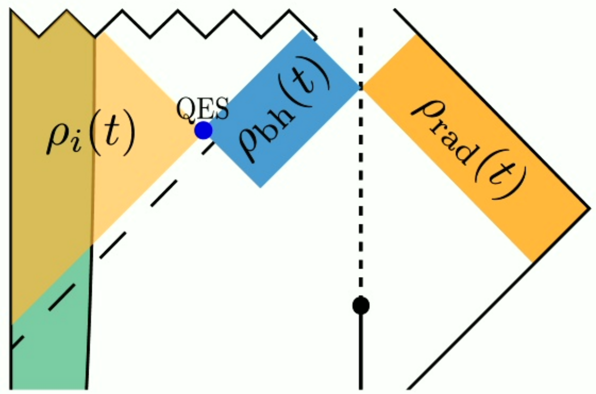
MIT

Based on:
2504.14586 with Elliott Gesteau

Quantum Information in Quantum Gravity '25

Some Conclusions from Evaporating Black Holes

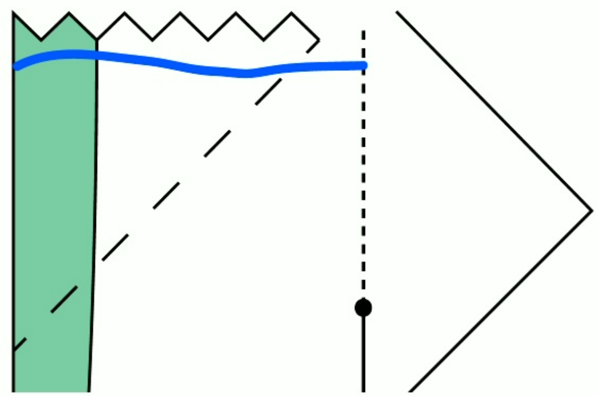
After the Page time, an evaporating black hole's interior lives in the entanglement wedge of the radiation Almheiri, NE, Marolf, Maxfield; Penington



The holographic bulk-to-boundary map acting on the black hole interior $V : \mathcal{H}_{eff} \to \mathcal{H}_{fund}$ is non-isometric (many-to-one) after the Page time. Akers, NE, Harlow, Penington, Vardhan

Some Conclusions from Evaporating Black Holes

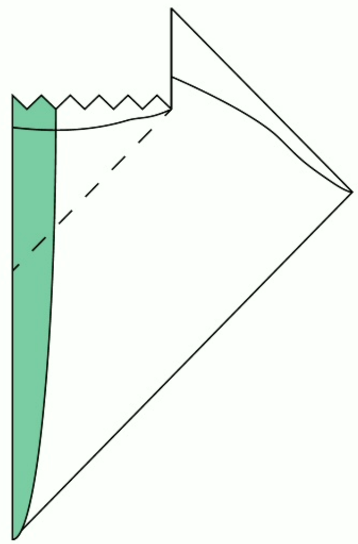
As we approach evaporation, the length of a nice slice keeps increasing, so $|\mathcal{H}_{eff}| \gg 1$; but $|\mathcal{H}_{\text{fund}}| \to 1$: almost all the states in $\mathcal{H}_{eff}$ are null states.



A slice that ends at the evaporation point is very similar to a slice of a baby universe. This analogy has been invoked to suggest that the fundamental Hilbert space of a baby universe is one-dimensional. Almheiri Mahajan, Maldacena, Zhao; PSSY

Other Arguments

The evaporating black hole involves a singular slice, which (to me) is questionable.



There are now other arguments that use the Maldacena-Maoz wormhole Usatyuk, Zhao to reach a similar conclusion, although it requires a nontraditional application of the AdS/CFT dictionary to a system without a Lorentzian AdS boundary.

Other Arguments

Recently, I started getting worried there might be something to these arguments.

Yes, worried.

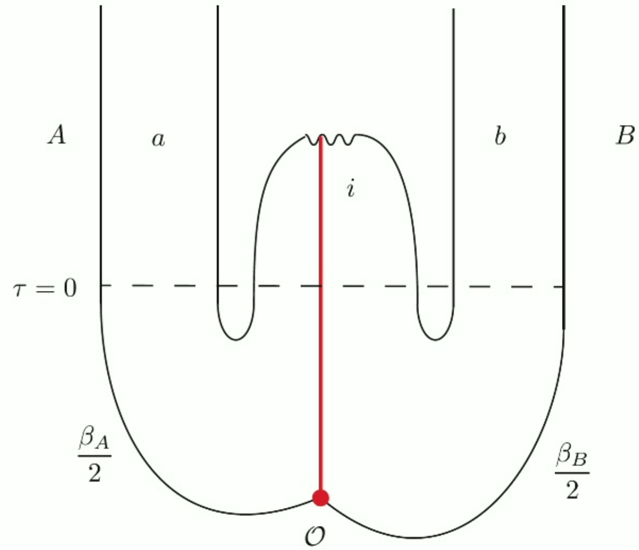
A one-dimensional Hilbert space does not seem consistent with our local observations, and we shouldn't be able to tell if our universe is closed or not based on local observations! recent work by Harlow, Usatyuk, Zhao; Abdalla, Antonini, Iliesiu, Levine suggests that we could fix this with an observer – see Daniel's talk this morning and Elliott's talk next.

In an effort to prove that it is a fallacy, we seem to have found the opposite.

Today, I'm going to begin by studying the baby universe using a particular – and standard – AdS/CFT construction by Antonini-Rath (AR).

The Antonini-Rath Setup

Consider two CFTs in a thermal state below Hawking-Page, and insert a heavy operator in Euclidean time Antonini, Sasieta, Swingle.



We have two boundary systems: A and B, and three bulk systems: a, b, and i. a and b are the causal wedges of A and B.

The Antonini-Rath Setup

The state prepared by the path integral is:

$$|\Psi\rangle_{AB} = \frac{1}{\sqrt{Z}} \sum_{m,n} e^{-\frac{1}{2}(\beta_A E_n + \beta_B E_m)} O_{m,n} |E_n\rangle |E_m\rangle ,$$

where $\beta_A, \beta_B > \beta_{\text{Hawking-Page}}$. AR truncate the tails so this lives in a large but $\mathcal{O}(1)$ microcanonical window, and large but $\mathcal{O}(1)$ entropy.

Let $|\psi^{(1)}\rangle_{aib}$ be the bulk state corresponding to the path integral preparation in the previous slide.

$$S_{\text{vN}}[\psi^{(1)}_{ab}] = S_{\text{vN}}[\psi^{(1)}_{i}] = \mathcal{O}(1)$$

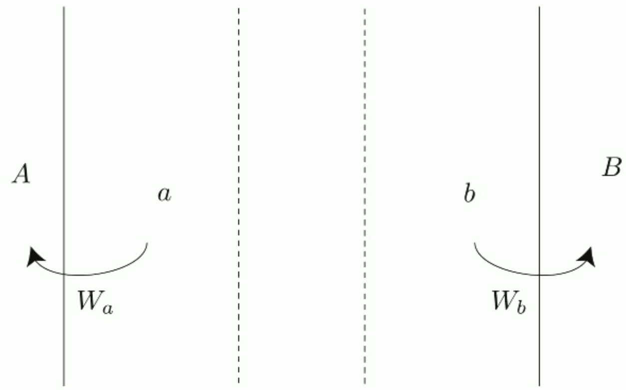
which is large but does not scale with N.

The Encoding Maps

Define encoding maps of the causal wedges:

$$W_a : \mathcal{H}_a \to \mathcal{H}_A$$
$$W_b : \mathcal{H}_b \to \mathcal{H}_B$$
$$V : \mathcal{H}_{ab} \to \mathcal{H}_{AB}$$

so that $V = W_a \otimes W_b$.



Because a, b are just causal wedges of A, B, these are just the HKLL map on various regions.

THE ANTONINI-RATH SETUP

Now rewrite $|\Psi\rangle_{AB}$ in a different basis: for simplicity consider a bulk theory with a single scalar ϕ dual to a single-trace operator Φ. Then, because $|\Psi\rangle_{AB}$ lives in an $\mathcal{O}(1)$ energy microcanonical window:

$$|\Psi\rangle_{AB} = \sum_{A_i B_j} c_{A_i B_j} \Phi_{A_i} \Phi_{B_j} |0\rangle_A |0\rangle_B \equiv \sum_{A_i B_j} c_{A_i B_j} |A_i\rangle |B_j\rangle ,$$

But there is a bijection between $|A_i\rangle$, $|B_j\rangle$ and the states obtained by acting with the local bulk field ϕ on the AdS vacuum. Banks, Douglas, Horowitz, Martinec

The Antonini-Rath Setup

So we can write:

$$|\Psi\rangle_{AB} = V\left(\sum_{a_i b_j} c_{a_i b_j} |a_i\rangle |b_j\rangle\right)$$

where V is the HKLL map on $\mathcal{H}_a \otimes \mathcal{H}_b$.

Define the pure state on ab:

$$|\psi^{(2)}\rangle_{ab} \equiv \sum_{a_i b_j} c_{a_i b_j} |a_i\rangle |b_j\rangle$$

Then

$$|\Psi\rangle_{AB} = V\left(|\psi^{(2)}\rangle_{ab}\right)$$

$\Rightarrow$ The holographic dual to $|\Psi\rangle_{AB}$ is a pure state on $\mathcal{H}_a \otimes \mathcal{H}_b$: $|\psi^{(2)}\rangle_{ab}$.

The Antonini-Rath Puzzle

Puzzle: Who is Holding the Baby?

The state $|\Psi\rangle_{AB}$ is dual to a pure state on $a \cup b$ by HKLL; but it is dual to a state on $a \cup b$ which is mixed (with i) by a path integral preparation.

AR suggest the following possibilities:

1. Ensemble averaging (doesn't make sense in higher dimensions);
2. AdS $\neq$ CFT.
3. The baby universe is not semiclassical. Agrees with other arguments that its Hilbert space is one-dimensional.

We'll give a new argument that (3) is correct.

Main Tool

The SWAP operator

Consider two copies of a quantum system with Hilbert space $\mathcal{H}$. The SWAP operator $\mathcal{S}$ on $\mathcal{H} \otimes \mathcal{H}$ is a low complexity operator:

$$\mathcal{S}(|\psi\rangle \otimes |\phi\rangle) = |\phi\rangle \otimes |\psi\rangle$$

And similarly for density matrices.

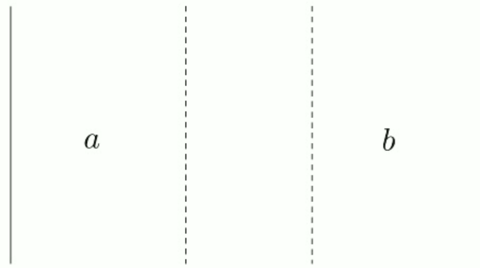
If ρ is a density matrix:

$$\langle \mathcal{S} \rangle_{\rho \otimes \rho} = \mathrm{tr}\left(\rho^2\right)$$

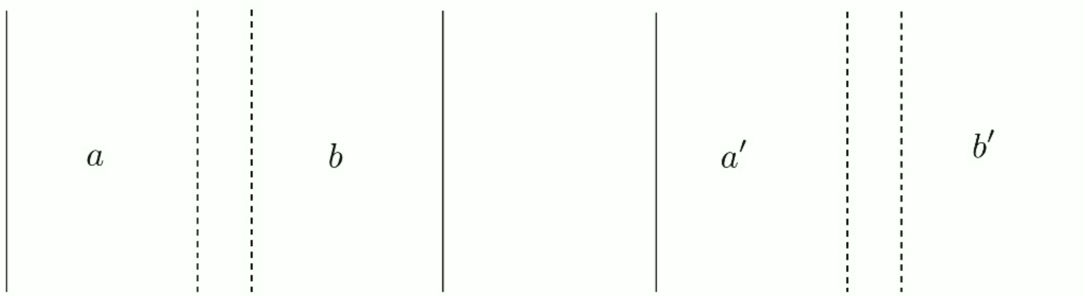
When ρ is pure, $\langle S \rangle$ is always 1; when ρ is maximally mixed, $\langle S \rangle$ is exponentially suppressed in $S_{\mathrm{vN}}[\rho]$.

The Bulk Swap Operator

Now take our quantum system to be the two causal wedges a and b.



We double the system, so we have a, b, and a', b': $\mathcal{H}_a \otimes \mathcal{H}_b \otimes \mathcal{H}_{a'} \otimes \mathcal{H}_{b'}$, where a, a' are identical and b, b' are identical.



The Bulk Swap Operator

We define $\mathcal{S}$ to be the SWAP operator on $\mathcal{H}_a \otimes \mathcal{H}_b \otimes \mathcal{H}_{a'} \otimes \mathcal{H}_{b'}$.

$\mathcal{S}$ has support exclusively in the causal wedge.

Compute $\mathcal{S}$ in $\psi_{ab}^{(1)}$ and $\psi_{ab}^{(2)}$:

$$\langle \mathcal{S} \rangle_{\psi_{ab}^{(1)} \otimes \psi_{a'b'}^{(1)}} = \mathrm{tr}\left[\left(\psi_{ab}^{(1)}\right)^2\right] \sim e^{-S[\psi_i^{(1)}]}$$

$$\langle \mathcal{S} \rangle_{\psi_{ab}^{(2)} \otimes \psi_{a'b'}^{(2)}} = \mathrm{tr}\left[\left(\psi_{ab}^{(2)}\right)^2\right] = 1.$$

Thus *there exists an $\mathcal{O}(1)$ complexity bulk operator localized purely in the causal wedge that can distinguish between $\psi_{ab}^{(1)}$ and $\psi_{ab}^{(2)}$.*

Because $\mathcal{S}$ is localized to the causal wedge, there is no ambiguity: it admits a boundary dual, and that boundary dual is given by the encoding map on the causal wedge.

Boundary Dual of $\mathcal{S}$

Since $\mathcal{S}$ acts on two copies of $\mathcal{H}_a \otimes \mathcal{H}_b$, we can find the boundary operator corresponding to $\mathcal{S}$ by two (tensored) applications of V:

$$S_\partial (V \otimes V) = (V \otimes V)\mathcal{S}$$

Now we remember that $|\Psi\rangle_{AB}$ can be written:

$$\Psi_{AB} = V \psi_{ab}^{(2)} V^\dagger .$$

So:

$$\begin{aligned} \langle \mathcal{S}_\partial \rangle_{\Psi_{AB} \otimes \Psi_{A'B'}} &= \langle \mathcal{S}_\partial \rangle_{V \psi_{ab}^{(2)} V^\dagger \otimes V \psi_{a'b'}^{(2)} V^\dagger} \\ &= \langle \mathcal{S} \rangle_{\psi_{ab}^{(2)} \psi_{a'b'}^{(2)}} \\ &= 1. \end{aligned}$$

Summary of Swap Argument

- We found an $\mathcal{O}(1)$ complexity bulk operator $\mathcal{S}$ in the causal wedge such that

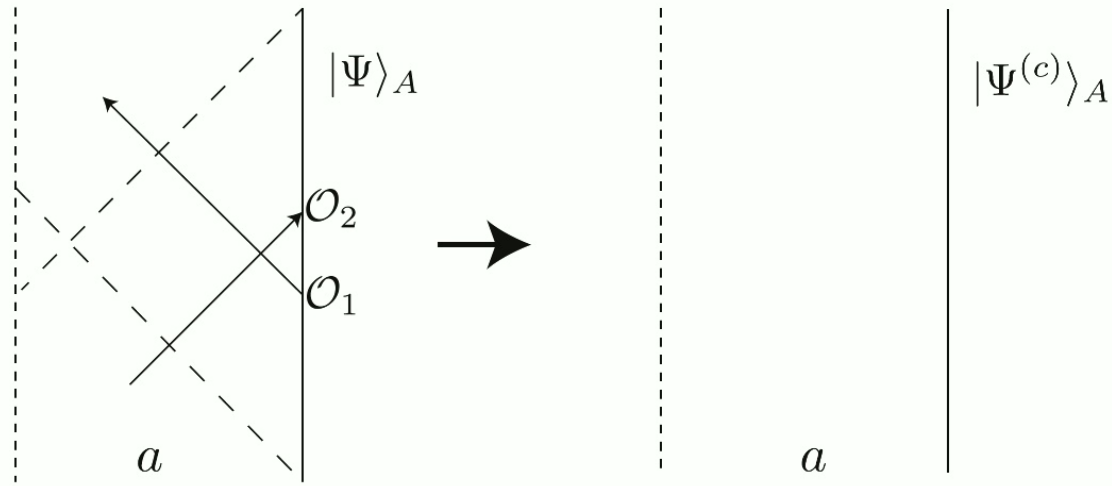
$$\langle \mathcal{S} \rangle_{\text{no baby universe}} = 1$$
$$\langle \mathcal{S} \rangle_{\text{w/ baby universe}} = e^{-S_{\text{vN}}[\phi_i^{(1)}]}$$

- Because operators localized purely in the causal wedge can be mapped to the boundary using an isometric encoding map V, we have a precise boundary dual to $\langle \mathcal{S} \rangle$ that preserves expectation values.
- The expectation of the boundary dual operator is always 1.
- Thus, for AdS$\neq$ CFT to be the explanation for AR, it has to be **at the level of the extrapolate dictionary**. That is, it has to be the case that the extrapolate dictionary **isn't part of the AdS/CFT duality**.

High Energy States

Key Observation

Introducing a lot of matter to the AdS regions won't add states to the baby universe.



For simple black holes, the construction goes through: this is not special to $\mathcal{O}(1)$ energy.

Upshot

- So what happens to the path integral preparation?
- The obvious resolution is that the baby universe is not semiclassical.
- Any entanglement between $a \cup b$ and the baby universe is with its state in its fundamental description.
- Any holographic description of the baby universe must then satisfy $1 = e^{-S_{\text{vN}}[\phi_i^{(1)}]}$. So the entropy of $\phi_i^{(1)}$ is bounded from above by 0.
- The holographic map throws out the baby with the bathwater.
- How can we fix it? (Observers?)

Thank you!