Title: On the reconstruction map in JT gravity

Speakers: Chris Akers

Collection/Series: QIQG 2025

Subject: Quantum Gravity, Quantum Information

Date: June 27, 2025 - 3:30 PM

URL: https://pirsa.org/25060019

Abstract:

A key question in holography is how to reconstruct bulk operators in the holographic dual. It is especially interesting to reconstruct operators inside the black hole interior, but also especially difficult to do explicitly. Recently, an explicit form for the bulk-to-boundary `holographic' map was proposed in JT gravity, by Iliesiu, Levine, Lin, Maxfield, and Mezei, who also proposed and studied an explicit `reconstruction' map on operators. In this talk, I will discuss various pros and cons of their reconstruction map, and propose an alternative map with perhaps nicer properties.

On the reconstruction map in JT gravity

Chris Akers

CU Boulder

Work with Andy Lucas and Amit Vikram, 2506.18975

Introduction to reconstruction

- Consider semiclassical gravity (SCG), defined as quantum field theory on a curved background, coupled to the metric in a perturbative G_N expansion.

Introduction to reconstruction

- Consider semiclassical gravity (SCG), defined as quantum field theory on a curved background, coupled to the metric in a perturbative G_N expansion.
- This is an effective theory for quantum gravity.
- As an effective theory, its predictions cannot always be trusted. Sometimes they receive large corrections from e^{-1/G_N} effects.
- AdS/CFT offers a path towards computing these effects: given an operator in AdS SCG, we *reconstruct* it in the CFT and evaluate it there.
- Doing this with the necessary precision is not yet possible in general. We don't understand enough about these reconstruction maps.
- Last year, a precise reconstruction map was proposed in *Jackiw-Teitelboim* (JT) gravity. [Iliesu-Levine-Lin-Maxfield-Mezei '24]
- I will explain this reconstruction map, some pros and cons, and a way we can modify it to remove some of the cons.

Jackiw-Teitelboim gravity

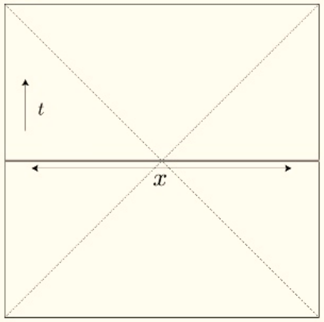
- JT gravity: [Teitelboim '83, Jackiw '85]

$$S_{\mathrm{JT}} \simeq S_0 \int d^2x\sqrt{-g}R + \int d^2x\sqrt{-g}\Phi(R+2)$$

- Consider $S_0 \to \infty$, called JT_∞. All solutions look like a wormhole. The only degree of freedom is its length. [Harlow-Jafferis 2018]
- In [Harlow-Jafferis 2018], it was demonstrated that the quantized theory is described by

$$\mathcal{H}_0 = \mathrm{L}^2(\mathbb{R}) \ , \qquad H_0 = -\frac{1}{2}\partial_x^2 + 2\mathrm{e}^{-x}$$

where x is the wormhole length.



- A typical solution has $\langle \hat{x} \rangle$ shrink for a while, then grow forever.
- This is analogous to "semiclassical gravity", $G_N \to 0$. Note the continuous energy spectrum.

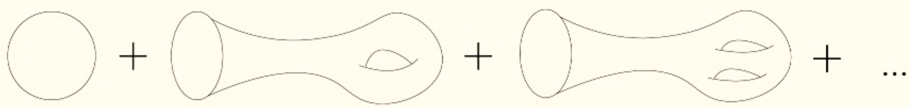
JT gravity at $S_0 < \infty$

- JT gravity:

$$S_{\mathrm{JT}} \simeq S_0 \int d^2x\sqrt{-g}R + \int d^2x\sqrt{-g}\Phi(R+2)$$

- At finite S_0, the topology can fluctuate. Computing with the gravitational path integral, e.g.

$Z =$



- This theory has a known dual description as a random matrix model [Saad-Shenker-Stanford '19]
- Each matrix is a Hamiltonian on Hilbert space $\mathcal{H}$, with a discrete spectrum

$$\{E_1, E_2, ...\}$$

typically with $\Delta E \sim O(e^{-S_0})$ and leading order density of states

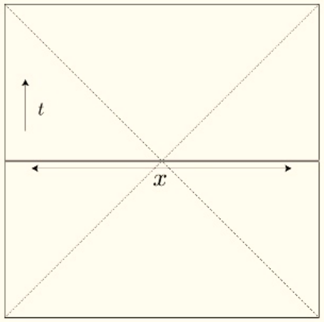
$$\rho_0(E) \simeq e^{S_0} \sinh\left(\sqrt{E}\right) \ .$$

JT_∞ as an effective theory

- Consider a single draw from the ensemble,

$$\{E_1, E_2, ...\}$$

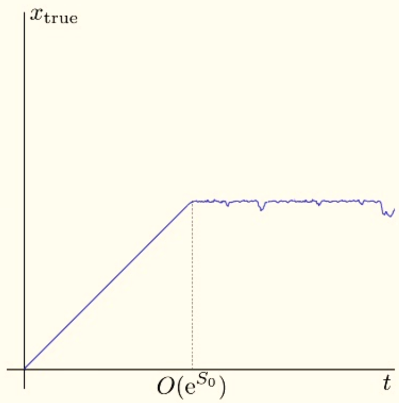
- We will follow [Iliesu-Levine-Lin-Maxfield-Mezei '24] and consider JT_∞ as an *effective* description of that *fundamental* theory.
- Good analogy for higher dimensions: $S_0 \to \infty$ is like $G_N \to 0$. SCG has continuous spectrum black holes, while QG discrete.
- Now we can ask how predictions of JT_∞ are corrected in the fundamental theory.
- For example, JT_∞ predicts wormhole length grows linearly forever:



- We'd like to ask: in the *fundamental* theory, what length $x_{\mathrm{true}}(t)$ do we expect as a fun of time?

Case study: length operator

- We expect $x_{\mathrm{true}}(t) \approx x(t)$ for a while. A short experiment can't resolve the energy differences.
- However, we know eternal growth can't happen in the fundamental theory at finite energy, because of the discrete spectrum. [Susskind '14, Iliesu-Mezei-Sarosi '22, Stanford-Yang '22, ...]



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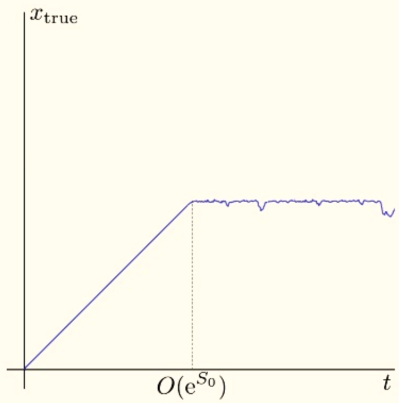


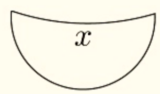
- It's one thing to know JT_∞ is wrong at late times. It's a harder thing to compute the corrections!
- Problem: what even is $\hat{x}_{\text{true}}$?

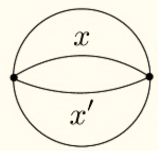
Reconstructing operators: a previous proposal

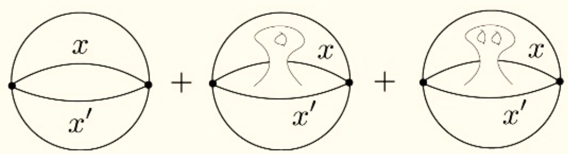
- Recently, there has been a proposal for $\hat{x}_{\text{true}}$ [Iliesu-Levine-Lin-Maxfield-Mezei '24].
- The starting point is the "holographic map"

$$V : \mathcal{H}_0 \to \mathcal{H}$$

which is deduced from studying $O(\mathrm{e}^{-S_0})$ corrections to the inner-product.

- $|x\rangle_0 =$ 

- $\langle x|x'\rangle_0 =$  $= \delta(x - x')$

- $\langle x|x'\rangle =$  $+ \; ... \; = \delta_{xx'} + O(e^{-S_0})$

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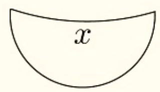
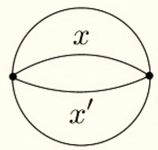
- $|x\rangle_0 =$ 

- $\langle x|x'\rangle_0 =$ 

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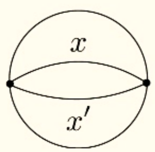
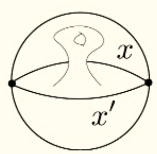
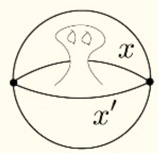
- $\langle x|x'\rangle =$ 

$+$ 

$+$ 

$+ \; ... \; = \delta_{xx'} + O(e^{-S_0})$

- They define a linear map $V : \mathcal{H}_0 \to \mathcal{H}$ such that

$$\langle x|x'\rangle = \langle x|V^\dagger V|x'\rangle_0$$

The holographic map, continued

- This V acts diagonally in the energy eigenbasis:

$$
\begin{aligned}
V\,|x\rangle_0 &= V \int dE\rho(E)\phi_E(x)\,|E\rangle_0 \\
&= e^{-S_0/2} \sum_n \phi_{E_n}(x)\,|E_n\rangle \\
&=: |x\rangle \ .
\end{aligned}
$$

- There is another way to motivate this V. It is the map that commutes with time evolution:

$$
\begin{array}{ccc}
|\psi\rangle_0 & \xmapsto{\ V\ } & |\widetilde{\psi}\rangle \\
e^{-itH_0} \Big\downarrow & & \Big\downarrow e^{-itH} \\
|\psi'\rangle_0 & \xmapsto{\ V\ } & |\widetilde{\psi}'\rangle
\end{array}
$$

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Reconstruction map

- So far, we have a map on states

$$V : \mathcal{H}_0 \to \mathcal{H}$$

- This induces a pullback on operators,

$$V^* : \mathrm{B}(\mathcal{H}) \to \mathrm{B}(\mathcal{H}_0)$$

$$\langle\psi|V^\dagger O V|\psi\rangle_0 = \langle\psi|V^*(O)|\psi\rangle_0$$

- V does *not* automatically define a *reconstruction* map

$$R^* : \mathrm{B}(H_0) \to \mathrm{B}(H) \ .$$

- They [Iliesu-Levine-Lin-Maxfield-Mezei '24] considered the reconstruction map

$$R^*(O_0) := V O_0 V^\dagger \ .$$

Nice features of their reconstruction map

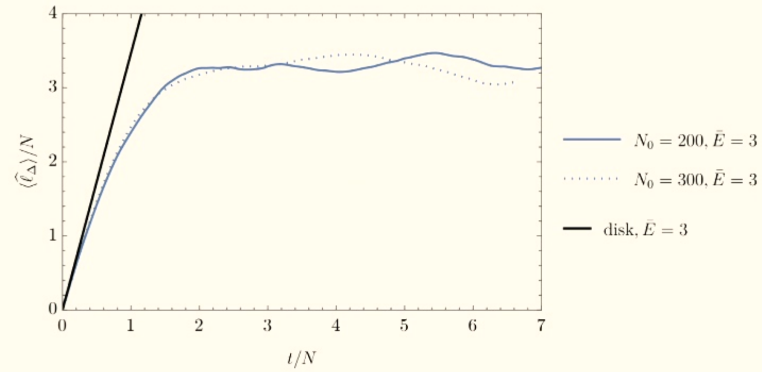
- This reconstruction map

$$R^*(O_0) := VO_0V^\dagger \ .$$

  has some good properties and some strange ones.
- It's very natural, given some $V : \mathcal{H}_0 \to \mathcal{H}$.
- If V is an isometry, then R^* inverts V^* in a natural way:

$$V^* \circ R^*(O_0) = V^\dagger V O_0 V^\dagger V = O_0 \ .$$

- As shown by [Iliesu-Levine-Lin-Maxfield-Mezei '24], this defines some reconstructions with behavior we like:



Perhaps undesirable features

- One peculiar feature of this R^* is that it does not reconstruct simple operators in a nice way, for example

$$R^*(\mathbb{1}_0) \neq \mathbb{1} \ .$$

- To see this,

$$\mathbb{1}_0 = \int_{-\infty}^{\infty} dx \, |x\rangle_0 \, \langle x|_0 \implies R^*(\mathbb{1}_0) = \int_{-\infty}^{\infty} dx \, |x\rangle \, \langle x|$$

but then

$$\langle x|R^*(\mathbb{1}_0)|x\rangle = \int_{-\infty}^{\infty} dx' | \langle x|x'\rangle |^2 = \infty \ .$$

- Similarly, $R^*(x)$ is ill defined.
- The authors [Iliesu-Levine-Lin-Maxfield-Mezei '24] knew this, and propose that we should simply focus on the operators with better reconstructions.

Our proposal

- Those features of R^* are perhaps undesirable because we would like JT_∞ to be a good effective description for certain "semiclassical" states and operators, in the sense

$$R^*(O_0)V\left|\psi\right\rangle_0 \approx VO_0\left|\psi\right\rangle_0$$

- We propose that we should look for a reconstruction map R^* that satisfies this.
- Specifically, we will demand it satisfies this to as good an approximation as possible for short times.
- Does such a map exist that also retains the nice features (natural, behavior we like,...)?
- Yes: we will construct such a reconstruction map!

Making our proposal more precise

- We look for an R^* satisfying to best approximation

$$R^*(O_0)V\left|\psi\right\rangle_0 \approx VO_0\left|\psi\right\rangle_0$$

- V commuted with time evolution, so

$$R^*(H_0) = H$$

- This does not yet fully specify R^*. To further constrain, we would like to decide how it acts on the conjugate to H_0

$$R^*(\delta) =?$$

- This δ satisfies

$$e^{iH_0t}\delta e^{-iH_0t} = \delta + t$$

- Technically this δ is not a well-defined self adjoint operator, but we can be more rigorous by working with action-angle operators instead. Qualitatively unchanged.

Reconstructing δ

- We want to decide what operator should be $R^*(\delta)$.
- Our guiding proposal instructs us to minimize the error on semiclassical states,

$$R^*(\delta)V\left|\psi\right\rangle_0 \approx V\delta\left|\psi\right\rangle_0$$

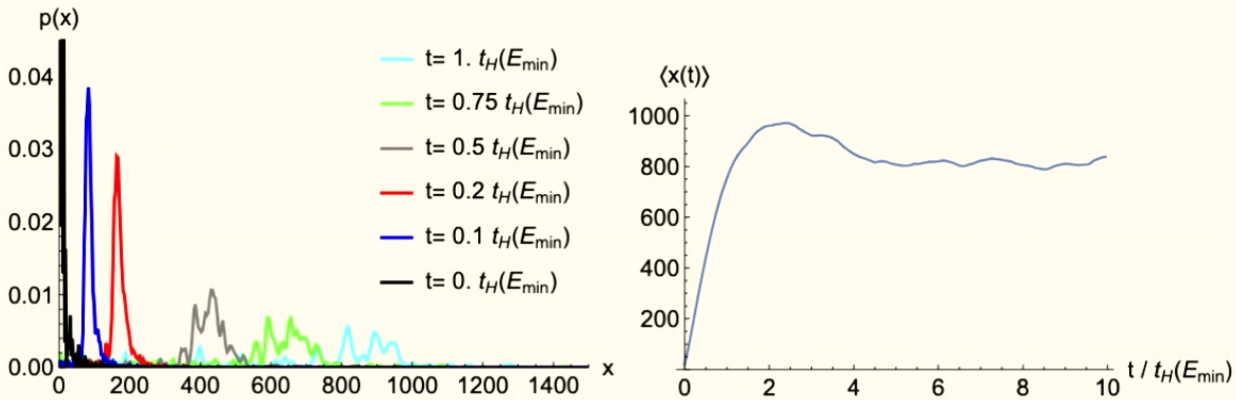
- For definiteness, I'll take "semiclassical states" to be the microcanonical Hartle-Hawking state and short time evolutions of it.
- We can do this for each energy window separately, which largely fixes $R^*(\delta)$.

Reconstructing other operators

- With $R^*(H_0)$ and $R^*(\delta)$, we can define the reconstruction of a general operator

$$A(H_0, \delta) \longrightarrow A(R^*(H_0), R^*(\delta))$$

- Applying this to the length operator x, we can evaluate $\langle R^*(x)(t) \rangle$ and find interesting physics,



Summary

- Starting with V, we designed an R^* based on the principle that semiclassical gravity is as valid as possible at short times.
- With this, we could write down explicit reconstructions, such as $R^*(x)$, and evaluate explicitly, finding novel behavior as a function of time.