Title: Holography and Pseudo (Entanglement) Entropy (Vision Talk)

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Collection/Series: QIQG 2025

Subject: Quantum Gravity, Quantum Information

Date: June 26, 2025 - 10:00 AM

URL: https://pirsa.org/25060017

Abstract:

The holographic calculation of entanglement entropy implies that space coordinate in gravity may emerge from quantum entanglement. The next key question will be to understand how time coordinate may emerge from quantum information. After we review recent developments of pseudo entropy, which is a generalization of entanglement entropy, and its holographic calculation, we will point out that this quantity looks like a useful starting point to understand the emergence of time, by studying explicit examples of time-like entanglement entropy, traversable wormhole and dS/CFT. In the final short time, we would also like to briefly discuss how much the AdS/CFT is efficient as a quantum computer, where the notion of pseudo entanglement in the context of pseudo random states, plays an important role.

QIQG 2025
JUNE 23-27, 2025

PERIMETER INSTITUTE

Vision talk:
Holography and
Pseudo (Entanglement) Entropy

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Grant-in-Aid for Transformative Research Areas A
Extreme Universe

① Introduction

Holography provides a key to understand quantum gravity.

Its connections to quantum information theory play crucial roles.

<u>Two different notions of *“pseudo entanglement”*</u>

[1] Generalization of holography to cosmological spacetimes

How does the time coordinate emerge from quantum information ?

➡ Pseudo (entanglement) entropy — Information theoretic

[cf. Entanglement wedge approach: Bousso’s talk; Hol. cosmology: Raamsdonk’s talk]

[2] The basic mechanism of holography (AdS/CFT)

How does the AdS/CFT work as a “quantum computer” ?

➡ Pseudo entanglement
~ fake entanglement — Computational

Contents

① Introduction

② Pseudo Entropy and Holography

③ Ex.1: Time-like Entanglement Entropy

④ Ex.2: Traversable AdS Wormhole

⑤ Ex.3: dS/CFT

⑥ AdS/CFT as a “Quantum Computer”

⑦ Conclusion

Refer to PRL essay: PRL 134, 240001 (2025)
[arXiv: 2506.06595]

Holographic Entanglement Entropy

[Ryu-TT 2006, Hubeny-Rangamani-TT 2007,
Derivation: Casini-Huerta-Myers 2009, Lewkowycz-Maldacena 2013]

In AdS/CFT, a generic Lorentzian asymptotically AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

The time-dependent entanglement entropy

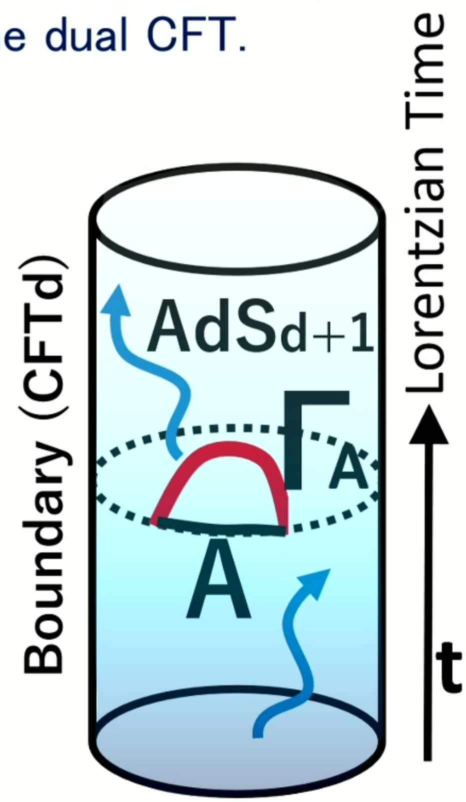
$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|]$$

$$\Rightarrow S_A(t) = -\mathrm{Tr}_A[\rho_A(t)\log\rho_A(t)],$$

is computed from an extremal surface area:

$$S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

$\partial A = \partial\Gamma_A$ and $A \sim \gamma_A$ (homologous) .



Question: More general formula ?

Minimal areas in *Euclidean time dependent* asymptotically AdS spaces

= What kind of QI quantity in CFT ?

➡ **The answer is Pseudo Entropy !**

[Nakata–Taki–Tamaoka–Wei–TT, 2020]

② Pseudo Entropy and Holography

(2-1) Definition of Pseudo (Renyi) Entropy

Consider two quantum states $|\psi\rangle$ and $|\varphi\rangle$, and define the *transition matrix*:

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

We decompose the Hilbert space as $H_{tot} = H_A \otimes H_B$.

and introduce the reduced transition matrix:

$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

⇨ **Pseudo Entropy** $$S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].$$

Renyi Pseudo Entropy $$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].$$

(2-2) Basic Properties of Pseudo Entropy (PE)

- In general, $\tau_A^{\psi|\varphi}$ is not Hermitian. Thus PE is complex valued.

(More generally, we call $S(\tau_A)$ pseudo entropy when τ_A is not hermitan.)

- If either $|\psi\rangle$ or $|\varphi\rangle$ has no entanglement (i.e. direct product state) , then

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0.$$

- We can show $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \left[S^{(n)}\left(\tau_A^{\varphi|\psi}\right)\right]^{\dagger}$.

- We can show $S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = S^{(n)}\left(\tau_B^{\psi|\varphi}\right)$.

→"SA=SB"

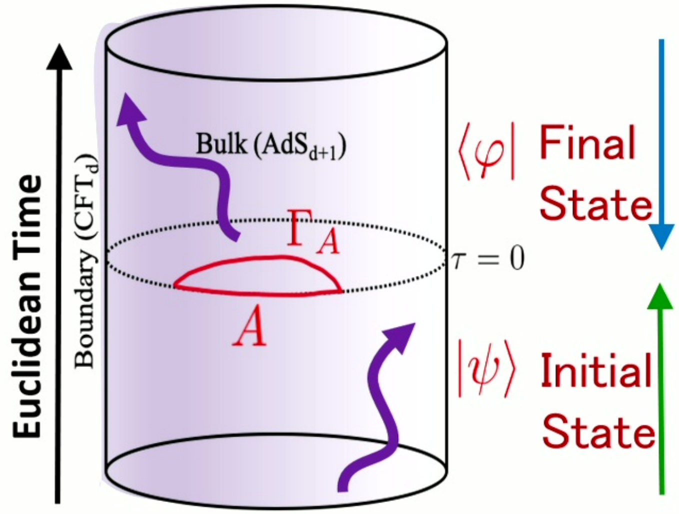
This implies a local holographic formula !

(2-3) Holographic Pseudo Entropy (HPE) Formula

[Nakata–Taki–Tamaoka–Wei–TT, 2020]

In **Euclidean time dependent setups**, the minimal surface area coincides with the pseudo entropy.

$$S\left(\tau_A^{\psi|\varphi}\right) = \mathrm{Min}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$



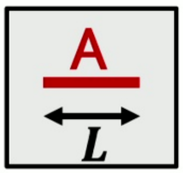
Below we will apply HPE to Lorentzian spacetimes, where **non-Hermitian density matrices** show up.
Key question: “Is the time coordinate encoded in QI quantity ?”

③ Ex. 1 Time-like Entanglement Entropy

[Doi–Harper–Mollabashi–Taki–TT 2022]

Consider a time-like version of entanglement entropy by rotating the subsystem A into a time-like one:

CFT on an infinite line



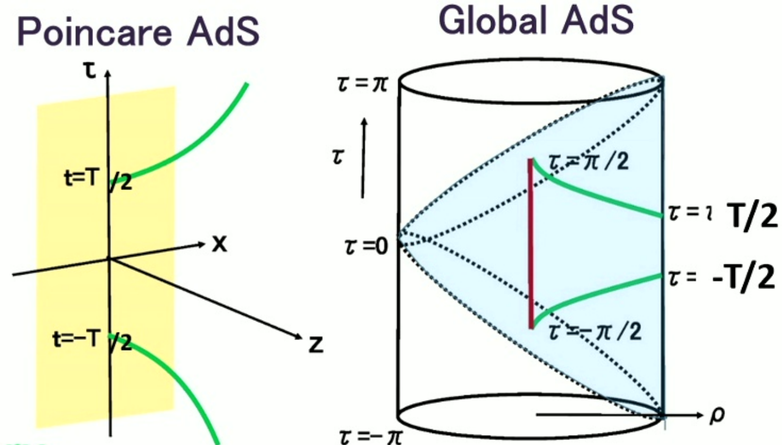
$$S_A = \frac{C}{3}\log\left[\frac{L}{\varepsilon}\right]$$

$L \to iT$

$$S_A = \frac{C}{3}\log\left[\frac{T}{\varepsilon}\right] + \frac{\pi}{6}iC$$

Imaginary part !

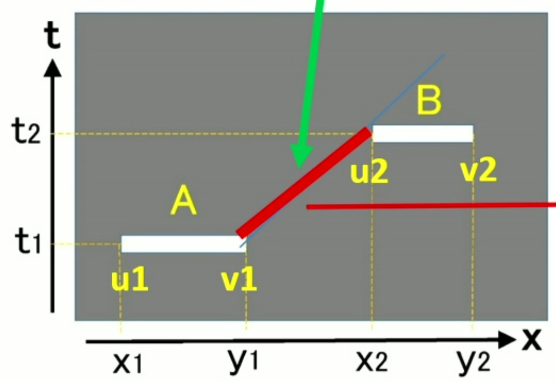
Holographic calculation



[For a systematic analysis of hol TEE, refer to Heller–Ori–Sereantes 2023]

We can find an essentially same phenomenon in a more standard setup of entanglement entropy for double intervals:

No longer time slice !



e.g. Free Dirac fermion CFT c=1

$$S_{AB} = \frac{c}{6}\log\frac{|v_1-u_1|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_1-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_1|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|u_1-u_2|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|v_1-v_2|^2}{\epsilon^2}.$$

If this interval is time-like, entropy gets complex valued !

The imaginary part of TEE is explained by the time-like geodesic in AdS.

[Kawamoto-Maeda-Nakamura-TT 25
refer also to Parzygnat-Fullwood 22]

ρ_{AB} is not Hermitian ⟺ A and B are causally connected

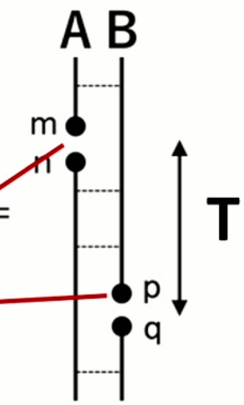
TEE is a special example of pseudo entropy.

A Toy Example: Coupled Harmonic Oscillators

$$H = \frac{1}{\sqrt{1-\lambda^2}}\left[a^\dagger a + b^\dagger b + \lambda(a^\dagger b^\dagger + ab) + 1 - \sqrt{1-\lambda^2}\right].$$

$$\lambda = \tanh 2\theta$$

$$(\rho_{AB})^{m\,p}_{n\,q} =$$

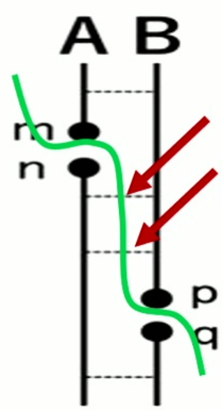


$$[\rho_{AB}]^{a_2,b_2}_{a_1,b_1} = \langle\Psi_0|_{12}\cdot(|b_2\rangle\langle b_1|)_2\cdot\mathcal{P}e^{-i\int_{t_1}^{t_2} dt H_{12}(t)}\cdot(|a_2\rangle\langle a_1|)_1\cdot|\Psi_0\rangle_{12},$$

$$\rho^\dagger_{AB} \neq \rho_{AB}$$

$$S^{(2)}_{AB} = \log\left[\frac{1+e^{-2iT}+(1-e^{-2iT})\cosh 4\theta}{2}\right].$$

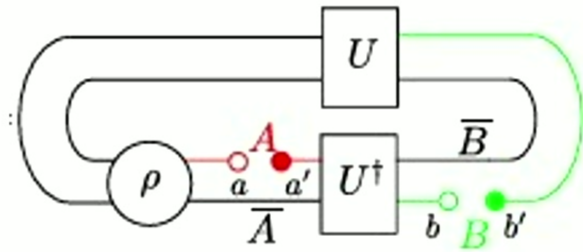
$$S(\rho_{AB}) \neq 0 \qquad \rho_{AB} = \text{mixed}$$



Purification needs extra Hilbert space

Recently, a clear theorem was shown by [Milekhin–Adamska–Preskill 2025]

$$(\rho_{AB})^{a'b'}_{ab} =$$



$$\langle [O_A(0), O_B(t)] \rangle = \mathrm{Tr}[(\rho_{AB} - \rho_{AB}^{\dagger}) O_A O_B]$$

Interactions between A and B

$$\frac{1}{\dim H_A} \left\| \rho_{AB} - \rho_{AB}^{\dagger} \right\|_2 \le \frac{\left| \langle [O_A(0), O_B(t)] \rangle \right|}{\left\| O_A \right\|_2 \cdot \left\| O_B \right\|_2} \le \left\| \rho_{AB} - \rho_{AB}^{\dagger} \right\|_2$$

Question: What is quantum information theoretic (e.g. resource theory) interpretation of pseudo entropy and time-like entanglement entropy ?

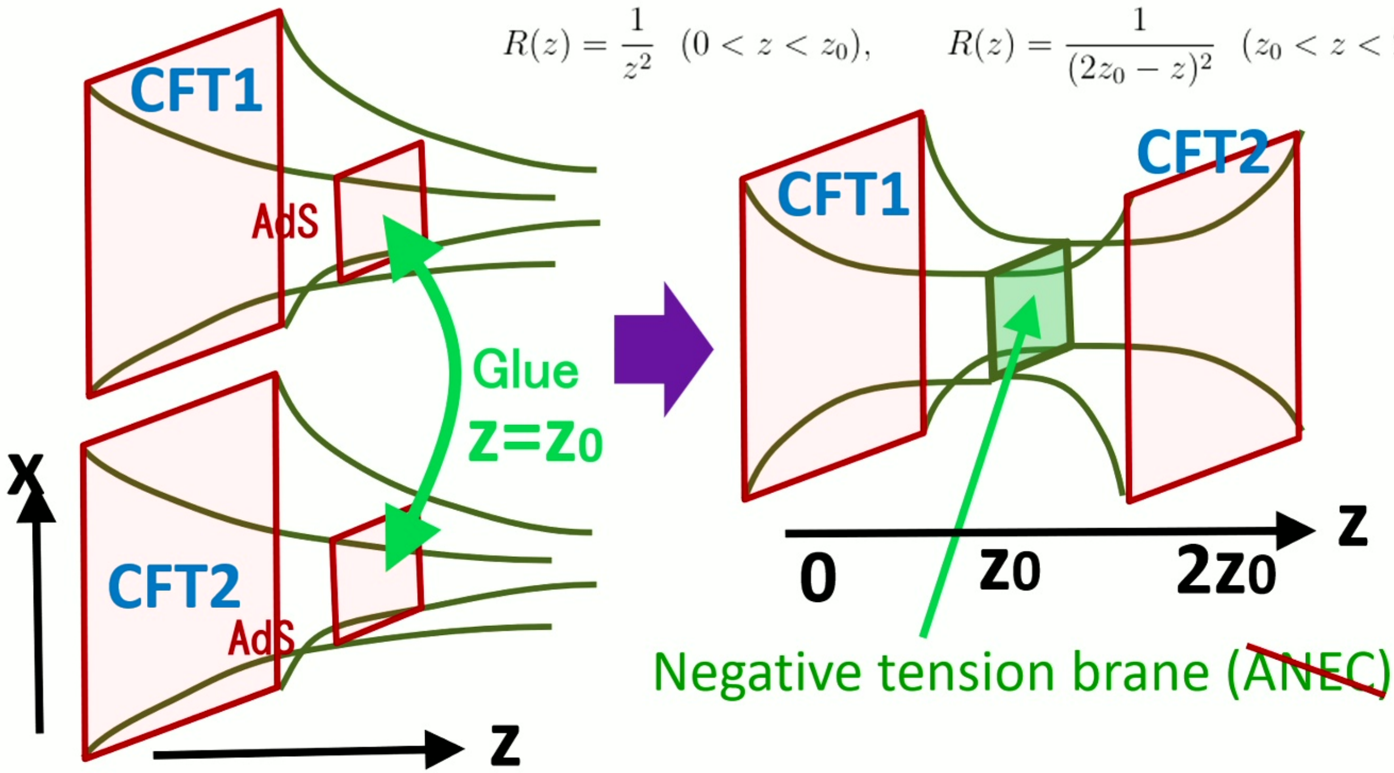
④ Ex.2 Traversable AdS Wormhole

(4-1) General argument

Consider a simple model of traversable AdS wormhole:

$$ds^2 = R(z)\left(dz^2 + \sum_{i=0}^{d-1} dx_i^2\right),$$

$$R(z) = \frac{1}{z^2} \quad (0 < z < z_0), \qquad R(z) = \frac{1}{(2z_0 - z)^2} \quad (z_0 < z < 2z_0).$$



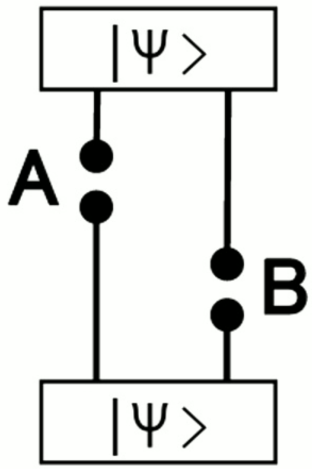
Two constructions of AdS Traversable wormhole

Traversable

Non-traversable

[Maldacena 01]

Thermofield double

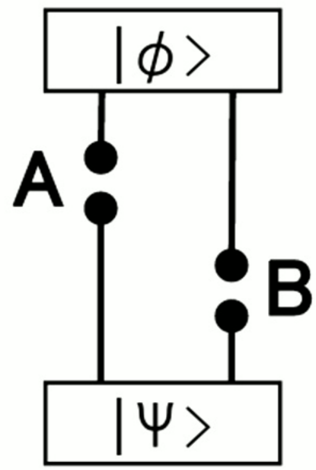


$\rho_{AB}^{\dagger} = \rho_{AB}$

$S(\rho_{AB}) = 0$

[Kawamoto-Maeda -Nakamura-TT 2025]

Model A (Janus)



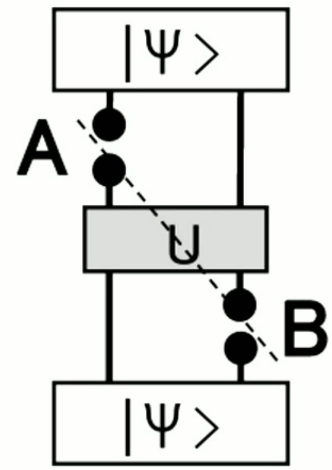
$\rho_{AB}^{\dagger} \neq \rho_{AB}$

$S(\rho_{AB}) = 0$

- ◆ No interactions between A and B
- ◆ H is non-hermitian

[Gao-Jafferis-Wall 2016]

Model B (Double trace)



$\rho_{AB}^{\dagger} \neq \rho_{AB}$

$S(\rho_{AB}) \neq 0$

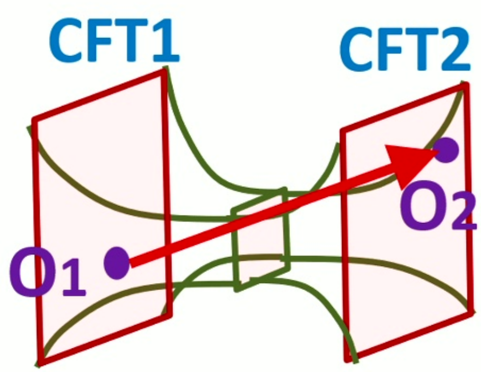
- ◆ ∃Interactions between A and B
- ◆ H is hermitian

Lorentzian 2pt functions of scalar operators

In Lorentzian signature x_0=it, the scalar two point function <O1O2> gets divergent at $-t^2+x^2+4z_0^2=0$ as two points are null separated:

$$\langle \mathcal{O}_1(t,x)\mathcal{O}_2(0,0)\rangle \sim \frac{1}{(-t^2+x^2+4z_0^2)^{d+2\nu-\frac{1}{2}}}.$$

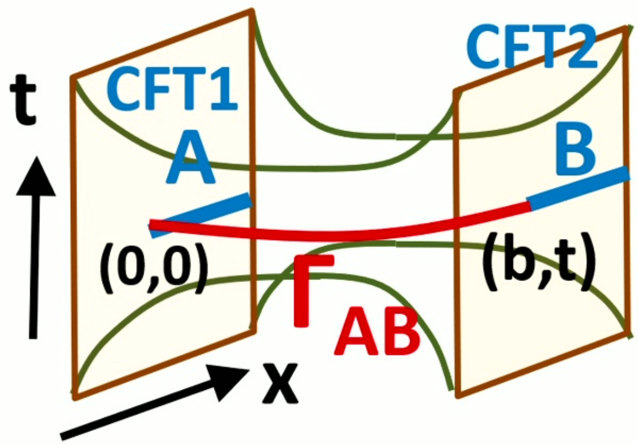
$$\nu = \sqrt{m^2+\frac{d^2}{4}}$$



A characteristic feature of traversable AdS black hole

Pseudo entropy (Time-like entanglement entropy)

How does SAB look like ?



When $t^2 < b^2 + 4z_0^2$:

$$S_{AB} = \frac{c}{3}\log\frac{\frac{b^2-t^2}{4}+z_0^2}{\epsilon z_0}.$$

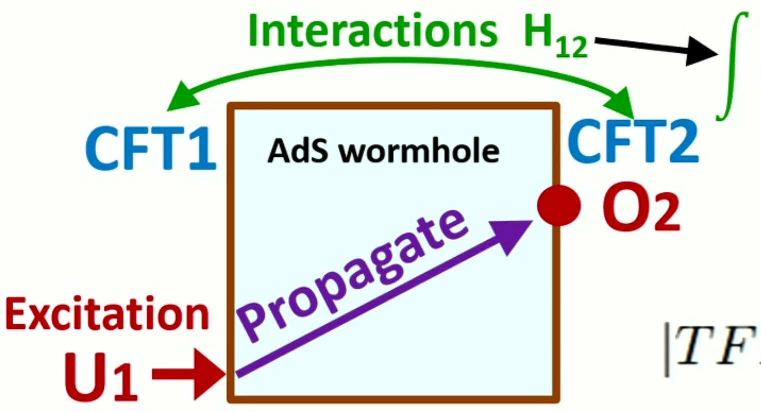
When $t^2 > b^2 + 4z_0^2$

$$S_{AB} = \frac{c}{3}\log\frac{\frac{t^2-b^2}{4}-z_0^2}{\epsilon z_0} + \frac{c}{3}\pi i.$$

Γ_{AB} can be time-like in a traversable wormhole.

⇨ S_{AB} becomes complex valued because $\rho_{AB}^{\dagger} \neq \rho_{AB}$.
Thus, S_{AB} should be regarded as pseudo entropy.

(4-2) Double trace deformation of External BH (Model B)



$\int dxdy\lambda(x,y)O_1(x)O_2(y)$

$H_{tot}=H_1+H_2+H_{12}$

$$|TFD\rangle = \mathcal{N}\sum_n e^{-\frac{\beta}{4}H_{tot}}|n\rangle_1|n\rangle_2$$

$$U_1 = e^{i\lambda O_1}$$

$$[\rho_{12}]_{ab}^{a'b'} = \langle TFD|e^{it_2H_{tot}}|b'\rangle\langle b|e^{-i(t_2-t_1)H_{tot}}|a'\rangle\langle a|e^{-it_1H_{tot}}|TFD\rangle,$$

$$\rho_2 = \mathrm{Tr}_1\left[e^{-it_2H_{tot}}\left(e^{it_1H_{tot}}U_1e^{-it_1H_{tot}}\right)|TFD\rangle\langle TFD|\left(e^{it_1H_{tot}}U_1^\dagger e^{-it_1H_{tot}}\right)e^{it_2H_{tot}}\right]$$

$$\langle O_2\rangle = \mathrm{Tr}[O_2\rho_2]$$
$$\simeq \langle TFD|O_2|TFD\rangle + i\lambda\langle TFD|[O_1(t),O_2]|TFD\rangle + O(\lambda^2).$$

Non-vanishing due to the interactions ↔ $\rho_{AB}^\dagger \neq \rho_{AB}$

(4-3) Wormhole via Janus deformation (Model A)

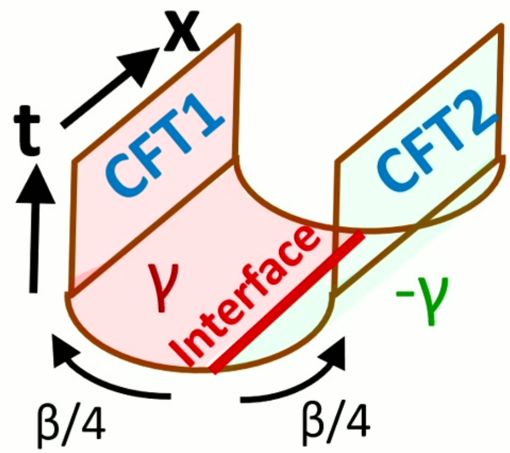
[Harper-Kawamoto-Maeda-Nakamura-TT, in preparation]

Janus deformation = asymmetric exactly marginal perturbations in a pair of CFTs

[Bak-Gutperle-Hirano 03]

$$S_{\mathrm{CFT}\,1} = S^{(0)}_{\mathrm{CFT}} + \gamma \int dx^d O_1(x)$$

$$S_{\mathrm{CFT}\,2} = S^{(0)}_{\mathrm{CFT}} - \gamma \int dx^d O_2(x)$$



◆We consider the TFD state of the doubled CFT for d=2.

◆In the standard Janus deformation, γ is real valued.
We will extend γ to imaginary values.

Explicit construction from Janus deformation

We start with 3D Janus BH solutions in [Bak-Gutperle-Hirano 2007].

The model is given by the 3d gravity action

$$I = \frac{1}{16\pi G_N} \int d^3x \left[R - g^{ab}\partial_a\phi\partial_b\phi + 2 \right].$$

The solution ansatz looks like

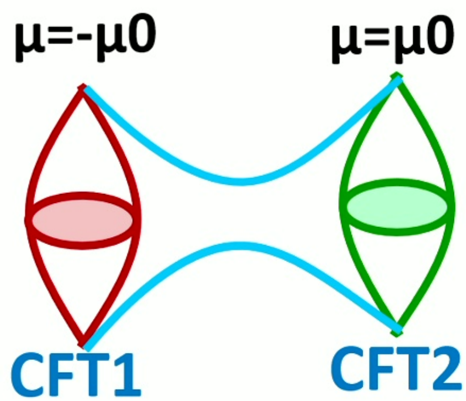
$$ds^2 = f(\mu)(d\mu^2 + ds^2_{AdS2}), \quad \phi = \phi(\mu).$$

$$ds^2_{AdS2} = -d\tau^2 + r_0^2 \cos^2\tau d\theta^2$$

γ is Janus deformation Parameter.

$$\frac{d\phi(\mu)}{d\mu} = \frac{\gamma}{\sqrt{f(\mu)}},$$

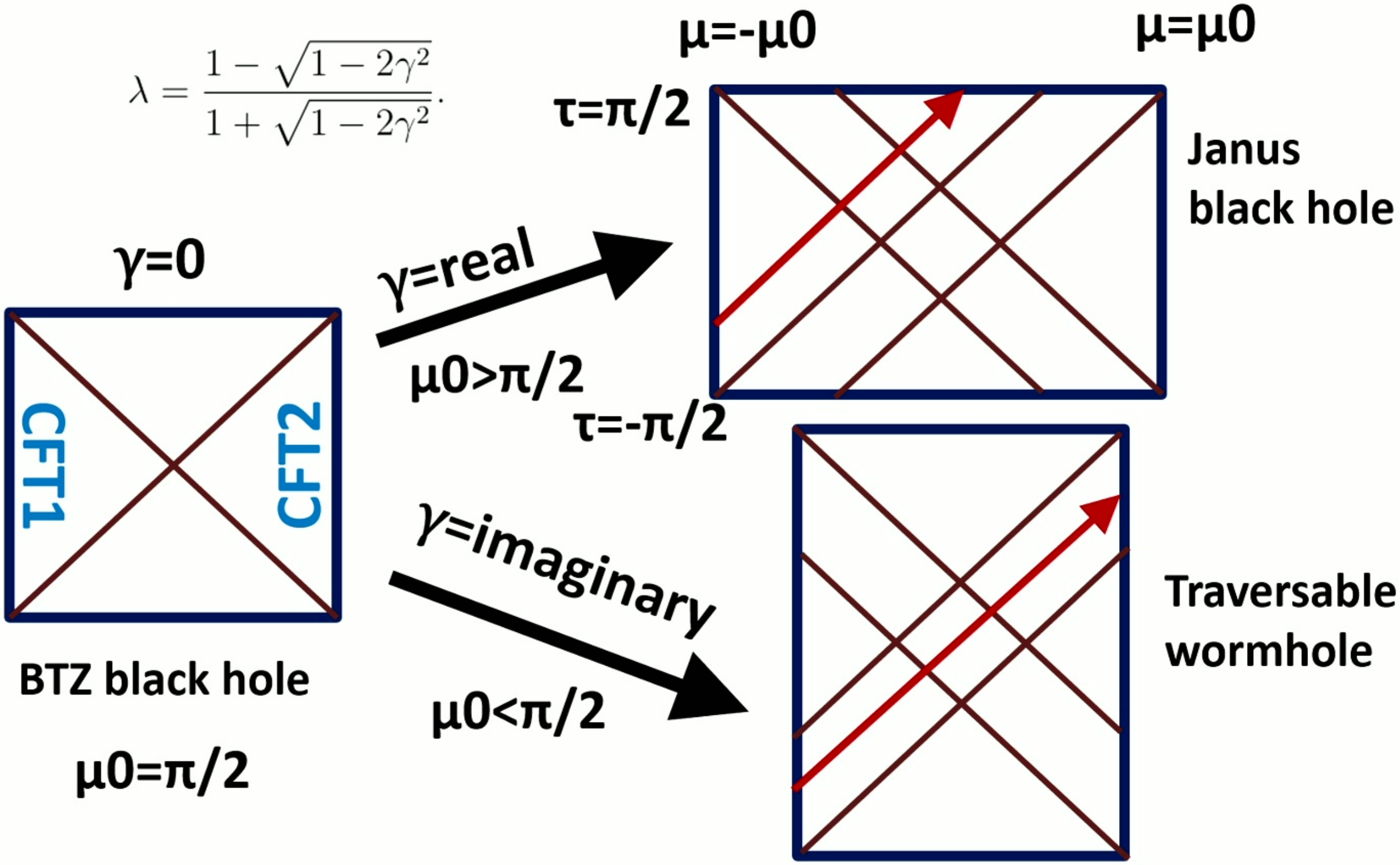
$$\frac{df(\mu)}{d\mu} = \sqrt{f(4f^2 - 4f + 2\gamma^2)}.$$



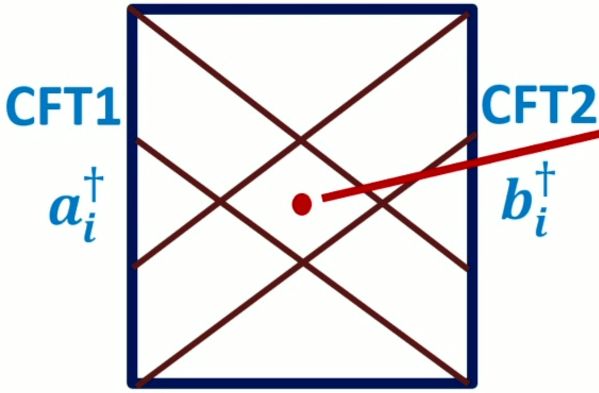
We now extend this solution to imaginary γ.

$$\mu_0 = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-\lambda x^2)}},$$

$$\lambda = \frac{1-\sqrt{1-2\gamma^2}}{1+\sqrt{1-2\gamma^2}}.$$



(Pseudo) Entanglement entropy between CFT1 and CFT2



$$S_1 = \frac{\pi}{6} c r_0 \sqrt{1+\sqrt{1-2\gamma^2}}$$

Area of throat

Monotonically decreasing function of γ^2

In the dual CFT, this is dual to the PE/EE in the deformed TFD state:

$$|\mathrm{TFD}(\beta,\gamma)\rangle = \tilde{\mathcal{N}} \exp\left[\sum_{i=1}^{\infty} e^{-\frac{\beta}{2}E_i}\left(\sin 2\theta\ a_i^\dagger b_i^\dagger + \cos 2\theta \left((a_i^\dagger)^2 - (b_i^\dagger)^2\right)\right)\right]|0\rangle$$

$$\langle \mathrm{TFD}(\beta,\gamma)| = \tilde{\mathcal{N}} \langle 0| \exp\left[\sum_{i=1}^{\infty} e^{-\frac{\beta}{2}E_i}\left(\sin 2\theta\ a_i b_i + \cos 2\theta \left((a_i)^2 - (b_i)^2\right)\right)\right] \quad \theta \equiv \frac{\pi}{4} + \gamma$$

S_1 becomes its maximum at θ=π/4 (i.e. no deformation) and decreases as γ^2 gets larger. For imaginary γ, it increases. This is consistent with the gravity dual.

Why traversable ?

The Hamiltonians H_1 and H_2 of CFT1 and CFT2 for γ=imaginary becomes non-Hermitian:

$$H_1 = H_0 + \gamma V, \quad H_2 = H_0 + \gamma^* V, \quad \text{such that} \quad H_1^\dagger = H_2$$

They have different eigen-vectors with complex eigen-values:

$$H_1 |n_+\rangle = E_n |n_+\rangle, \qquad H_2 |n_-\rangle = E_n^* |n_-\rangle,$$

$$\langle n_+ | H_2 = \langle n_+ | E_n^*, \qquad \langle n_- | H_1 = \langle n_- | E_n \ ,$$

where we introduce their (Hermitian) conjugations by $|n_\pm\rangle^\dagger = \langle n_\pm|$.

They satisfy $\langle n_\mp | m_\pm \rangle = \delta_{n,m}, \quad \sum_n |n_\pm\rangle\langle n_\mp| = 1.$

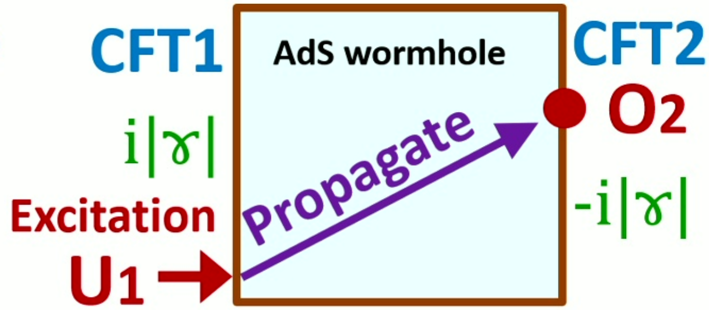
This motivates us to define the modified conjugation ‡ by

$$\langle n_+ | O | m_- \rangle^* = \langle m_+ | O^\ddagger | n_- \rangle.$$

The initial and final TFD state look like

$$|\mathrm{TFD}\rangle = \sum_n e^{-\frac{\beta}{4}(H_1+H_2)} |n_+\rangle_1 |n_+\rangle_2 \quad ,$$

$$\langle\overline{\mathrm{TFD}}| = \sum_n \langle n_-|_1 \langle n_-|_2 \, e^{-\frac{\beta}{4}(H_1+H_2)} \quad .$$



The density matrix $\rho = |\mathrm{TFD}\rangle\langle\overline{\mathrm{TFD}}|$ is not Hermitian $\rho^\dagger \neq \rho$.

However, it satisfies $\rho^\ddagger = \rho$, implying ‡ is good for the conjugation.

An observer in CFT2 probes the state:

$$\rho_2 = e^{-it_2 H_-^{(2)}} \mathrm{Tr}_1 \left[(e^{-it_1 H_+^{(1)}} e^{i\alpha O^{(1)}} e^{it_1 H_+^{(1)}}) |TFD\rangle\langle\overline{TFD}| (e^{-it_1 H_+^{(1)}} e^{-i\alpha O^{(1)\ddagger}} e^{it_1 H_+^{(1)}}) \right] e^{it_2 H_-^{(2)}} .$$

When the CFT1 is excited by $U_1 = e^{i\alpha O^{(1)}}$, the CFT2 observer sees

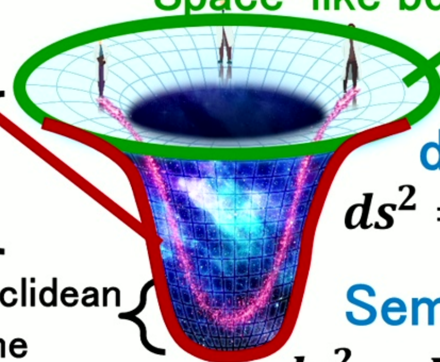
$$\begin{aligned} \langle O^{(2)}(t_2)\rangle &= \mathrm{Tr}[\rho_2 O^{(2)}] \\ &\simeq \langle\overline{TFD}|O^{(2)}|TFD\rangle + i\alpha\langle\overline{TFD}| \left(O^{(2)}(t_2)O^{(1)}(t_1) - O^{(1)\ddagger}(t_1)O^{(2)}(t_2) \right) |TFD\rangle , \end{aligned}$$

We have $[O_1,O_2]=0$, but this is non-vanishing !

⑤ Ex.3 dS/CFT correspondence

A Sketch of dS/CFT [Strominger 2001, Witten 2001, Maldacena 2002,⋯.]

d+1 dim. Lorentzian de−Sitter spacetime ⟺ Dual ⟺ Euclidean d. dim CFT on S^d



Time

Lorentzian time

Euclidean time

Time emerges from Euclidean CFT !

de Sitter

$$ds^2 = L_{dS}^2(-dt^2 + \mathrm{Cosh}^2 t\, d\Omega^2)$$

Semi sphere

$$ds^2 = L_{dS}^2(d\theta^2 + \mathrm{Sin}^2\theta\, d\Omega^2)$$

Ψ [dS gravity]=Z [CFT]

Why dS/CFT is much more difficult than AdS/CFT ?

[1] Dual Euclidean CFTs should be exotic and non-unitary !

A "standard" Euclidean CFTs is dual to gravity on hyperbolic space.

e.g. dS3/CFT2 ➔ ***Imaginary valued*** central charge $c \approx i\frac{3L_{dS}}{2G_N}$!

Unsual conjugation: $(L_n)^\dagger = (-1)^{n+1}\widetilde{L_n}$ [Doi-Ogawa-Shimyo-Suzuki-TT 2024 *Refer to Shinmyo's poster*]

[2] Time should emerge from Euclidean CFT !

From a usual Euclidean CFT, a space-like direction will emerge as RG scale.

How does a ***time-like direction emerge*** from a Euclidean CFT ?

[3] "Entanglement entropy" looks complex valued !

Extremal surfaces in dS which end on its boundary are ***time-like*** !

Non-unitary CFT dual of 3 dim. dS

[Hikida–Nishioka–Taki–TT, 2021]

Large c limit of $SU(2)_k \times SU(2)_{-k}$ WZW model (a 2dim. CFT)

= **Einstein Gravity on 3 dim. de Sitter** (radius L_{ds})

Level $k \approx -2 + \frac{4iG_N}{L_{dS}}$ → **Central charge** $c = \frac{3k}{k+2} \approx i\frac{3L_{dS}}{2G_N}$

$$Z[S^3, R_j] = \left|S_j^0\right|^2 \approx e^{\frac{\pi L_{ds}}{2G_N}\sqrt{1-8G_N E}}$$

CFT partition function — De Sitter Entropy

This non-unitary CFT is equivalent to the Liouville CFT

at $b^{-2} \approx \pm\frac{i}{4G_N}$. $\quad I_{CFT}[\phi] = \int d^2x \left[\frac{1}{4\pi}(\partial_a\varphi\partial_a\varphi) + \mu e^{2b\varphi}\right].$

[Hikida–Nishioka–Taki–TT, 2022] — complex !

The same Liouville CFT appears in [Verlinde–Zhang 2024] via DSSYK.

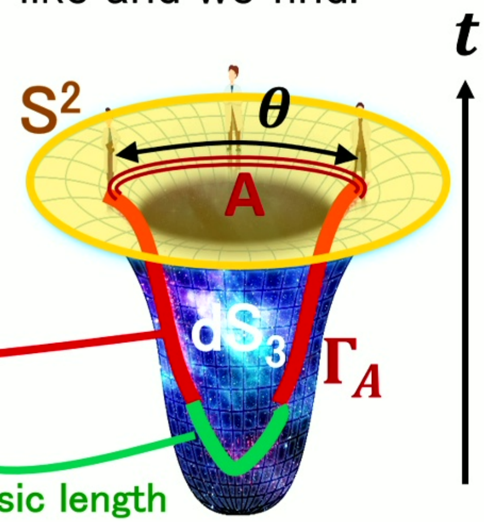
→Why two different holographic constructions lead to the same CFT ?

Holographic Entanglement Entropy in dS3/CFT2 ?

[Doi-Harper-Mollabashi-Taki-TT 2022]

In dS3/CFT2, the geodesic Γ_A becomes time-like and we find:

$$S_A = \frac{\mathrm{L}(\Gamma_A)}{4G_N} = i\frac{C_{ds}}{3}\log\left(\frac{2}{\epsilon}\mathrm{Sin}\frac{\theta}{2}\right) + \underbrace{\frac{C_{dS}}{6}\pi}_{S_{dS}/2}.$$



Time-like geodesics length
➔imaginary part !

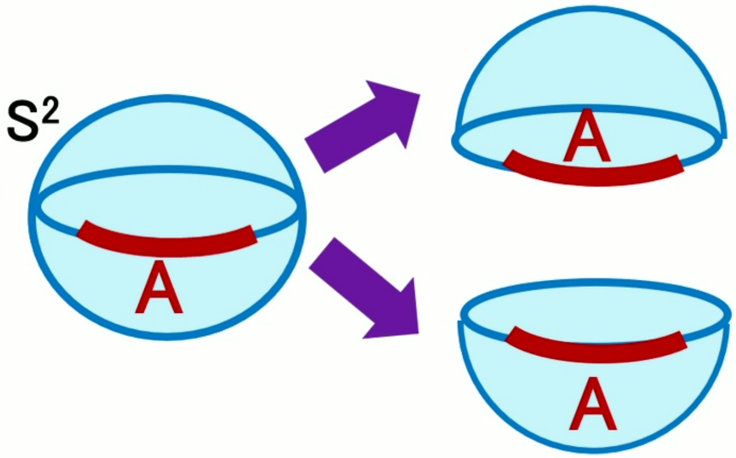
Space-like geodesic length
➔Real part

Complex valued entropy !
(should not be EE !)

We argue it is more properly considered as pseudo entropy (PE).

This is because the reduced density matrix ρ_A **is not Hermitian !**

$\rho_A =$



$\langle \varphi | \sim \int_{S^2_+} D\varphi \, e^{-I_{CFT}[\varphi]}$

Different states !

$| \psi \rangle \sim \int_{S^2_-} D\varphi \, e^{-I_{CFT}[\varphi]}$

2D CFT on the space with the metric: $h_{ab} = e^{2\phi}\delta_{ab}$,

$$I_{CFT}[\phi] = i\frac{C_{ds}}{24\pi}\int d^2x[(\partial_a\phi)^2 + e^{2\phi}].$$

→ $\rho_A \neq \rho_A^\dagger$

Note: the emergent time coordinate = imaginary part of PE.

⑥ AdS/CFT as a "Quantum Computer"

Quantum Problems
e.g. Strongly coupled gauge theories

Quantum answers

Classical computers

It is very intriguing question to ask

(1) Is the AdS/CFT more powerful compared with quantum computers ?

(2) What is the basic mechanism of computation behind the AdS/CFT ?

Useful input to extend AdS/CFT to general spaces

Below I would like to give some idea on them. [I am very grateful to Ryu Hayakawa and Tomoyuki Morimae for valuable discussions.]

Quantum Extended Church-Turing thesis (qECT)

All physical processes, including quantum gravity, are efficiently simulable on a quantum computer.

→ within polynomial time poly(n)
→ complexity class BQP

Fact In AdS/CFT, we can calculate computational complexity easily via the "complexity=volume" formula [Stanford-Susskind 2014]. However, quantum computer cannot measure complexity within polynomial time in general. [Bouland-Fefferman-Vazirani 2019]

➡ Therefore qECT might be violated !

Harr random state C=exp(n) ⬌ **Pseudo random state C=poly(n)**

Quantum computer cannot distinguish

TFD state after a short time evolution

Pseudo entanglement

Quantum computers cannot compute entanglement entropy within polynomial time, though the AdS/CFT can do easily.

Quantum computer cannot distinguish
➔ Pseudo entanglement

Harr random state S=O(n) ⟺ **Pseudo random state S=O(log n)**

[Aaronson-Bouland-Fefferman-Ghosh-Vazirani-Zhan-Zhou 2022]

Moreover, [Gheorghiu-Hoban 2020] argued that the measurement of entanglement entropy for a generic quantum state generated by the O(1) depth quantum circuit is more difficult than LWE.

Hamiltonian complexity [Kitaev-Shen-Vyalyi 2002,…]

Calculating the ground state (GS) energy of the local Hamiltonian:

$$\boldsymbol{H} = \sum_i \boldsymbol{H}_i, \quad ||\boldsymbol{H}_i|| \leq \mathbf{1}, \quad \boldsymbol{H}_i \text{ acts only on k qubits.}$$

up to the error O(1/poly(n)),

is known to be in the complexity class **QMA** in general, which is quantum version of NP. Thus quantum computer cannot solve this.

➡ However, in AdS/CFT, we can calculate the energy Immediately once we know the metric !

Can AdS/CFT do better than quantum computer even though we use classical computers for its calculations ?

Useful Recent Progress

[Cade–Folkertsma–Gharibian–Hayakawa–Le Gall–Morimae–Weggemans 2022.]

❶ If we know a guided state $|\alpha\rangle$ such that $\langle \mathrm{GS}|\alpha\rangle = O(1)$, then the GS energy problem gets BQP. **➔Quantum computers can do !**

❷ If we further relax the allowable error to O(1) from O(1/Poly(n)), then the problem becomes BPP. **➔Classical computers can do !**

Implications for AdS/CFT

[Hayakawa–Morimae–TT, work in progress]

This looks consistent with the calculation of energy in AdS/CFT !

❶ We normally start with a basic solution (e.g. pure AdS), which is obtained from Euclidean (imaginary time) method.➔ Guided state

❷ In the classical gravity approximation, we can compute physical quantities which scale as $O(N^2)$ ignoring O(1) contributions.

Thus, the procedure how we compute the energy for a given new quantum state in AdS/CFT seems to be naturally reduced to BPP.

Key point: * The Euclidean path-integral plays an important role to prepare the initial state.
*** The large N limit makes the calculations classical.**

The further questions will be what AdS/CFT is actually doing for

- Calculations of entanglement entropy
- Calculations of computational complexity
- Calculations of correlations functions

:

Many future problems !

International workshop
“Extreme Universe 2025”
Oct.27-Nov.1
@YITP, Kyoto

Registration Deadline: June 30

Thank you !

⑦ Conclusions

Key ideas

Is gravity the fastest "quantum computer" ?

Holography **→New insights into quantum matter, quantum computation and quantum cryptography**

Universe = Collection of Qubits (=Strings?)

Does gravitational spacetime emerge from qubits？

Emergence **→New approach to quantum gravity**

In this talk we emphasized the use of pseudo entropy (PE).

- PE has a clear gravity dual via holography.
- PE is a useful geometric probe of non-Hermitian dynamics.

e.g. time-like entanglement, wormholes, and de Sitter spaces…

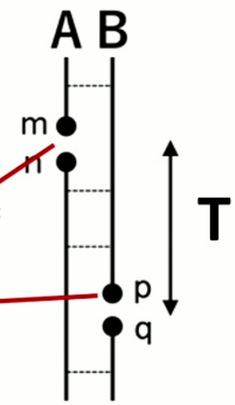
Imaginary part of Pseudo entropy→Emergence of Time
(but what is quantum informational meaning of PE ?)

A Toy Example: Coupled Harmonic Oscillators

$$H = \frac{1}{\sqrt{1-\lambda^2}}\left[a^\dagger a + b^\dagger b + \lambda(a^\dagger b^\dagger + ab) + 1 - \sqrt{1-\lambda^2}\right].$$

$$\lambda = \tanh 2\theta$$

$$(\rho_{AB})^{m\,p}_{n\,q} =$$

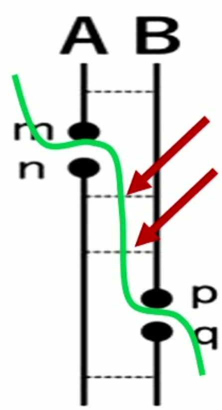


$$[\rho_{AB}]^{a_2,b_2}_{a_1,b_1} = \langle\Psi_0|_{12}\cdot(|b_2\rangle\langle b_1|)_2\cdot\mathcal{P}e^{-i\int_{t_1}^{t_2}dtH_{12}(t)}\cdot(|a_2\rangle\langle a_1|)_1\cdot|\Psi_0\rangle_{12},$$

$$\rho^\dagger_{AB} \neq \rho_{AB}$$

$$S^{(2)}_{AB} = \log\left[\frac{1+e^{-2iT}+(1-e^{-2iT})\cosh 4\theta}{2}\right].$$

$$S(\rho_{AB}) \neq 0 \qquad \rho_{AB} = \text{mixed}$$



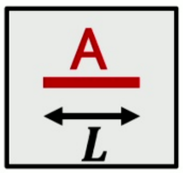
Purification needs extra Hilbert space

③ Ex. 1 Time-like Entanglement Entropy

[Doi–Harper–Mollabashi–Taki–TT 2022]

Consider a time-like version of entanglement entropy by rotating the subsystem A into a time-like one:

CFT on an infinite line



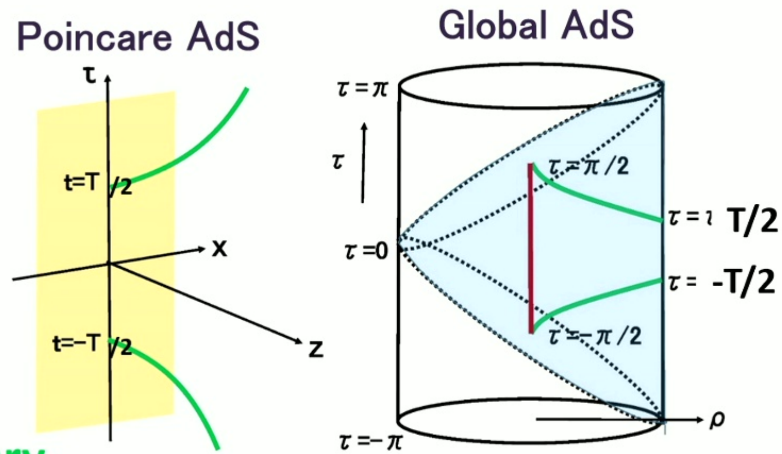
$$S_A = \frac{C}{3}\log\left[\frac{L}{\varepsilon}\right]$$

$L \to iT$

$$S_A = \frac{C}{3}\log\left[\frac{T}{\varepsilon}\right] + \frac{\pi}{6} iC$$

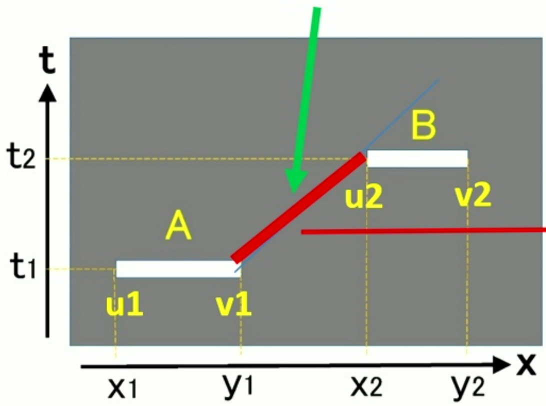
Imaginary part !

Holographic calculation



[For a systematic analysis of hol TEE, refer to Heller–Ori–Sereantes 2023]

We can find an essentially same phenomenon in a more standard setup of entanglement entropy for double intervals:



e.g. Free Dirac fermion CFT c=1

$$S_{AB} = \frac{c}{6}\log\frac{|v_1-u_1|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_1-u_2|^2}{\epsilon^2} + \frac{c}{6}\log\frac{|v_2-u_1|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|u_1-u_2|^2}{\epsilon^2} - \frac{c}{6}\log\frac{|v_1-v_2|^2}{\epsilon^2}.$$

If this interval is time-like, entropy gets complex valued !

The imaginary part of TEE is explained by the time-like geodesic in AdS.

[Kawamoto-Maeda-Nakamura-TT 25
refer also to Parzygnat-Fullwood 22]

ρ_{AB} is not Hermitian ⟺ A and B are causally connected

TEE is a special example of pseudo entropy.

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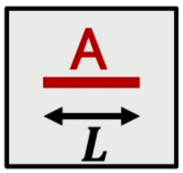

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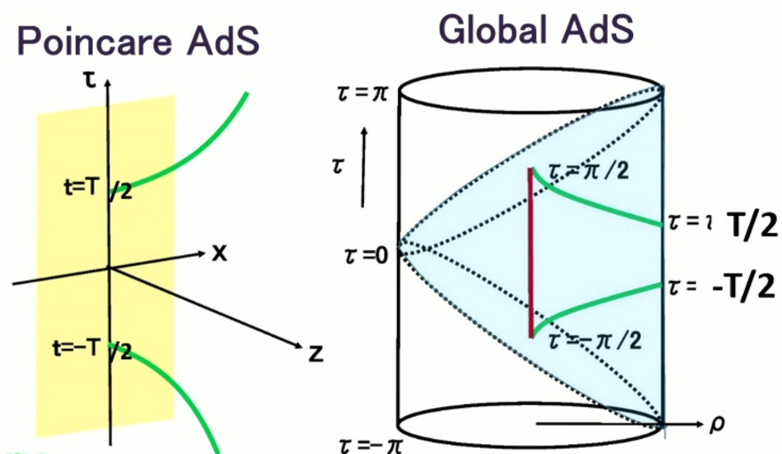

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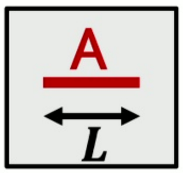

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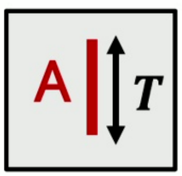

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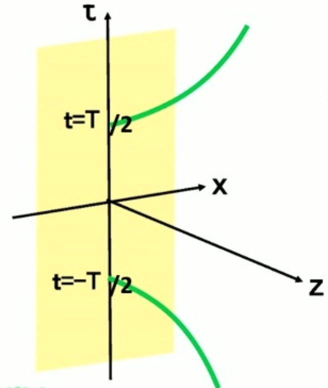
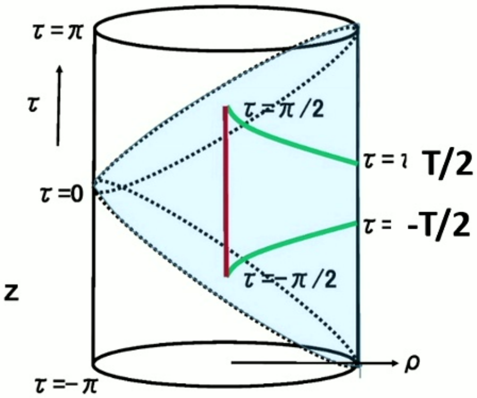
Imaginary part !

Holographic calculation

Poincare AdS



Global AdS



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