Title: Holographic pseudoentanglement and the complexity of the AdS/CFT dictionary

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Abstract:

The `quantum gravity in the lab' paradigm suggests that quantum computers might shed light on quantum gravity by simulating the CFT side of the AdS/CFT correspondence and mapping the results to the AdS side. This relies on the assumption that the duality map (the `dictionary') is efficient to compute. In this talk, I will argue that the complexity of the AdS/CFT dictionary is surprisingly subtle: there might be cases in which one can efficiently apply operators to the CFT state (a task we call 'operator reconstruction') without being able to extract basic properties of the dual bulk state such as its geometry (which we call 'geometry reconstruction'), and vice versa. In order to reason about the complexity of geometry reconstruction we construct examples of holographic pseudoentanglement: that is, pairs of ensembles of states that obey the Ryu-Takayanagi formula for different geometries but which are nevertheless computationally indistinguishable. This result should be compared with existing evidence that operator reconstruction is generically easy in AdS/CFT. A useful analogy for the difference between these two tasks is quantum fully homomorphic encryption (FHE): this encrypts quantum states in such a way that no efficient adversary can learn properties of the state, but operators can be applied efficiently to the encrypted state. I will show that quantum FHE can separate the complexity of geometry reconstruction vs operator reconstruction, which raises the question whether FHE could be a useful lens through which to view AdS/CFT.

Holographic pseudoentanglement and the complexity of the AdS/CFT dictionary

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Joint work with Chris Akers, Adam Bouland, Lijie Chen, Tony Metger & Umesh Vazirani

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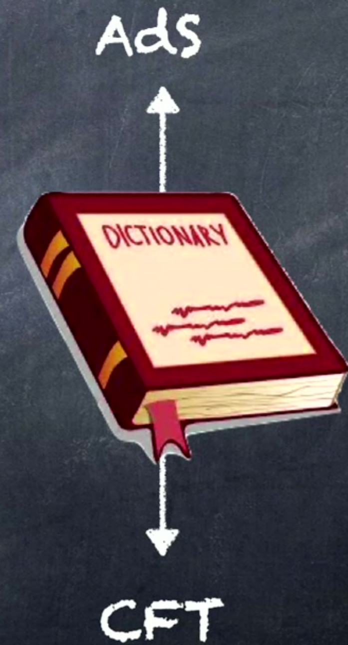
Goal: understand the AdS/CFT dictionary

- What is the complexity of the AdS/CFT dictionary?

- Could it be efficiently implemented on a quantum computer [NLBGLSSSW'21]?



AdS/CFT dictionary

AdS	CFT
Geometry	Entanglement

Easy to compute – 'feelable'

Hard to compute – not 'feelable'

AdS/CFT dictionary

AdS	CFT
Geometry	Entanglement
Wormhole volume	Circuit complexity

Easy to
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'feelable'

Hard to
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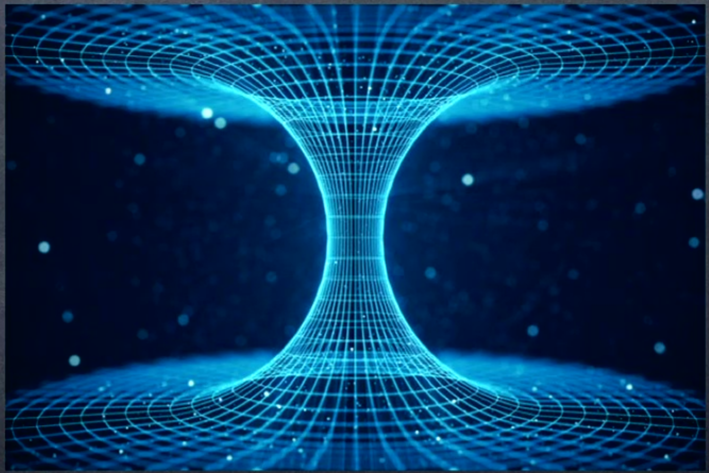
Complexity & wormholes

- Conjecture: volume of wormhole = complexity of CFT states [Susskind'14]
- Complexity theory: circuit complexity should be hard to measure in general due to pseudorandomness
- [BFV'19]: can construct quantum pseudorandomness out of wormholes using 'shocks' - this implies the AdS/CFT dictionary is exponentially hard to compute



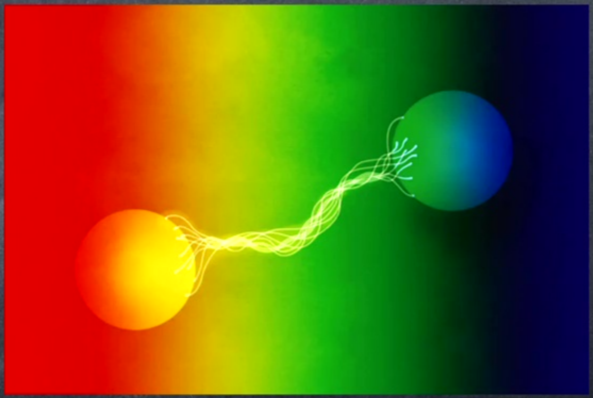
Is exponential complexity possible without horizons?

- The BFV argument only works in geometries with black holes
- This matches the physical intuition that bulk reconstruction should be easy outside of black holes
- But what about entanglement - does that imply reconstruction can be hard even outside of black holes?

Pseudoentanglement [GH'20], [ABFGVZZ'24]

Pseudoentangled state ensembles are:

- efficiently preparable on a quantum computer
- not highly entangled
- indistinguishable from a (efficiently preparable) highly entangled state ensemble to computationally bounded observers



Pseudoentanglement [GH'20], [ABFGVZZ'24]

Key question: how much entanglement can you hide?

- Entanglement is on a scale of 0 to n (number of qubits)
- [GH'20]: there exist pseudoentangled states with gap n vs $n - O(1)$ across a single bipartition
- [ABFGVZZ'24]: there exist pseudoentangled states with gap n vs $\omega(\log n)$ across all bipartition
- This isn't enough to imply that the dictionary is complex - initial constructions don't obey the RT formula

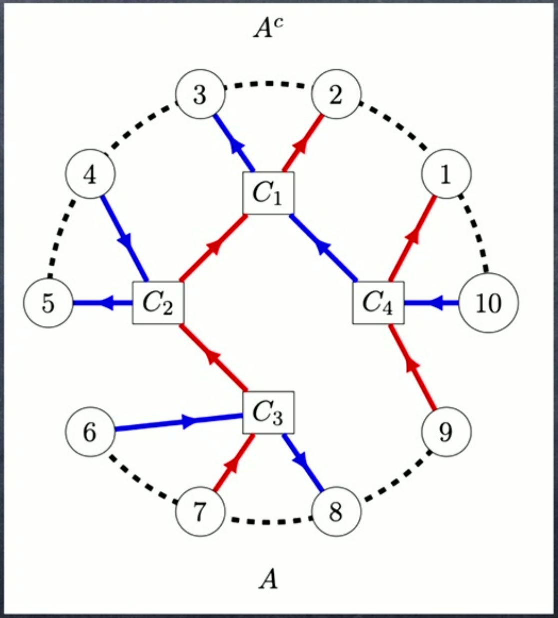
Holographic Pseudoentanglement

Holographic Pseudoentanglement

- [ABCKMV'24]: it's possible to construct pseudoentangled states which obey the R-T formula
  - See also [EFLVY'24b] & [CFI'24]
- Two constructions: one uses tree tensor networks + pseudo entangled link states, the other uses HQECC + non-entangling pseudorandom unitaries [SHH'24]
- This demonstrates that the R-T formula is not a barrier to pseudoentanglement

Holographic pseudoentanglement via pseudo entangled link states

- Take an efficiently constructible tree tensor network with RT entanglement scaling
- Replace one of the maximally entangled link states in the tensor network by a pseudo entangled state



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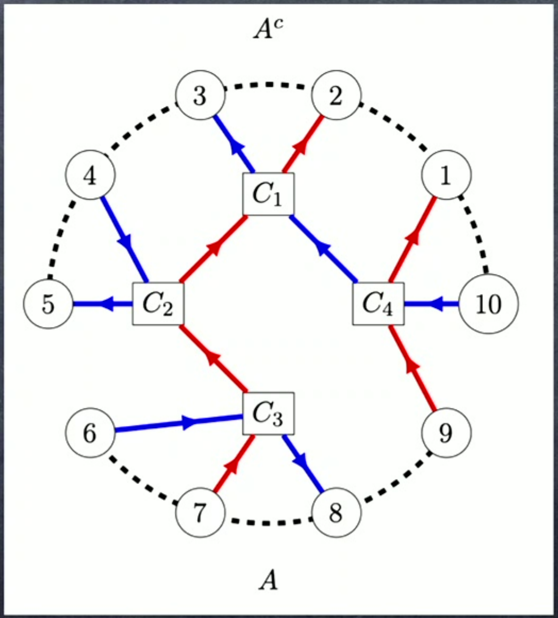
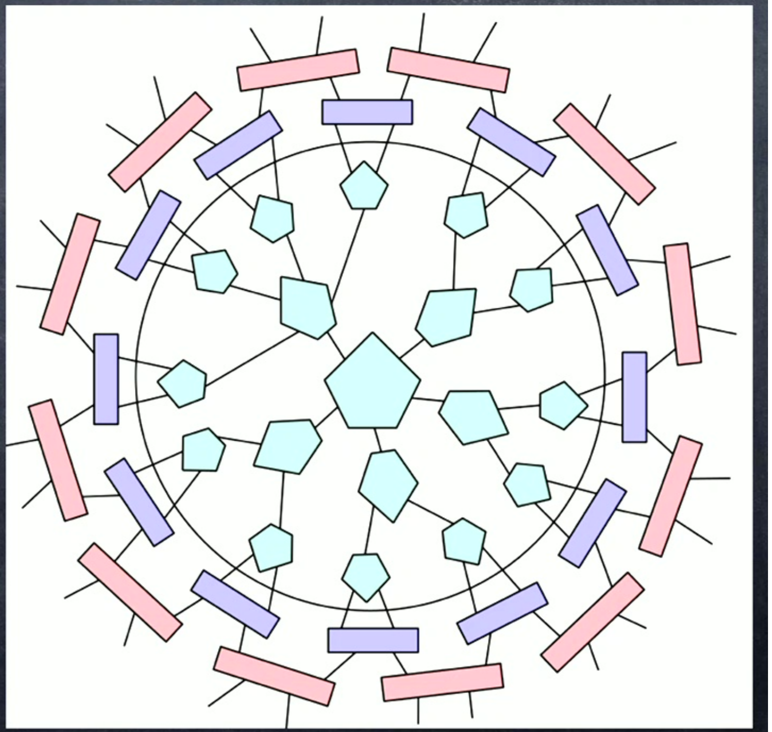


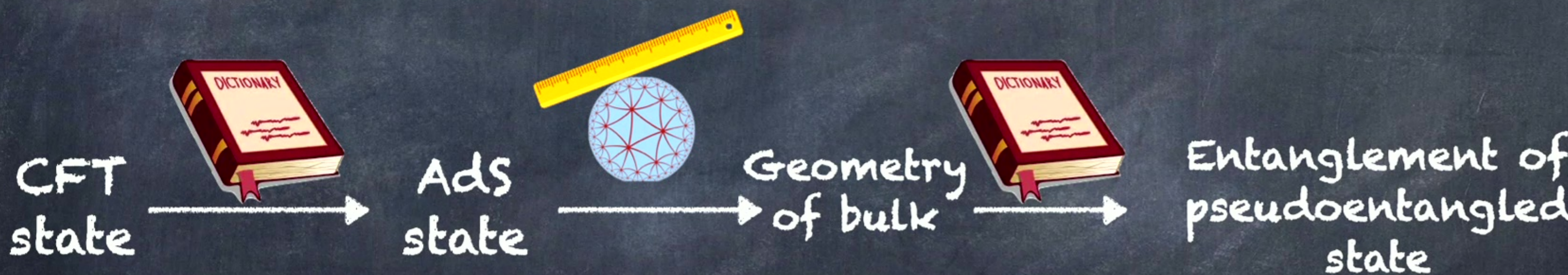
- Two boundary states, both obey RT for different geometries, but the states are indistinguishable

Holographic pseudoentanglement via PRUs

- Take an efficiently constructible tensor network toy model of a holographic state
- Apply a non-entangling pseudorandom unitary to the boundary
- The boundary state is now indistinguishable from a random state
- Can apply to arbitrary geometries to give indistinguishable boundaries



What does this mean for the complexity of the dictionary?



This is a toy model, but applied to 'real' AdS/CFT it implies that some step in this procedure cannot be done efficiently on a quantum computer

The Python's lunch conjecture

Python's lunch conjecture [BGPS'19]

The (only?) source of exponential complexity in reconstructing the AdS/CFT dictionary is a 'Python's lunch geometry'

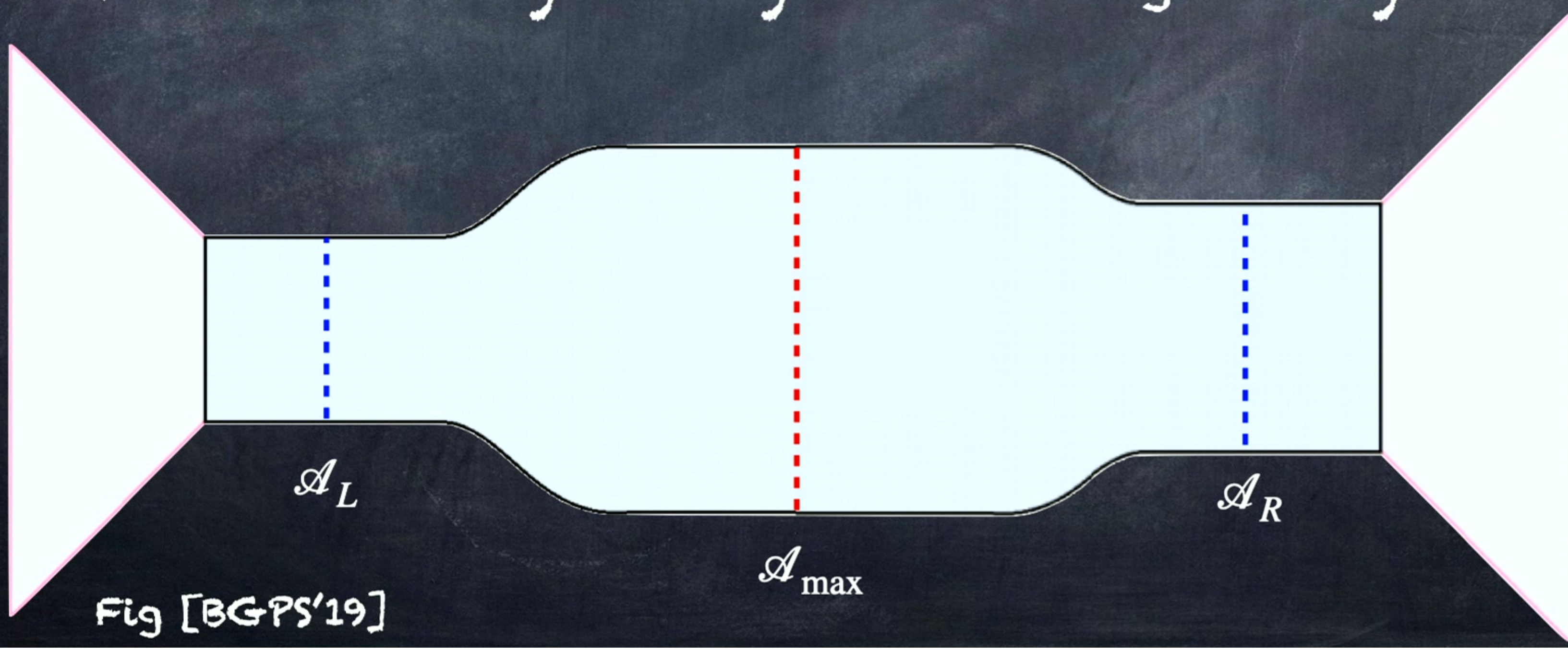
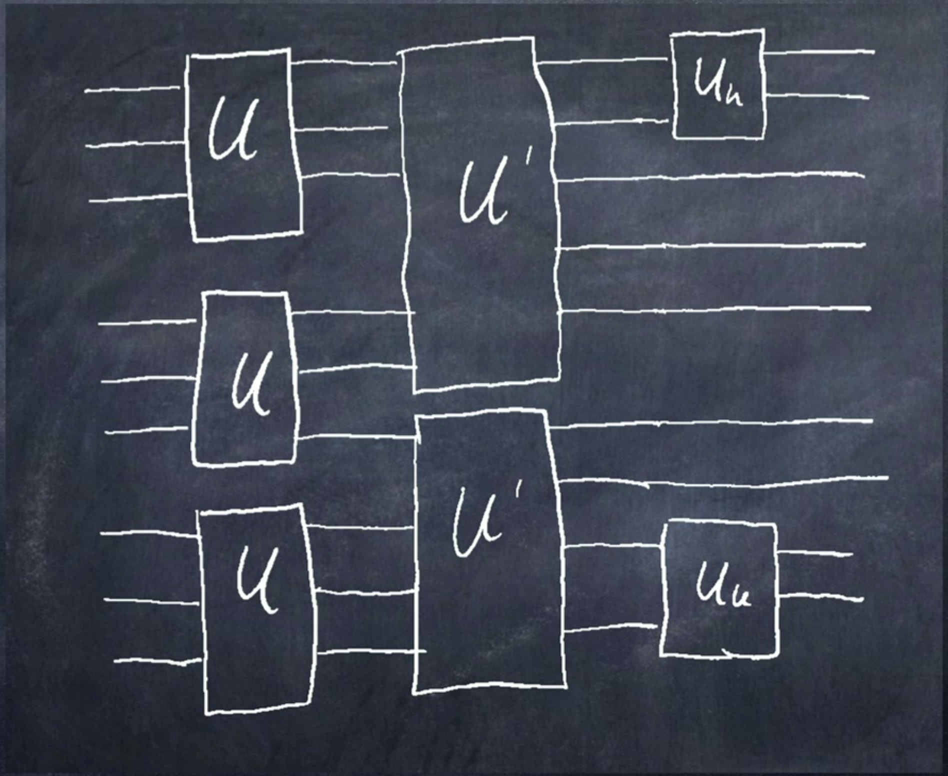


Fig [BGPS'19]

Python's lunch in our construction

- On initial inspection our constructions don't appear to contain a Python's lunch



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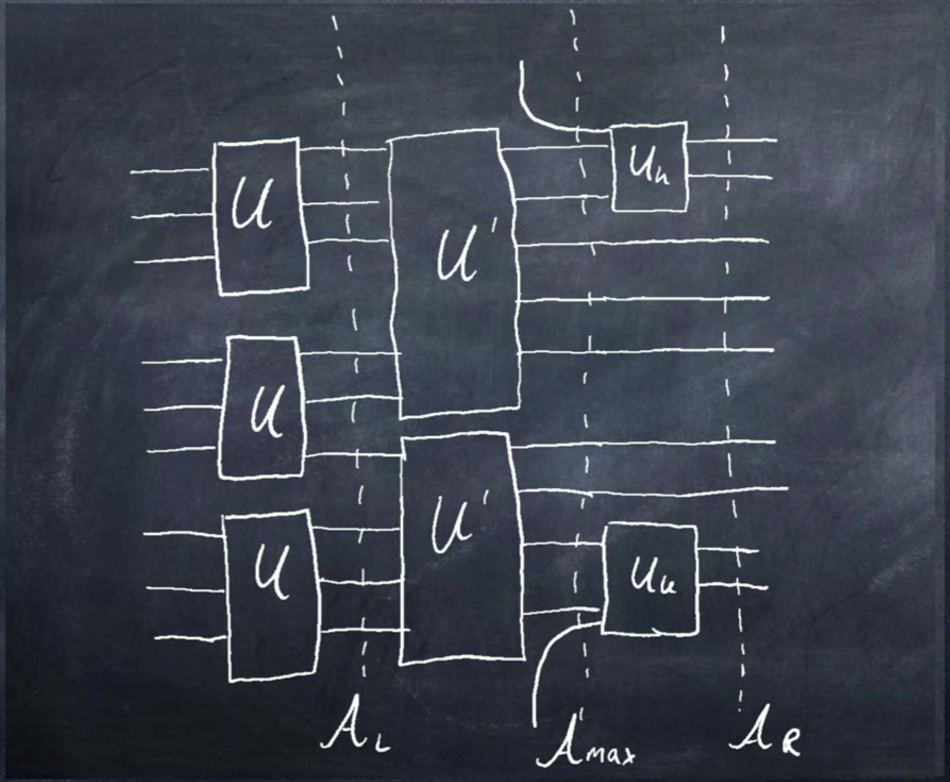
- But once the randomness needed for the PRUs / pseudoentangled link state is taken into account [EPS'21] a Python's lunch does appear



Python's lunch in our construction

- The pseudoentangled link state construction inevitably introduces a Python's lunch once the key is included due to the geometry of the network - we've added an input to a link state, no way to modify the network to remove the Python's lunch
- Our PRU construction require a private key - information has to be 'thrown away', this inevitably leads to a Python's lunch.

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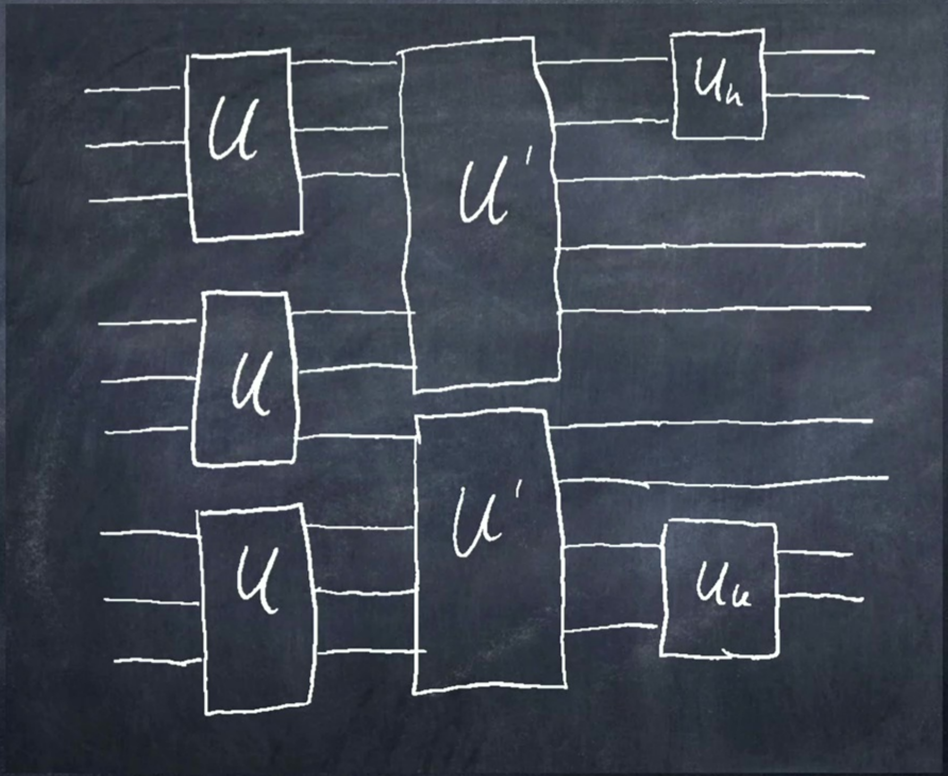

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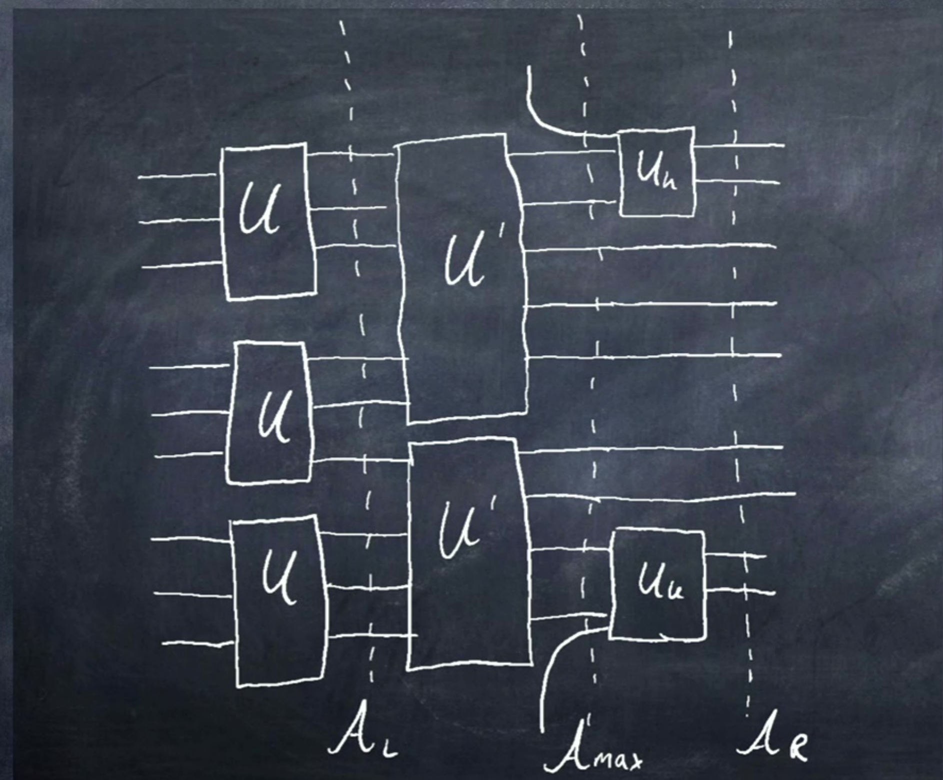

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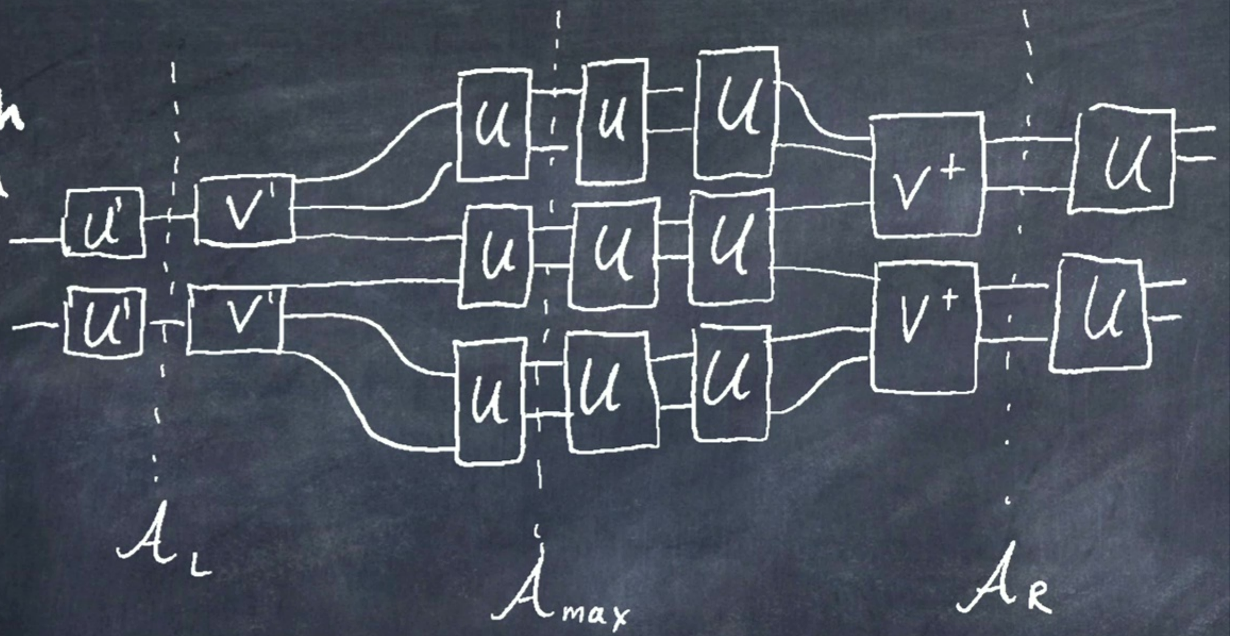
Does the Python's lunch conjecture apply to entanglement?

- In tensor networks if there's no Python's lunch operator reconstruction is easy [BGPS'19]
- There's no clear reason why a Python's lunch changes the complexity of measuring entanglement



Does the Python's lunch conjecture apply to entanglement?

- Is the complexity of implementing bulk operators on the boundary the same as the complexity of geometry reconstruction in AdS/CFT?
- In QI extracting information from states can have different complexity to applying unitaries to states - homomorphic encryption

$$|\psi\rangle \xrightarrow[\text{with key}]{\text{encrypt}} \rho_\psi \xrightarrow[\text{without key}]{\text{apply unitaries}} \tilde{U}\rho_\psi\tilde{U}^\dagger$$

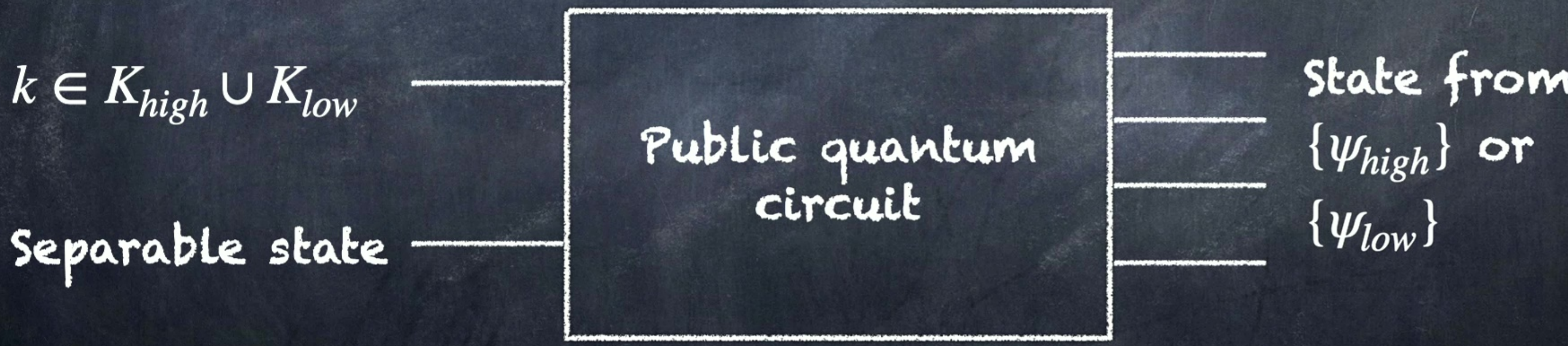
Need key to extract info!

Does the Python's lunch conjecture apply to entanglement?

- Counterpoint!
- In [EFLVY'24b] it's shown that under certain assumptions about an area operator in AdS/CFT the strong Python's lunch conjecture would imply that pseudoentanglement is only possible in geometries with a Python's lunch

Holographic pseudoentanglement without a Python's lunch?

- There are public key pseudoentanglement constructions where the circuit constructing the state & all inputs to the circuit are public knowledge [Bouland et al, 2023]



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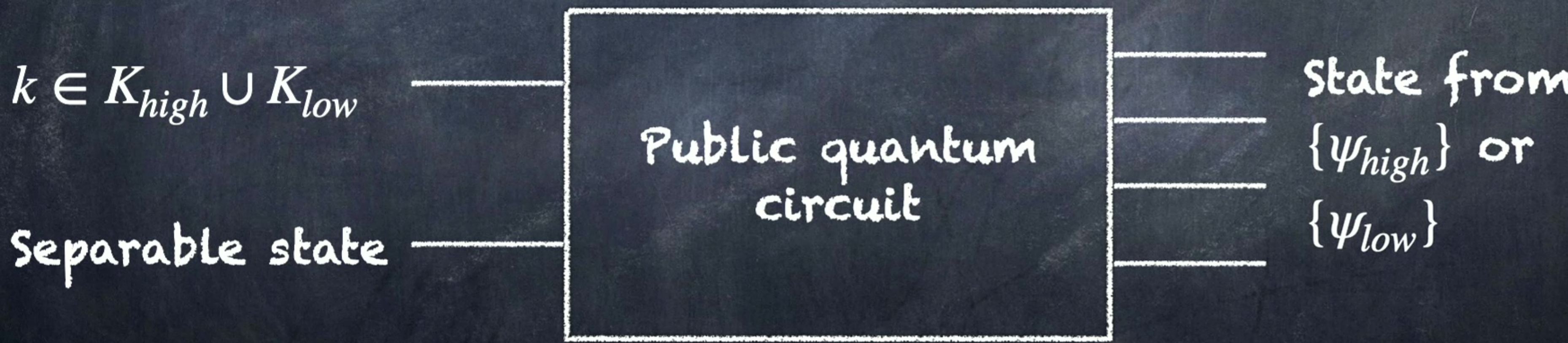

- Can we make a holographic version of this construction?

Open questions

- Can we construct a toy model of holographic pseudoentanglement with provable security & no Python's lunch?
- Can we construct holographic pseudoentanglement with a known boundary Hamiltonian?
- OR: can we prove that the Python's lunch conjecture does apply to entanglement and give a QI algorithm to compute entanglement in holographic states that don't contain a Python's lunch

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