Title: Randomizing excitations of half-BPS states gives near-extremal black holes

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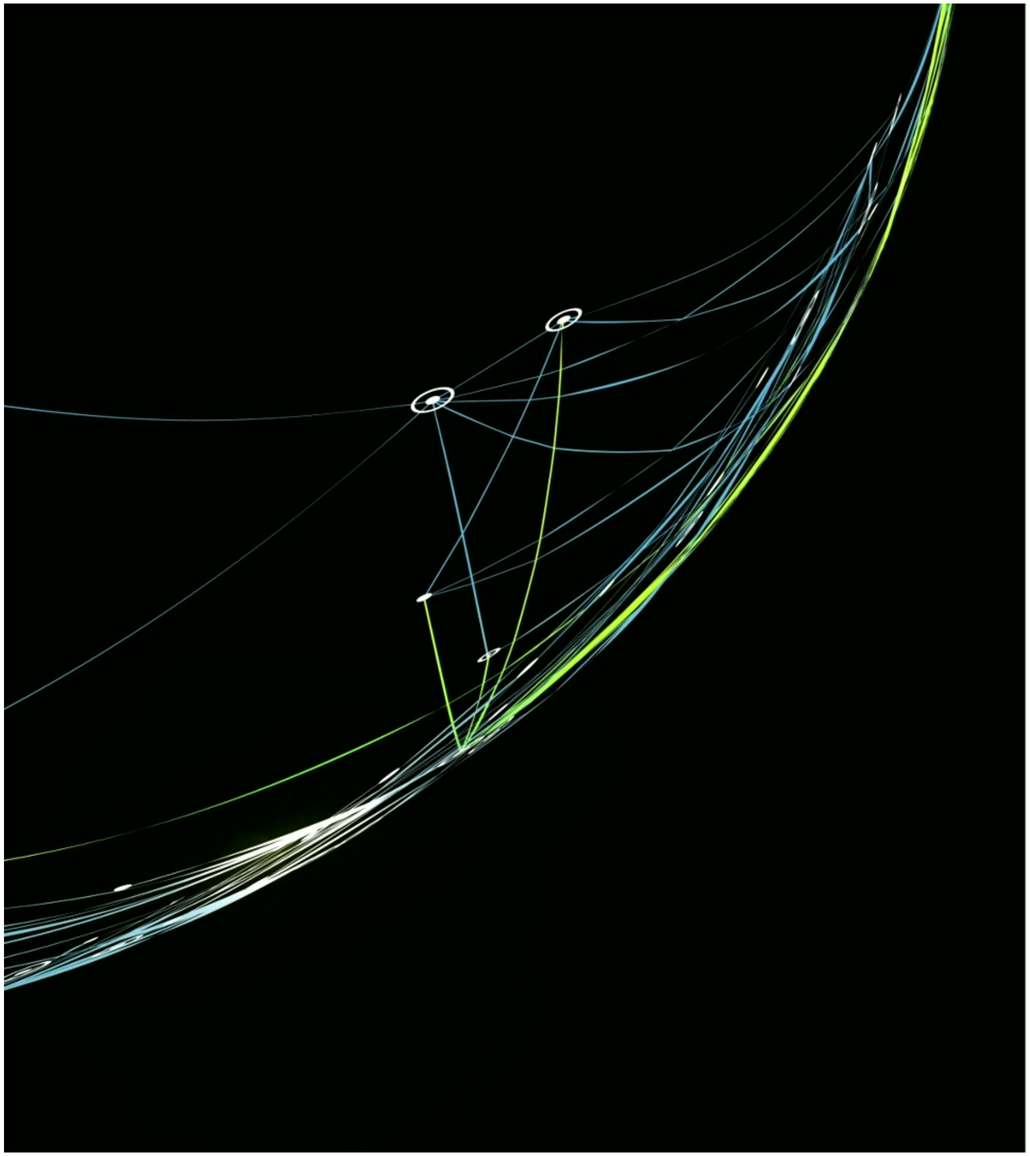
Abstract:

I will discuss a concrete realization in N=4 SYM of the mechanism of cryptographic censorship: that sufficiently random time evolution in a holographic CFT incurs an event horizon in the bulk dual. I will show that perturbing half-BPS states by randomly distributing a large number of defects on them corresponds in the gravitational dual to exciting an extremal (horizonless) black hole to near-extremality. This random distribution of defects corresponds to acting with a typical random isometry on the half-BPS subspace. This allows us to interpret this process in the field theory as an extension of cryptographic censorship.



Randomizing excitations of ½-BPS states gives near-extremal black holes

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QIQG 2025

based on 2402.03425 w/ N. Engelhardt, Å. Folkestad, A. Levine & L. Yang, and w.i.p. w/ V. Balasubramanian

A brief history

- Goal: formulate and prove a "quantum gravity" version of weak cosmic censorship:

  singularities resulting from initial data evolution should generically be hidden behind horizons [Penrose '69]

- Intermediate step: a QI diagnostic for event horizon formation in holography:

  cryptographic censorship [Engelhardt-Folkestad-Levine-EV-Yang '24]

- Feedback: ok nice but can you make it concrete

  randomizing excitations of ½-BPS states in $\mathcal{N}=4$ SYM gives near-extremal AdS_5 BHs [Balasubramanian-EV, wip]

Plan

1. Cryptographic Censorship
2. The setup: ½-BPS subspace
3. Near-extremal AdS_5 black holes
4. Randomizing excitations of ½-BPS states
5. Discussion

Cryptographic Censorship

- There exists a protocol for causal wedge reconstruction [Hamilton-Kabat-Lifschytz-Lowe '06]
- Existence of event horizon in the bulk $\longleftrightarrow$ failure of HKLL for some operator
- View bulk reconstruction as a learning algorithm $\mathcal{A}_{\mathrm{HKLL}}$. We proved that when the fundamental time evolution is sufficiently (pseudo)random, there exists a bulk operator Q such that $\langle Q \rangle$ cannot be computed by $\mathcal{A}_{\mathrm{HKLL}}$.
- Therefore, the causal wedge is not the entire spacetime.

Cryptographic Censorship: (Pseudo)random dynamics guarantees event horizon formation in typical* (pseudo)random states. [Engelhardt-Folkestad-Levine-**EV**-Yang '24]

*states for which measure concentration applies. For small *G*: nearly all states

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The ½-BPS subspace

- $\mathcal{N}$=4 SU(N) SYM fields: gauge field, three scalars (*X*, *Y*, *Z*), four Weyl fermions

- ½-BPS states ($\Delta = J$): multi-traces built out of *Z*

- We are interested in the high-energy regime $\Delta \sim N^2$. Multi-trace basis becomes highly overcomplete: trace relations kick in

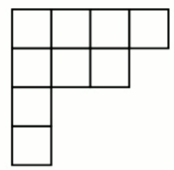
- One solution is to diagonalize to N one-dim fermions in a harmonic oscillator potential w/ energy above the ground state [Corley-Jevicki-Rangoolam '01, Berenstein '04]

$$r_i = \frac{1}{\hbar\omega}(E_i - E_i^g) = e_i - i + 1\,, \qquad i = 1, \ldots, N$$

A basis of Schur polynomials

- These are ordered integers $r_N \geq \ldots \geq r_1$: represent states using Young diagrams, e.g.

$$\{4,3,1,1\} \quad \Leftrightarrow$$



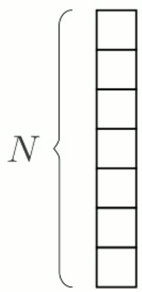
- A ½-BPS state is represented by a Young diagram of Δ boxes and no more than N rows

- Schur polynomials form a complete basis that takes this into account:
[Corley-Jevicki-Rangoolam '01, Berenstein '04]

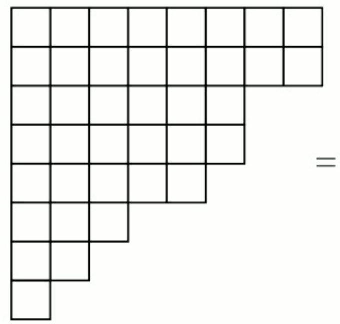
$$\mathcal{O}_R(Z) = c_R \sum_{\sigma \in S_\Delta} \chi_R(\sigma)\, \mathrm{tr}\big(\sigma Z^{\otimes \Delta}\big)$$

LLM geometries and the superstar

- The ½-BPS states are dual to horizonless and regular LLM ("bubbling") Type IIB supergravity solutions in AAdS$_5$ [Lin-Lunin-Maldacena '04]
  - Small excitations above the ground state: gravitons on empty AdS$_5$



= giant graviton [McGreevy-Susskind-Toumbas '00]



= bound state of giant gravitons

- For $\Delta \sim N^2$, a typical LLM solution is not a smooth classical geometry
- It has been suggested [Balasubramanian-de Boer-Jejjala-Simon '05] that in this regime the LLMs are well-approximated by the so-called superstar [Myers-Tafjord '01]

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Near-extremal AdS$_5$ black holes

- Single R-charge black holes in AdS$_5$ supergravity (uplift to BHs in Type IIB)
[Cvetic et. al. '99, Myers-Tafjord '01]

$$ds^2 = -H^{-2/3} f \mathrm{d}t^2 + H^{1/3}\left(f^{-1}dr^2 + r^2 d\Omega_3^2\right)$$
$$H = 1 + \tfrac{q}{r^2}\ , \qquad f = 1 - \tfrac{\mu}{r^2} + \tfrac{r^2 H}{L^2}$$

- Here $\mu = L^2\hat{\mu}$ is a non-extremality parameter; mass and charge are

$$M = \tfrac{\pi L^2}{4G_5}\left(\tfrac{3}{2}\hat{\mu} + \hat{q}\right)\ , \qquad Q = L^2\sqrt{\hat{q}(\hat{q}+\hat{\mu})}$$

- The horizon area and entropy near extremality are

$$4r_h^2 \simeq 4L^2\frac{\hat{\mu}}{1+\hat{q}}\,, \qquad S_{\mathrm{BH}} \simeq N^2\hat{\mu}\frac{\sqrt{\hat{q}}}{1+\hat{q}}$$

- Finite area, classical horizon for $O(1) \sim \hat{\mu} \ll \hat{q} \sim O(1)$

Near-extremal AdS_5 black holes (2)

- Dual field theory states have dimension and R-charge given by

$$\Delta = ML \simeq N^2 \hat{q} \left(1 + \frac{3}{2}\frac{\hat{\mu}}{\hat{q}}\right),$$

$$J = \frac{QL}{G_5} \simeq N^2 \hat{q} \left(1 + \frac{1}{2}\frac{\hat{\mu}}{\hat{q}}\right).$$

- Extremal ($\hat{\mu} = 0$) solution is BPS. It is the superstar of [Myers-Tafjord '01]: backreaction of $\hat{q}N$ giant gravitons

- Microstates of near-extremal black holes can be interpreted as susy-breaking deformations of the extremal 1/2-BPS states [Balasubramanian-de Boer-Jejjala-Simon '07]

- Goal: to understand this process in the language of Cryptographic Censorship

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Randomness incurs the horizon

Outline of argument:

1. Randomly distributing impurities on top of the ½-BPS states is well-approximated by Haar random isometry $V : \mathcal{H}_{\rm BPS} \to \mathcal{H}_{\rm def}$

   $$V\,|\psi\rangle = U(|\psi\rangle \otimes |\psi_0\rangle)$$

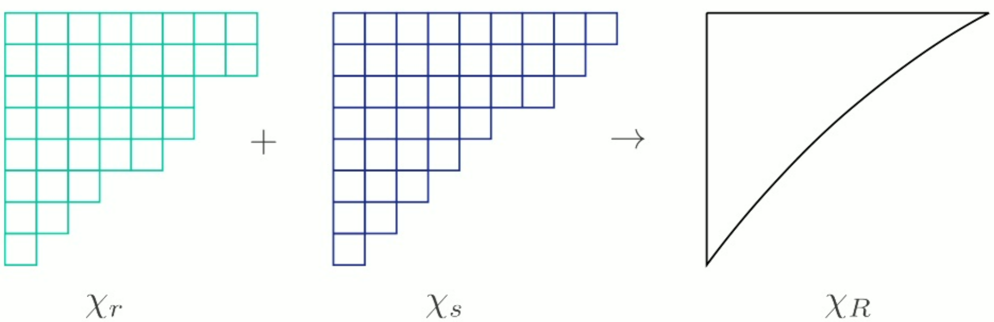
2. Extend cryptographic censorship to isometries b/w static subspaces
   - New hardness of learning result for Haar random isometries

3. By this extension, a horizon must exist in the dual spacetime

4. Counting the number of excited states reproduces the near-ext BH entropy

Randomly distributing impurities

- Distribute $\varepsilon\Delta$ number of *Y* defects on top of a heavy ($\Delta \sim N^2$) ½-BPS operator built out of *Z's* (we will take $\varepsilon = \hat{\mu}/\hat{q}$)
- A basis for observables of *n Z* and *m Y* fields is given by restricted Schur polynomials $\mathcal{O}_{R,(r,s)}(Z,Y)$ [Bhattacharyya-Collins-de Mello Koch '08]. Their distribution concentrates around the 'VKLS' limit shape [Vershik-Kerov '77, Logan-Shepp '77]



- In the ½-BPS subspace, this is implemented by a typical Haar random isometry $V : \mathcal{H}_{\mathrm{BPS}} \to \mathcal{H}_{\mathrm{def}}$

Cryptographic censorship for static subspaces

Theorem 1 (Cryptographic Censorship for static subspaces, informally):

Let $V : \mathcal{H}_1 \to \mathcal{H}_2$ a typical Haar random isometry and let $|\psi\rangle \in \mathcal{H}_1$ a typical Haar random state that has a geometric bulk dual. For sufficiently large $\dim \mathcal{H}_{1,2}$, if $V|\psi\rangle \in \mathcal{H}_2$ has a geometric bulk dual *(M,g)*, then there is an event horizon in *(M,g)*.

Proof (informal):

- By assumption, there exists an efficient algorithm, $\mathcal{A}_{\rm cw}$, that can predict the expectation values of all causally accessible operators (intuition from [Hamilton-Kabat-Lifschytz-Lowe '06] + explicit algorithm of [Huang-Chen-Preskill '22])
- Proved that for any efficient quantum algorithm, and so also for $\mathcal{A}_{\rm cw}$, there exists an operator *Q* whose expectation value in the state $V|\psi\rangle$ cannot be predicted*
- Thus *Q* has support outside the causal wedge. An event horizon exists. □

*assuming $d_{1,2}$ are large enough

Is there indeed an event horizon?

- Can compute $\dim \mathcal{H}_{\text{def}}$ using restricted Schur polynomials: multiplicity of $\mathcal{O}_{R,(r,s)}$ is largest for VKLS shape and has been bounded [Pak-Ganova-Yeliussizov '19]. For $n+m$ boxes:

$$e^{-c\sqrt{n+m}} \binom{n+m}{n}^{1/2} \leq C(n,m) \leq \binom{n+m}{n}^{1/2}, \quad c = \pi(1+\sqrt{2})/\sqrt{6}$$

- This gives an entropy of order ($n = \Delta = N^2$, $m = \varepsilon\Delta = N^2\hat{\mu}$, where $\varepsilon \ll 1$):*

$$S \approx N^2 \hat{\mu} \log \hat{q}/\hat{\mu} + N^2 \hat{\mu}$$

- Reproduces near-extremal entropy up to *O(1)* multiplicative factor:

$$S_{\text{BH}} \approx N^2 \hat{\mu} \frac{\sqrt{\hat{q}}}{1+\hat{q}} \quad \Rightarrow \quad r_h \sim O(1)$$

*So $d_{\text{def}} = \exp(N^2)$, vs. $d_{\text{BPS}} = N^N$. These are large enough for Theorem 1 to apply.

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Summary & Discussion

- Randomizing excitations of ½-BPS states should incur an event horizon in the bulk dual via the mechanism of cryptographic censorship, and indeed we reproduce the right entropy scaling

- Subtleties: overcounting of the entropy; what happens if we add fewer defects?

- Relation to BPS chaos? [Chen-Lin-Shenker '24]
  - ½-BPS subspace not (strongly) chaotic, in line with CC
  - We break susy – could project back to BPS sector and study properties of transfer matrix between initial and final BPS state