

Title: Chaos in deformed SYK models

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Abstract:

The SYK model has attracted significant interest for its maximal chaos, connections to two-dimensional holography, and relevance to strange metals.

Two signatures of chaos in this model are out-of-time-ordered correlators (OTOCs) and Krylov complexity, both exhibiting early-time exponential behaviours characterized by the Lyapunov and Krylov exponents, respectively. In this talk, I explore these quantities in a class of relevant deformations of the SYK model, including flows which interpolate between two regions of near-maximal chaos and flows that lead to a nearly-integrable behaviour at low temperatures. I present both analytic and numerical results showing that the Krylov exponent consistently upper-bounds the Lyapunov exponent. Notably, while the Lyapunov exponent varies non-monotonically with temperature, the Krylov exponent remains smooth and monotonic, showing no clear signatures across chaotic transitions. This challenges the effectiveness of Krylov complexity as a diagnostic of chaos in quantum mechanical systems. I will also comment on the potential relevance of SYK flows for de Sitter holography and on connections to other recent works.



Chaos in deformed SYK models

Shira Chapman, Ben-Gurion University

QIQG 2025: Quantum Information in Quantum Gravity

26-06-2025



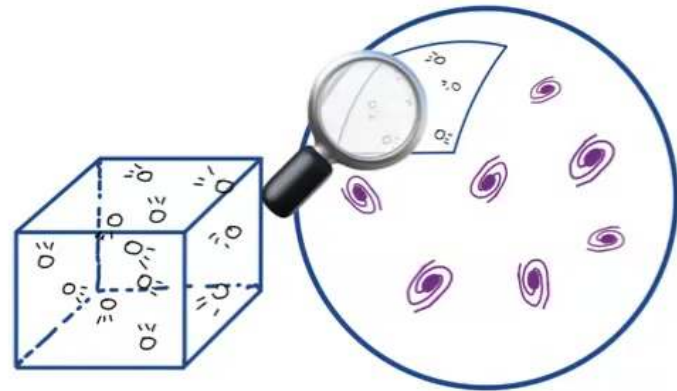
SC, Saskia Demulder, Damian Galante, Sam Sheorey, Osher Shoval [PRB 2025']

Motivation



- Better understanding many-body systems.
- Signatures of quantum chaos.
- OTOC and Krylov Complexity

- Microscopic models of spacetime?
- The role of chaos in holography.
- Potential for dS holography?



Outline

- Motivation [V]
- SYK model and its deformations
- Signatures of Chaos
- Results – signatures of chaos in SYK deformations
- Summary and Outlook: hints for dS holography?

SYK model: ensemble of 1D many-body QM

$$H_q = i^{\frac{q}{2}} \sum_{1 \leq i_1 < i_2 < \dots < i_q \leq N} J_{i_1 i_2 \dots i_q} \psi_{i_1} \psi_{i_2} \dots \psi_{i_q}, \quad \{\psi_i, \psi_j\} = \delta_{ij}$$

↑
↑
 random q-body interactions
 N Majorana fermions
(i ∈ 1 ... N)

$$\langle J_{i_1 i_2 \dots i_q} \rangle = 0, \quad \langle J_{i_1 i_2 \dots i_q}^2 \rangle = \frac{2^{q-1} \mathcal{J}^2 (q-1)!}{q N^{q-1}}$$



[Sachdev and Ye (1993); Maldacena and Stanford (2016); Kitaev and Suh (2018)]
 [Review: Chowdhury, Georges, Parcollet, Sachdev]

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Appealing properties:

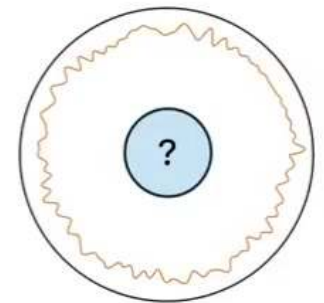
- **Exactly solvable** at large N, in terms of effective action for fermion bilinears, whose equations of motion are the **Schwinger-Dyson** equations.
- Further simplification when q also large (ratio $\lambda \equiv 2q^2/N$ fixed: **DSSYK**, we take $\lambda \rightarrow 0$).
- Exhibits **chaos** for $q \geq 4$; Special case: $q = 2$ is integrable.
- Used to model **strange metals** (many-body quantum states without quasiparticles).



[Sachdev and Ye (1993); Maldacena and Stanford (2016); Kitaev and Suh (2018)]
 [Review: Chowdhury, Georges, Parcollet, Sachdev]

The low temperature limit and gravity

- At low energies, the SYK model is approximated by a conformal field theory where the fermions obtain a conformal dimension of $\Delta_\psi = 1/q$.
- The effective action: $S_{\text{eff}} = S_{\text{CFT}} + S_{\text{Sch}} + \dots$
with a leading Schwarzian correction making it nearly conformal.
- Maximally chaotic at low temperatures (and yet tractable!)

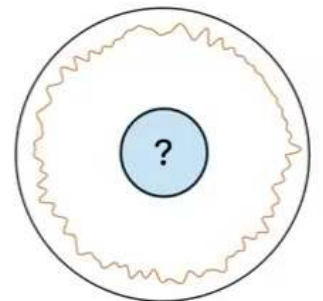


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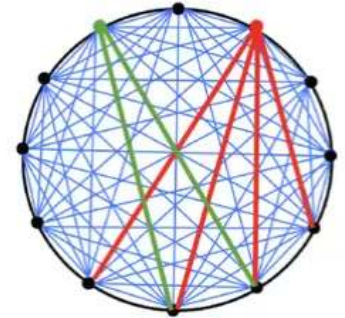
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Dual gravitational description JT gravity (gravity + dilaton)

- Bulk is rigid AdS_2 and dynamics reduces to that of the dilaton boundary mode [Maldacena, Stanford, Yang (2016)]
- Can we obtain more general gravity backgrounds from SYK deformations? with different IR?



Deformations of the SYK model



- Relevant deformation of SYK – hard to find!
- Why? In the IR the fermion acquired a dimension

$$H_q = i^{\frac{q}{2}} \sum J_{i_1 i_2 \dots i_q} \psi_{i_1} \psi_{i_2} \dots \psi_{i_q} \rightarrow \Delta_\psi = 1/q$$

- Naive additions to the Hamiltonian - vanishing $\mathcal{O} = \sum_i \psi_i \psi_i$ or irrelevant: $\mathcal{O} = \sum_i \psi_i \partial_\tau^{2n+1} \psi_i$
- Consider disordered operators, like the Hamiltonian itself

$$H_{\text{def}} = H_q + s H_{\tilde{q}}, \quad s > 0, \quad q \geq \tilde{q}$$

- *Naive scaling of deformation around IR fixed point $\Delta = \tilde{q}/q < 1$.
- *Becomes important at very low energies (deep IR)

[Jiang and Yang (2019); Anninos, Galante (2020), Anninos, Galante, Sheorey (2022)]

[Garcia Garcia et al. (2018); Lunkin et al. (2020); Nandy et al. (2022)]

$q = \#$ of fermions in each interaction term

$$H_{\text{def}} = H_q + sH_{\tilde{q}} , \quad s > 0 , \quad q \geq \tilde{q} .$$

- Large N & large q techniques can be generalized to this case.
- Effective theory in terms of bilinear fields (fermion bi-linear G and self-energy Σ):

$$G(\tau_1, \tau_2) = \frac{1}{N} \sum_{i=1}^N \langle T \psi_i(\tau_1) \psi_i(\tau_2) \rangle, \quad \Sigma(\tau_1, \tau_2) = \text{self energy}$$

- Deformed **Schwinger-Dyson** equations (EOM):

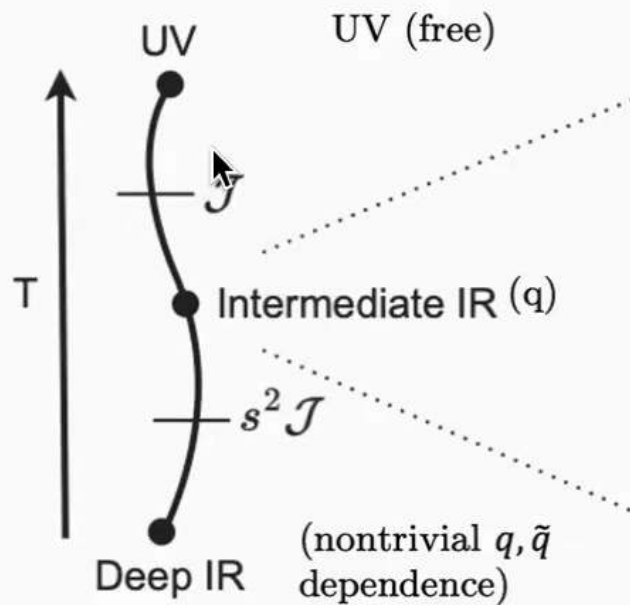
$$G^{-1}(\tau_1, \tau_2) = \delta(\tau_1 - \tau_2) \partial_{\tau_2} - \Sigma(\tau_1, \tau_2) ,$$

$$\Sigma(\tau_1, \tau_2) = \mathcal{J}^2 \left(\frac{2^{q-1}}{q} G(\tau_1, \tau_2)^{q-1} + s^2 \frac{2^{\tilde{q}-1}}{\tilde{q}} G(\tau_1, \tau_2)^{\tilde{q}-1} \right) .$$

- Large N and large q ($q, \tilde{q} \rightarrow \infty$ with $n \equiv q/\tilde{q}$ fixed), have analytic solution



Regimes along the RG flows



$$H_{\text{def}} = H_q + sH_{\tilde{q}}, \quad s > 0, \quad q \geq \tilde{q}$$

- Renormalization group flows away from the near-fixed point of the Sachdev-Ye-Kitaev (SYK) model
- Expect two different regimes of nearly maximal chaos, can we see them?

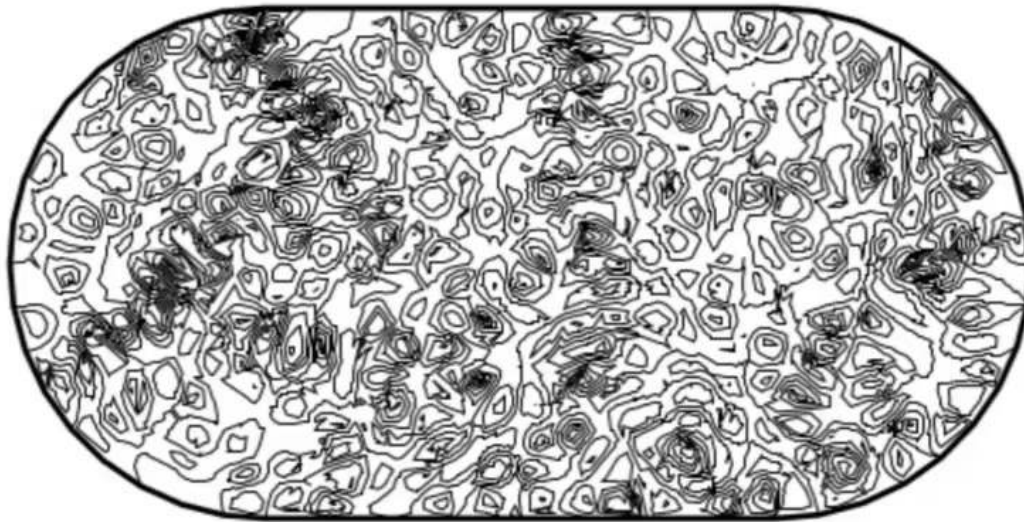
[Jiang, Yang]

[Anninos, Galante, Sheorey,]

*If not $s \ll 1$ go directly to deep IR

Quantum Chaos

(and how to recognize it)





Signature 1 - Out of time ordered correlation functions (OTOCs)

- Quantum version of hypersensitivity to initial conditions.

$$\text{OTOC}(t) = \frac{1}{N^2} \sum_{i,j=1}^N \text{Tr} \left(\rho^{\frac{1}{4}} \psi_i(t) \rho^{\frac{1}{4}} \psi_j(0) \rho^{\frac{1}{4}} \psi_i(t) \rho^{\frac{1}{4}} \psi_j(0) \right), \quad \rho = \frac{1}{Z(\beta)} e^{-\beta H}$$

$$\boxed{\text{OTOC} = f_0 - \frac{f_1}{N} \exp(\lambda_{\text{OTOC}} t) + \dots}$$

← OTOC Lyapunov exponent

- A bound on chaos [Maldacena, Shenker, Stanford (2016)]: $\lambda_{\text{OTOC}} \leq \frac{2\pi}{\beta}$
- Need semiclassical limit to have separation between the dissipation and scrambling time.
- In SYK – relevant signature when N is large.

Signature 2 – Krylov Complexity

- No need for large N in this approach
- How fast does an operator spread with time $\mathcal{O}(t) = e^{iHt}(\mathcal{O}_{local})e^{-iHt}$?
- More precisely – how complex does it become with time?
- Construct a basis for the time evolution:

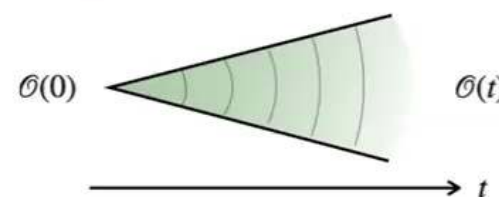
$$\mathcal{H}_{\mathcal{O}} = \text{span}\{\mathcal{O}, [H, \mathcal{O}], [H, [H, \mathcal{O}]], \dots\} \longrightarrow \mathcal{H}_{\text{Krylov}} = \text{span}\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3 \dots\}$$

Simple \leftrightarrow complicated

- Measure of complexity of the time-evolved operator:

$$C_K(t) = \sum_n n \times \left(\begin{array}{c} \text{amplitude} \\ \text{to be at } \mathcal{O}_n \end{array} \right) = \exp(\lambda_K t)$$

Krylov exponent



- Conjecture: exponential spreading in **Chaotic systems**

- Computable from the thermal two point function - easier.

[Parker, Cao, Avdoshkin, Scaffidi, Altman (2019)]

Relation between the two exponents

- Bounds on Krylov complexity

[Parker, Cao, Avdoshkin, Scaffidi, Altman (2019);
Avdoshkin, Dymarsky (2020); Gu, Kitaev, Zhang (2022)]

$$\lambda_{\text{OTOC}} \leq \lambda_K \leq \frac{2\pi}{\beta}$$

- The bound turned out to be tight $\lambda_{\text{OTOC}} = \lambda_K$ for SYK at large N , large q , at any value of the temperature.
- Conjecture: the two are always equal. Easier way to calculate OTOC exponent?

Questions

- What are λ_{OTOC} and λ_K in simple and in deformed SYK models?
- Are they generally equal?
- Can they both diagnose chaos transitions along RG flows?

Answers:

- The two exponents are not equal! Can already see at next order in $1/q$ expansion for a single SYK:

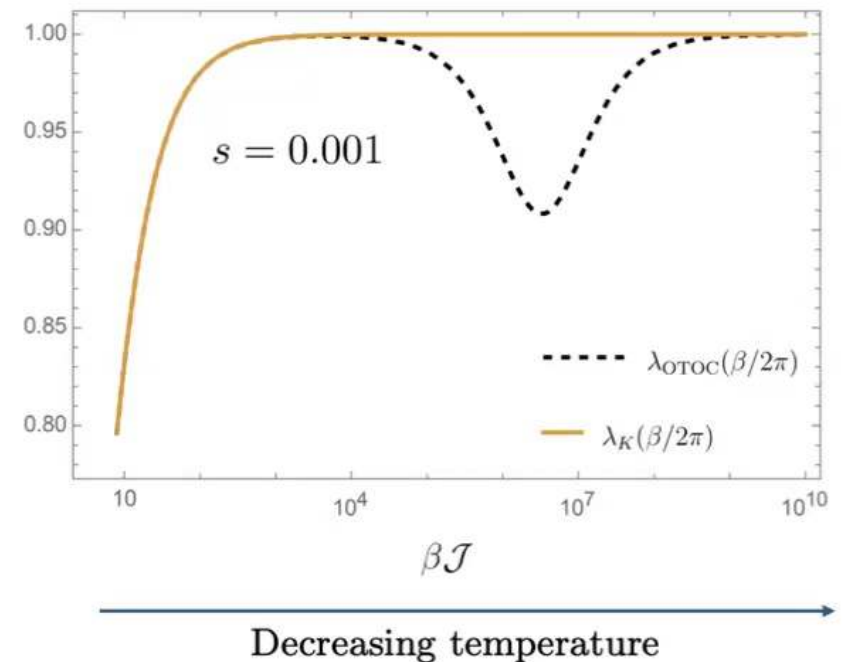
$$\frac{\beta}{2\pi}(\lambda_K - \lambda_{OTOC}) = \frac{4\pi^2}{3q\beta\mathcal{J}} + \dots$$

- Only λ_{OTOC} can diagnose chaos, λ_K cannot!

Flows - Chaos to Chaos

$$H_{\text{def}} = H_q + sH_{q/2}, \quad q \rightarrow \infty$$

- Two regimes of nearly maximal chaos.
- Chaos transition at $\beta\mathcal{J} \sim 1/s^2$
- OTOC feels the transitions
- Krylov does not diagnose chaos transitions!
- Krylov complexity correctly (but poorly!) bounds (above) the Lyapunov exponent.



Large N and finite q – Numeric strategy

Outline of the numerical procedure:

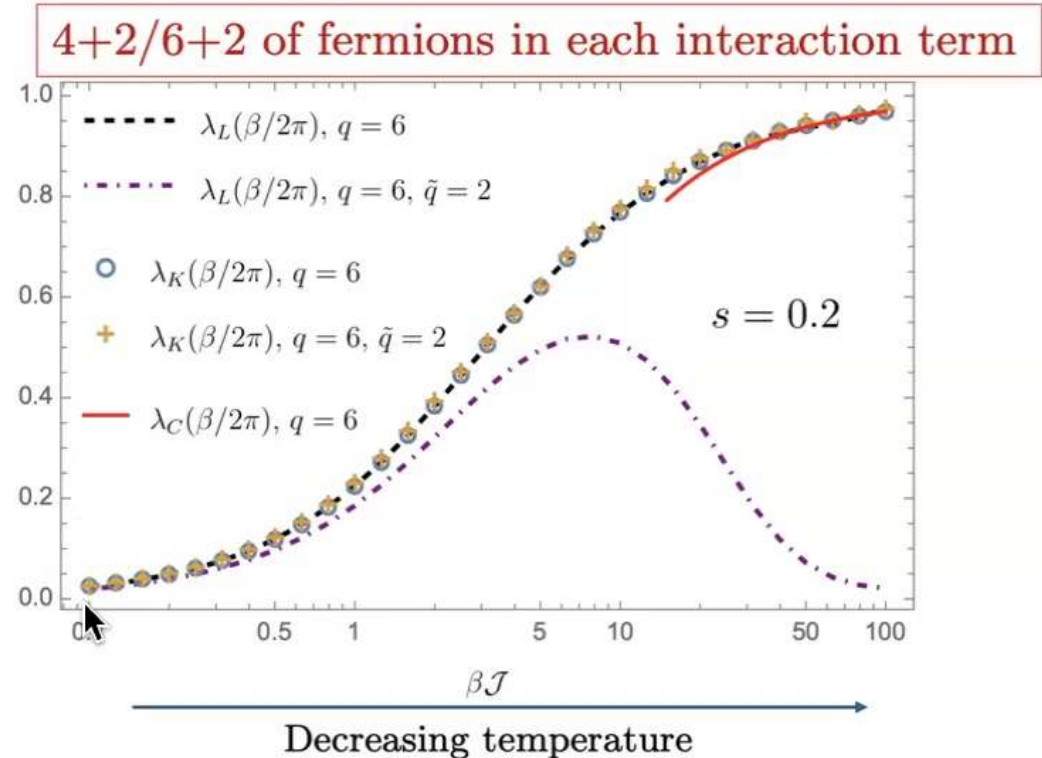
- 1) Solve the the Schwinger-Dyson equation iteratively in Fourier space to extract the fermion 2pt function.
- 2) Evaluate the Krylov exponent from the moments of the 2pt functions
- 3) Solve a Kernel equation for the OTOC, as a matrix diagonalization in discretized time.
- 4) Extract the OTOC Lyapunov exponent.
- 5) Numerical difficulties associated with low temperatures (many coding hours of our talented students & postdocs)

$$K(t_1, t_2, t_3, t_4) = G^R(t_{13})G^R(t_{24})\mathcal{J}^2 \left(\frac{2^{q-1}}{q}(q-1)G^W(t_{34})^{q-2} + s^2 \frac{2^{\tilde{q}-1}}{\tilde{q}}(\tilde{q}-1)G^W(t_{34})^{\tilde{q}-2} \right)$$
$$F(t_1, t_2) = \int dt_3 dt_4 K(t_1, t_2, t_3, t_4) F(t_3, t_4)$$
$$\text{OTOC}(t_1, t_2) = F_0(t_1, t_2) + \frac{1}{N}F(t_1, t_2) + \dots .$$

Flows - Chaos to integrable

$$H_{\text{def}} = H_6 + sH_2$$

- OTOC exponent sees the transition
- Krylov seems unable to diagnose the chaotic properties of the flows



Other studies of chaos-integrable transitions: [Berkooz, Brukner, Jia, Mamroud, 2024; Garcia-Garcia, Loureiro, Romero-Bermudez, Tezuka, 2018; Kim, Cao, 2021; Lunkin, Kitaev, Feigel'man, 2020; Nandy, Gadez, Ditz, Andrianov, Rosa 2022; Menzler, Jha 2024...]

Summary

- We saw deviations between the two exponents (Krylov vs. OTOC) already for the single SYK.
 - For flow SYK, Krylov did not provide a good tool for diagnosing chaos transitions along RG flows
 - In all examples, the Krylov exponent grew monotonically (when it could be defined)
- ❖ Some authors have suggested that it is the covariances of the Lanczos coefficients, that should be used to diagnose chaos [Balasubramanian, Magan, Wu, 2023']

Outlook: SYK Flows and dS Holography

- SYK at low energies dual to near-AdS₂
- Deformations of SYK: IR deformations of nearly-AdS₂ [Anninos, Galante (2020); Anninos, Galante, Sheorey (2022)]
- In 2D gravity, dS₂ bubbles can be embedded inside AdS₂ → enables dS holography with timelike boundary [Anninos, Hofman (2017)]
- Open question: Can SYK flows capture these dS-AdS configurations?



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$$H_{\text{def}} = H_q + sH_{\bar{q}} \Rightarrow H_q + isH_{\bar{q}}$$
 - Non-Hermitian Hamiltonian interpretation?
 - dS dual to open quantum system? (dS has open boundary at future infinity!)
- Chaotic signatures differ from the usual AdS₂. [Anninos, Galante, Hofman 2019; SC, Galante, Kramer 2022] For example, the OTOC oscillates. Compare to signatures of chaos in open systems? [Sá Et al. 2021; Bhattacharya et al. 2022; Liu, et al. 2023; Bhattacharjee et al. 2023; Srivatsa et al. 2024...]



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- Explore relation to other proposals DSSYK/dS? [Narovlansky, Verlinde; Sekino, Susskind...]



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Lots to explore!