

Title: Topological Resummation in Quantum Gravity

Speakers: Sergio Hernandez-Cuenca

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Abstract:

The gravitational path integral remains one of the most elusive constructs in quantum gravity, particularly in regimes dominated by topological fluctuations. In this talk, I will present recent progress in Jackiw-Teitelboim (JT) gravity, where the sum over geometries can be made precise. We demonstrate that in a low-temperature regime where semiclassical approximations fail, the all-genus thermal partition function of JT gravity admits a resummation into an effective description on a single geometry with a nonlocal deformation. This construction gives formal and geometric realization to long-standing ideas from the 1980s on wormhole-induced nonlocality, linking them to ensemble interpretations and topological expansions. The resulting theory, defined on the disk with conical defect operators, captures the complete topological expansion through an emergent nonlocal interaction, providing a precise geometric window into strongly quantum gravitational dynamics.



Massachusetts Institute of Technology

Center for Theoretical Physics

TOPOLOGICAL RESUMMATION IN QUANTUM GRAVITY

Sergio Hernández-Cuenca

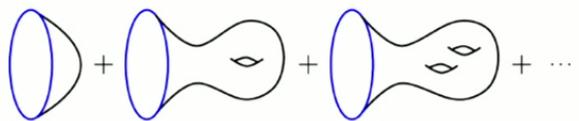
based on [2412.08799] with Wayne W. Weng and Nico Valdes-Meller

Quantum Information in Quantum Gravity
Perimeter Institute

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THE GRAVITATIONAL PATH INTEGRAL

- ▶ A popular approach to quantum gravity formulates a gravitational path integral (GPI) over geometries consistent with given rules and boundary conditions.
- ▶ In its most general form, the GPI sums over all topological manifolds and associated metric structures:

$$Z = \sum_{\text{topologies}} \int \mathcal{D}g \mathcal{D}\Phi e^{-S} = \text{[sphere]} + \text{[torus]} + \text{[genus 2 surface]} + \dots$$


- ▶ While quantum-mechanically well motivated, the GPI is rarely well defined (e.g. in Euclidean signature we have the conformal factor problem)

THE GRAVITATIONAL PATH INTEGRAL

- ▶ Only in low-dimensional models do we have well-defined measures over metrics and complete classifications of topological manifolds.
- ▶ In more general cases, studies of the GPI are limited to semiclassical regimes, where a dominant saddle-point geometry (or sum thereof) yields the proxy:

$$Z \approx \# \exp(-S_{\text{saddle}}) + \text{corrections}$$

- ▶ But in many important settings, no single saddle dominates (e.g. geometric phase transitions) or even exists (e.g. multiboundary wormholes).
- ▶ How can we address such strongly quantum gravitational regimes in richer, higher-dimensional theories? Can we recover an effective geometric description when topological fluctuations are large?

WORMHOLE INTUITION

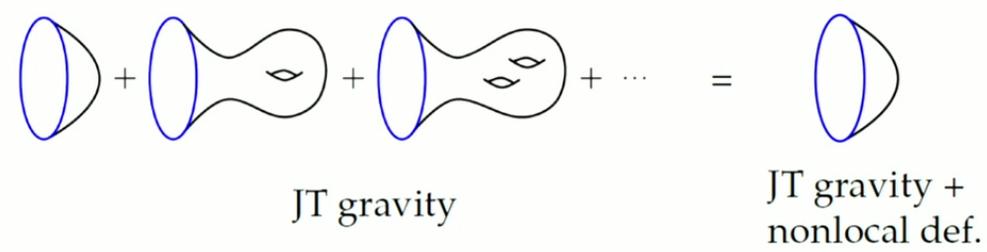
- ▶ Interesting explorations of effective implications of topological fluctuations on our universe started already in the 80's [Hawking '82; Coleman '88; Strominger-Giddings '88]
- ▶ Their key intuition was that such fluctuations would characteristically induce nonlocal interactions among quantum fields mediated by wormholes
- ▶ Because nonlocal interactions lead to an inherent uncertainty in the couplings of any local approximation, these ideas have recently resurfaced in the context of low-dimensional models of gravity in connection with ensemble interpretations [Saad-Shenker-Stanford'19, Marolf-Maxfield'20, Balasubramanian-Heckman-Lipeles-Turner'21, SHC'24]
- ▶ Here we want to be more explicit about this mechanism from the 80's and attempt a resummation over topologies to directly derive the actual effective description
- ▶ Our goal is to realize this mechanism in order to open up an effectively geometric window into highly quantum gravitational regimes, where features such as scale-invariant nonlocality become manifestly unavoidable (cf. α parameters)

MAIN RESULT

We address this question in Jackiw-Teitelboim (JT) quantum gravity, a theory where the GPI is sufficiently well-defined for our calculations to be under control

Our main result is as follows:

In a low-temperature regime where topological fluctuations escape semiclassics, the all-genus thermal partition function of JT gravity resums into an effective description involving JT gravity with a nonlocal deformation on a single geometry



MOTIVATION
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JT GRAVITY
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TOPOLOGICAL RESUMMATION
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EFFECTIVE DESCRIPTION
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CONCLUSION
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MOTIVATION

JT GRAVITY

TOPOLOGICAL RESUMMATION

EFFECTIVE DESCRIPTION

CONCLUSION

ACTION

- ▶ The action of JT in Euclidean AdS is

$$I_{\text{JT}} = -\frac{S_0}{2\pi} \left[\frac{1}{2} \int_M R + \int_{\partial M} K \right] - \frac{1}{2} \int_M \phi(R+2) - \int_{\partial M} \phi(K-1)$$

- ▶ By Gauss-Bonnet, for the first term we get the Euler characteristic

$$\frac{1}{2\pi} \left[\frac{1}{2} \int_M R + \int_{\partial M} K \right] = \chi(M) = 2 - 2g - n$$

with M an n -boundary Riemann surface of genus g

- ▶ For thermal boundary conditions on an asymptotic boundary at inverse temperature β one obtains the boundary Schwarzian action,

$$\int_{\partial M} \phi(K-1) = \int_0^\beta d\tau \{t(\tau), \tau\} = I_{\text{Sch}} \quad (1)$$

GPI

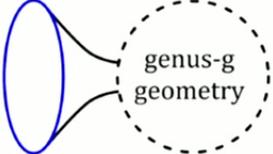
- ▶ The GPI for JT on a fixed topological manifold M thus reads

$$Z_M = e^{\chi(M) S_0} \int \mathcal{D}g \mathcal{D}\phi \exp \left[\frac{1}{2} \int_M \sqrt{g} \phi (R + 2) - I_{\text{Sch}} \right]$$

- ▶ Integrating out ϕ localizes the GPI to hyperbolic manifolds with $R = -2$,

$$Z_M = e^{\chi(M) S_0} \int \mathcal{D}g \delta(R + 2) e^{-I_{\text{Sch}}}$$

- ▶ The full GPI then involves fixing boundary conditions for the M manifolds, but otherwise summing over all possible genus- g hyperbolic geometries,

$$Z = \sum_{g=0}^{\infty} e^{(2-2g-n)S_0} \text{genus-}g \text{ geometry}$$


- ▶ This is an asymptotic expansion in the genus counting parameter e^{-2S_0}

THERMAL PARTITION FUNCTION

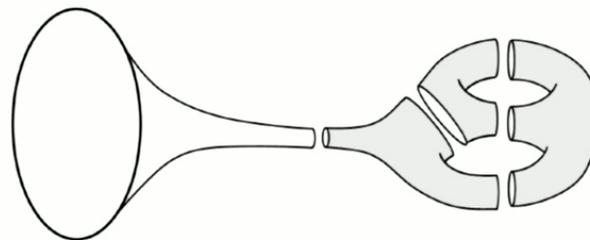
- ▶ The single-boundary thermal partition function at inverse temperature β is

$$\langle Z(\beta) \rangle_{JT} = \sum_{g=0}^{\infty} e^{(1-2g)s_0} Z_g(\beta) = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

- ▶ The $g = 0$ case is semiclassical: the disk topology admits a saddle-point geometry

$$e^{s_0} Z_0(\beta) = \frac{e^{s_0 + 2\pi^2/\beta}}{\sqrt{2\pi}\beta^{3/2}} = \text{[Diagram 4]} \quad \text{[Stanford-Witten '17]}$$

- ▶ No higher topology with $g \geq 1$ admits a semiclassical analysis; however, they can all be constructed by cutting and gluing of pairs of pants, e.g. for $(g, n) = (2, 1)$



[Saad-Shenker-Stanford '19]

WEIL-PETERSSON VOLUMES

- ▶ The GPI for any $g \geq 1$ can be decomposed into a ‘trumpet’ contribution

$$Z_T(\beta, b) = \frac{e^{-b^2/2\beta}}{\sqrt{2\pi\beta}} = \text{trumpet}(b)$$

which carries the Schwarzian mode, glued to and weighted by the Weil-Petersson (WP) volume of moduli space $\mathcal{M}_{g,n}$ of hyperbolic surfaces of genus g and $n = 1$ geodesic boundary of length b ,

$$V_{g,n=1}(b) = \sum_{\ell=0}^{3g-2} \frac{\langle \omega^\ell \psi^{3g-2-\ell} \rangle_{g,1}}{2^{3g-2-\ell} \ell! (3g-2-\ell)!} b^{2(3g-2-\ell)} = b \text{ genus-}g \text{ surface}(b)$$

- ▶ Gluing with a measure bdb that accounts for twists leads to the prescription

$$Z_g(\beta) = \int bdb Z_T(\beta, b) V_{g,1}(b) = \int bdb \text{trumpet}(b) \text{genus-}g \text{ surface}(b)$$

which explicitly yields a sum over inverse powers of β ,

$$Z_g(\beta) = \frac{\beta^{3g}}{\sqrt{2\pi\beta^{3/2}}} \sum_{\ell=0}^{3g-2} \frac{\beta^{-\ell}}{\ell!} \langle \omega^\ell \psi^{3g-2-\ell} \rangle_{g,1} = \text{trumpet-genus-}g \text{ surface}$$

TOPOLOGICAL EXPANSIONS

- ▶ Having understood the Z_g that go into the JT topological expansion, we would now like to explore regimes with non-negligible topological fluctuations
- ▶ We find an interesting interplay between β and e^{S_0} in the sum over g ,

$$\langle Z(\beta) \rangle_{\text{JT}} = \frac{e^{S_0}}{\sqrt{2\pi}\beta^{3/2}} \sum_{g=0}^{\infty} \sum_{\ell=0}^{3g-2} \left(e^{-2S_0} \beta^3 \right)^g \frac{\beta^{-\ell}}{\ell!} \left\langle \omega^\ell \psi^{3g-2-\ell} \right\rangle_{g,1}$$

- ▶ By inspecting the double series, we observe three different regimes of interest:
 - $\beta \sim 1$: The $g = 0$ saddle dominates; $\langle Z(\beta) \rangle_{\text{JT}}$ is perturbative in e^{-2S_0} and organized by genus. At each $g \geq 1$, every ℓ contributes at the same order
 - $\beta \sim e^{2S_0/3}$: The $\ell = 0$ term dominates; $\langle Z(\beta) \rangle_{\text{JT}}$ is perturbative in $1/\beta$ and organized by ℓ . At each $\ell \geq 0$, every genus g contributes at the same order
 - $\beta \sim e^{S_0}$: The expansion diverges; $\langle Z(\beta) \rangle_{\text{JT}}$ is non-perturbatively dominated by an instanton effect, and only the free energy $\log \langle Z(\beta) \rangle_{\text{JT}}$ admits a topological expansion [Saad-Shenker-Stanford '19, Okuyama '19, SHC '24]

LOW-TEMPERATURE EXPANSION

- ▶ In the interesting regime $x \equiv e^{-2S_0} \beta^3 \sim 1$ all topologies contribute equally, giving

$$\langle Z(\beta) \rangle_{\text{JT}} = \frac{1}{\sqrt{2\pi x}} \sum_{\ell=0}^{\infty} \frac{\beta^{-\ell}}{\ell!} F_{\ell}(x)$$

$$F_{\ell}(x) \equiv (2\pi^2)^{\ell} + \sum_{g=\lceil(\ell+2)/3\rceil}^{\infty} x^g \left\langle \omega^{\ell} \psi^{3g-2-\ell} \right\rangle_{g,1}$$

- ▶ The pertinent intersection numbers take the general form [Okuyama '19]

$$\left\langle \omega^{\ell} \psi^{3g-2-\ell} \right\rangle_{g,1} = \frac{(2\pi^2)^{\ell}}{24^g g!} P_{\ell}(g),$$

where the $P_{\ell}(g)$ are degree- (2ℓ) polynomials in g normalized to $P_{\ell}(0) = 1$.

- ▶ This guarantees convergence of the infinite series for F_{ℓ} at each order in $1/\beta$, i.e., at each ℓ we have expansion with large but resummable topological fluctuations!

LEADING-ORDER RESUMMATION

- ▶ The leading term in the $1/\beta$ expansion corresponds to $\ell = 0$, for which $P_0(g) = 1$ and the intersection numbers involve only ψ -classes [Witten '91, Kontsevich '92]

$$\langle \psi^{3g-2} \rangle_{g,1} = \frac{1}{24^g g!}$$

- ▶ The genus sum for this term is straightforward and yields

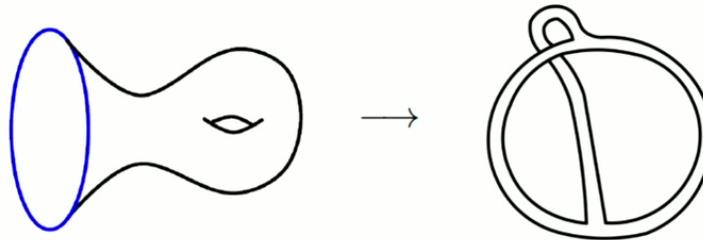
$$\langle Z(\beta) \rangle_{\text{JT}} \stackrel{\ell=0}{=} \frac{F_0(x)}{\sqrt{2\pi x}} = \frac{e^{x/24}}{\sqrt{2\pi x}} = \frac{e^{S_0}}{\sqrt{2\pi} \beta^{3/2}} \exp\left(\frac{\beta^3 e^{-2S_0}}{24}\right)$$

- ▶ Subleading corrections in $1/\beta$ correspond to $\ell \geq 1$, and can be similarly computed using known results for the $P_\ell(g)$ polynomials [Okuyama '19, '20]
- ▶ We have all the desired features for the kind of effective theory we are after:
 - Every F_ℓ is a convergent sum over a highly quantum superposition of topologies
 - There is a leading contribution captured by $\ell = 0$, which we hope can be made geometric albeit potentially nonlocal as argued in the 80's

EFFECTIVE DESCRIPTION

GEOMETRIC INTUITION

- ▶ To obtain an effective description on a single geometry, we want to understand what happens to our hyperbolic geometries at large $\beta \sim e^{2S_0/3}$
- ▶ Large β effectively favors a large geodesic b . But by Gauss-Bonnet the volume of a hyperbolic geometry is topological and independent of b , so geometries with large b must be shrinking to thin ribbons

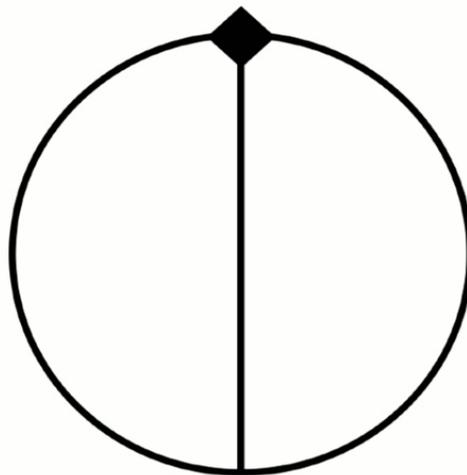


- ▶ These are the $\ell = 0$ intersection numbers for the Airy volumes of topological gravity described by Kontsevich's ribbon geometries [Witten '91, Kontsevich '92]

$$V_{g,1}(b) \stackrel{\text{large } b}{\approx} V_{g,1}^{\text{Airy}}(b) = \frac{(b^2/2)^{3g-2}}{(3g-2)!} \left\langle \psi^{3g-2} \right\rangle_{g,1}$$

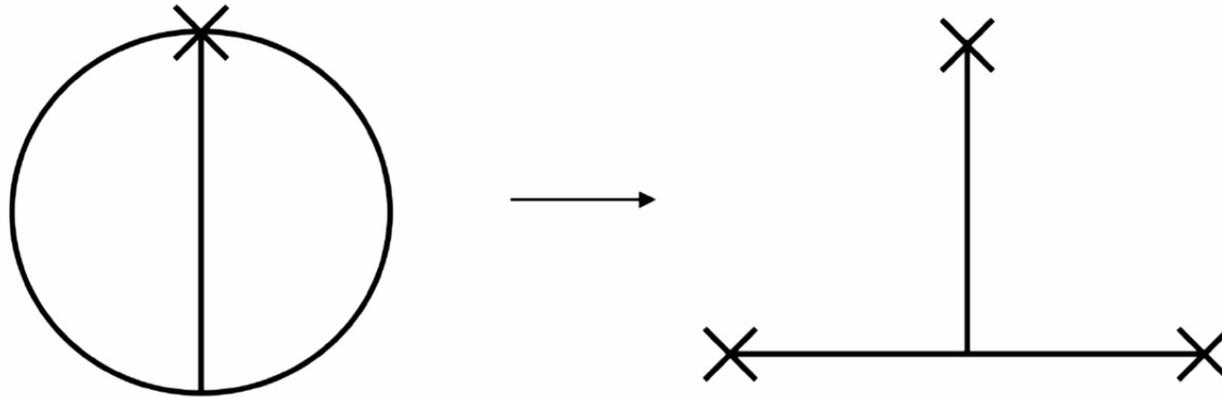
- ▶ The strict large- β limit giving $\ell = 0$ corresponds to the degeneration of these ribbon geometries into one-dimensional trivalent graphs; the resulting vertices are singular points where the original geometry suffers from a three-way pinch-off

HEURISTIC PICTURE



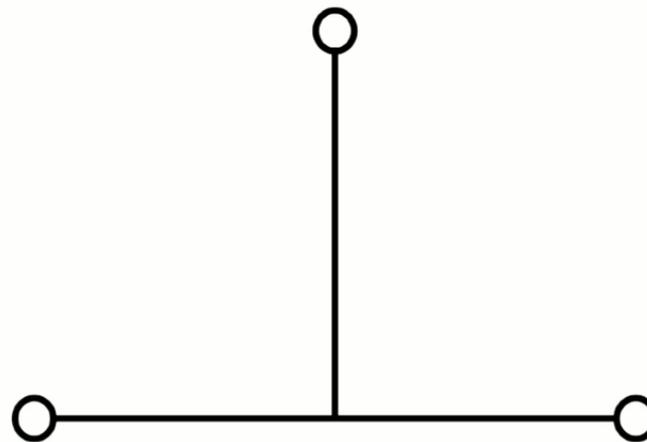
Graph for $(g, n) = (1, 1)$

HEURISTIC PICTURE



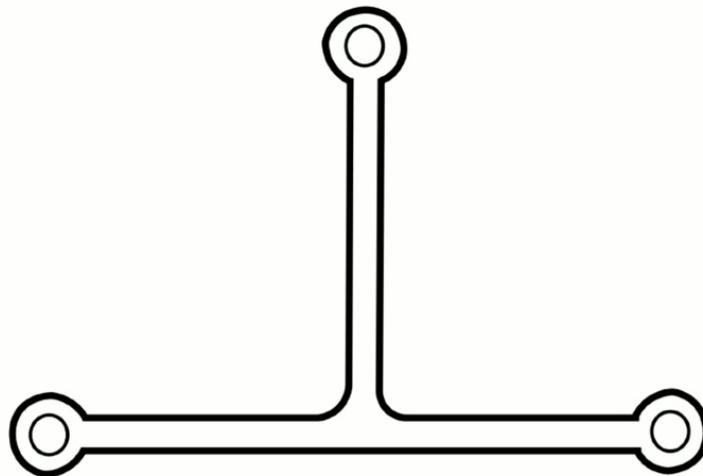
Splitting $(1, 1) \rightarrow (0, 4)$

HEURISTIC PICTURE



Graph for $(g, n) = (0, 4)$

HEURISTIC PICTURE



Ribbon for $(g, n) = (0, 4)$

FORMAL EQUIVALENCE

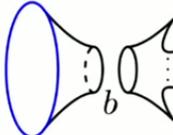
- ▶ This heuristic picture extends to higher genus and boundary numbers, suggesting a relation between $(g, 1)$ and $(0, 1 + 3g)$ ribbon graphs
- ▶ This relation is combinatorially realized by matching dimensions between the moduli spaces $\mathcal{M}_{g,1}$ and $\mathcal{M}_{0,1+3g}$
- ▶ More explicitly, the WP volumes are found to satisfy the large- b identity

$$V_{g,1}(b) \approx \frac{1}{24^g g!} V_{0,1+3g}(b, 0^{3g}),$$

[SHC-Valdes-Meller-Weng '24]

i.e., the WP volumes of $(g, 1)$ surfaces are identically reproduced by the WP volumes of $(0, 1 + 3g)$ surfaces where genera degenerate into 3 cusps/genus

- ▶ In light of this result, it will be useful to define a cusp partition function

$$Z_0(\beta, 0^k) \equiv \int_0^\infty b db Z_T(\beta, b) V_{0,1+k}(b, 0^k) = \int b db$$


in terms of which the relation above at the level of partition functions gives

$$Z_g(\beta) \stackrel{\ell=0}{=} \frac{1}{24^g g!} Z_0(\beta, 0^{3g})$$

DEFECT OPERATORS

- ▶ Geodesic boundaries of zero length, or cusps, correspond to conical defects of zero opening angle
- ▶ Conical singularities are easily sourced in JT by the insertion of exponential operators of the form [Maxfield-Turiaci '20; Witten '20; Turiaci-Usatyuk-Weng '20]

$$Q \equiv q_\alpha \int d^2x \sqrt{g} e^{-2\pi\alpha\phi(x)}$$

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- ▶ With the insertion of Q^k , the path integral over ϕ now localizes the geometry to

$$R + 2 = 2\pi\alpha \sum_{i=1}^k \delta(x - x_i),$$

i.e., a hyperbolic surface with k cone points of opening angle $2\pi(1 - \alpha)$

- ▶ Using these operators with $\alpha = 1$, we can realize the cusp partition function via

$$e^{S_0} Z_0(\beta, 0^k) = \int_{\text{Disk}} \mathcal{D}g \mathcal{D}\phi e^{-I_{\text{JT}}} Q^k = \left. \begin{array}{c} \text{Diagram of a hyperbolic surface with } k \text{ cusps} \\ \vdots \\ \end{array} \right\} k \text{ defects}$$

GENUS AS CUSPS

- ▶ By the equivalence between genus and cusp partition functions, we thus obtain a representation of genus g surfaces in terms of operators on the disk,

$$e^{S_0} Z_g(\beta) \stackrel{\ell \equiv 0}{\equiv} \frac{1}{24^g g!} Z_0(\beta, 0^{3g}) = \frac{1}{24^g g!} \int_{\text{Disk}} \mathcal{D}g \mathcal{D}\phi e^{-I_{\text{JT}} Q^{3g}}$$

- ▶ Defining $\lambda \equiv e^{-2S_0}/24$, in this representation the genus sum exponentiates

$$\langle Z(\beta) \rangle \stackrel{\ell \equiv 0}{\equiv} e^{S_0} \sum_{g=0}^{\infty} \frac{\lambda^g}{g!} Z_0(\beta, 0^{3g}) = \int_{\text{Disk}} \mathcal{D}g \mathcal{D}\phi e^{-I_{\text{JT}+Q}},$$

where we have absorbed the operator deformation as a redefinition of the action

$$I_{\text{JT}+Q} \equiv I_{\text{JT}} + \lambda Q^3$$

- ▶ This deformed JT theory captures the full topological expansion of ordinary JT at $\beta \sim e^{2S_0/3}$ upon evaluation on just the disk topology:

$$\langle Z(\beta) \rangle_{\text{JT}+Q, \text{disk}} \approx \langle Z(\beta) \rangle_{\text{JT}, \text{all genus}}$$

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- ▶ Crucially, the effective theory $I_{\text{JT}+Q}$ that results from integrating out wormholes involves a strongly nonlocal interaction Q^3 , just as we hoped to find

HIGHER CORRECTIONS

- ▶ So far we have focused on the leading large- β term $\ell = 0$; actually, all higher $\ell \geq 1$ corrections can also be captured perturbatively in the effective theory
- ▶ At leading order we start with the previous $\ell = 0$ relation,

$$24^g g! V_{g,1}^{(0)}(b) = V_{0,1+3g}^{(0)}(b, 0^{3g})$$

which can be lifted to the following relation up to order $\ell = 1$

$$24^g g! V_{g,1}^{(1)}(b) = V_{0,3g}^{(1)}(b, 0^{3g}) - \frac{21\pi^2}{5} g(g-1) V_{0,3g-1}^{(0)}(b, 0^{3g-1})$$

- ▶ Iterating, it is possible to reconstruct all genus volumes in terms of cusp volumes, obtaining the following general relation which holds at all orders:

$$24^g g! V_{g,1}^{(\ell)}(b) = \sum_{k=0}^{\ell} \frac{1}{k!} \left(-\frac{21\pi^2}{5} \right)^k H_k(g) V_{0,3g-k}^{(\ell-k)}(b, 0^{3g-k}),$$

where $H_k(g)$ are degree- $(2k)$ polynomials which, importantly, are independent of ℓ

PERTURBATIVE EFFECTIVE THEORY

- ▶ Using $\langle \cdot \rangle_{\text{JT, disk}}$ as a shorthand for the GPI of ordinary JT on the disk, these perturbative relations capture the genus partition functions to all orders in $1/\beta$,

$$Z_g(\beta) = \frac{1}{24^g g!} \sum_{k=0}^{3g} \frac{1}{k!} \left(-\frac{21\pi^2}{5} \right)^k H_k(g) \langle Q^{3g-k} \rangle_{\text{JT, disk}}.$$

- ▶ Upon genus resummation, the inclusion of these higher order terms results in perturbative corrections around the effective theory previously obtained

$$I_{\text{JT}+Q} = I_{\text{JT}} - \sum_{k=3}^{\infty} \lambda_k Q^k, \quad \lambda_k \sim \lambda^{\lfloor k/2 \rfloor} \sim (e^{-2S_0})^{\lfloor k/2 \rfloor}$$

- ▶ That the perturbative expansion of the effective theory takes this nice form (e.g. no inverse powers of Q), relies on properties of the $H_k(g)$ polynomials arising from very nontrivial relations among intersection numbers
- ▶ We have been able to verify these up to $\ell = 50$; proving these to all orders may require making use of Mirzakhani's recursion relations among WP volumes

CONCLUSION

SUMMARY & OPEN QUESTIONS

We have derived an effective description of the all-genus thermal partition function of JT gravity in terms of a nonlocal deformation of JT gravity on a single geometry.

There remain various open questions:

- ▶ Here we focused on the single-boundary observable $Z(\beta)$. Are there similar effective descriptions for other observables, such as multiboundary correlators?
- ▶ What does our deformed JT theory tell us about the quantum geometry of the higher-dimensional black hole horizon at non-perturbatively low temperatures?
- ▶ The cusp theory shifts the edge of the leading spectral density relative to JT by

$$\sqrt{E} \rightarrow \sqrt{E - E_0}, \quad E_0 \equiv -\beta^2 e^{-2S_0},$$

where E_0 is precisely the location of the eigenvalue instanton that arises at $\beta \sim e^{S_0}$. But $\beta \sim e^{2S_0/3}$ is an intermediate regime where E_0 is only perturbatively away from the main cut by $O(e^{-2S_0/3})$, and both contribute equally to $\langle Z(\beta) \rangle$

Is there an effective description of the instanton phase $\beta \sim e^{S_0}$ in terms of branes? (cf. a D-brane description of interacting strings at strong coupling)?

Thank you!