Title: The gravitational path integral from an observer's point of view

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Abstract:

One of the fundamental problems in quantum gravity is to describe the experience of a gravitating observer in generic spacetimes. In this talk, I will describe a framework within which we can analyze non-perturbative physics relative to an observer using the gravitational path integral. We apply our proposal to an observer that lives in a closed universe and one that falls behind a black hole horizon. We find that the Hilbert space that describes the experience of the observer is much larger than the Hilbert space in the absence of an observer. In the case of closed universes, the Hilbert space is not one-dimensional, as calculations in the absence of the observer suggest. Rather, its dimension scales exponentially in 1/G_N. Similarly, from an observer's perspective, the dimension of the Hilbert space in a two-sided black hole is increased and this drastically changes what an observer sees when falling past the horizon of a black hole at late times.

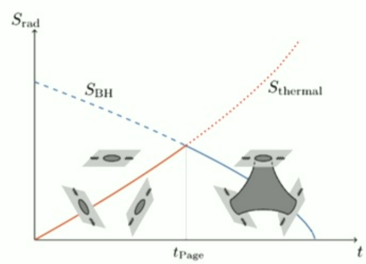
The gravitational path integral from an observer's point of view

Luca V. Iliesiu

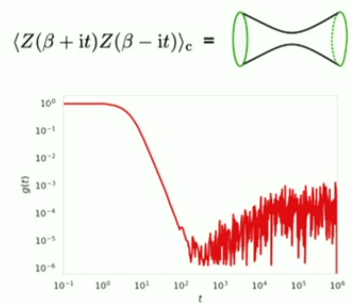
February 21, 2025

In recent years, the GPI has successfully explained why gravitational theories and the objects within them (e.g., black holes) are described by ordinary quantum systems.

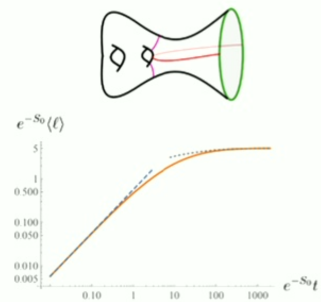
Done by computing overlaps between black hole states in extreme regimes:



Page curve

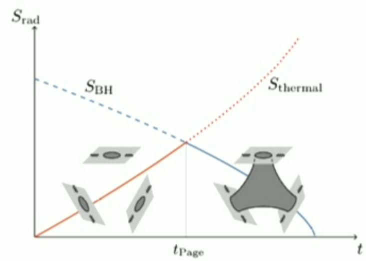


Energy
discreteness

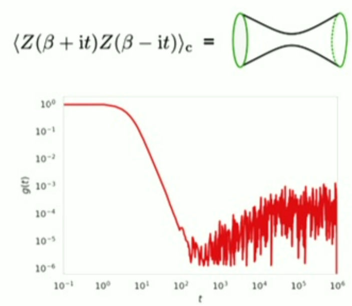


Probes of the
BH interior at late times

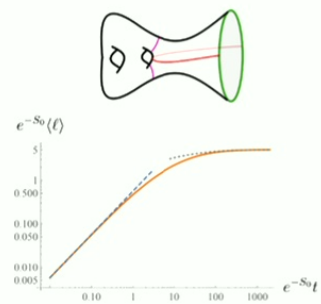
The unifying theme of these results is that the naive GR contribution breaks down in extreme regimes (when some parameter scales with G_N^{-1}).



Page curve



Energy discreteness



Probes of the BH interior at late times

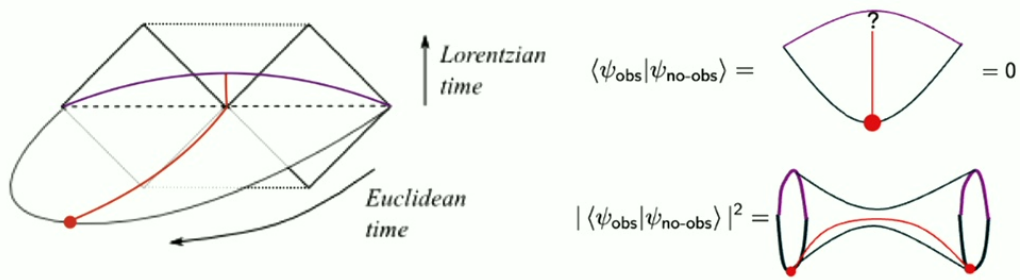
In such regimes, non-perturbative corrections (e.g., wormholes) in the GPI become critical and dominate over the conventional GR contributions.

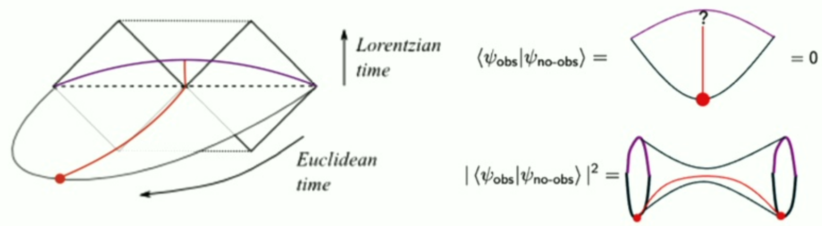
Nevertheless, all these probes have not successfully described the experience of a gravitating observer, i.e., a subsystem of the bulk.

Since we are all gravitating observers (subsystems), answering the following questions is critical in order to understand quantum gravity in our own universe.

- Can such an observer probe that GR breaks down in extreme regimes? What happens to them in wormhole backgrounds?
- Within AdS/CFT, can bulk observations be mapped to the boundary? Can the CFT describe the observer's experience?
- Is the experience of a gravitating observer even described by quantum mechanics, or do we need a different framework?

- First, is a gravitating observer even well defined?
- In QFT on a fixed curve spacetime, at weak coupling, the answer is yes. However, this question is complicated by the non-perturbative corrections that were so important in recent years. States that were naively orthogonal become non-orthogonal.
- How can we see this? Problem: $\langle \psi_{\text{no-obs}} | \psi_{\text{obs}} \rangle \neq 0$.
  In the GPI, states and operators are described by bdy. conditions.
  E.g., in a two-sided black hole:





Contribution guaranteed from the boundary POV, by the Eigenstate Thermalization Hypothesis (ETH). Like all others, the simple operator that inserts the observer should satisfy the ETH ansatz.

Two-sided BH: $$|TFD\rangle \sim \sum_E e^{-\frac{\beta E}{2}} |E, E\rangle$$

Two-sided BH with "observer": $$|TFD_O\rangle \sim \sum_{E_1, E_2} e^{-\frac{\beta(E_1+E_2)}{4}} O_{E_1,E_2} |E_1, E_2\rangle$$

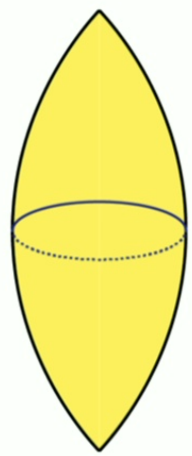
$$|\langle TFD | TFD_O \rangle|^2 \sim \sum_{E,E'} e^{-\frac{\beta(E+E')}{2}} O^*_{E,E} O_{E',E'} \approx \sum_E e^{-\beta E} |\overline{O_{E,E}}|^2 \neq 0$$

This reproduces the wormhole contribution, assuming the ETH ansatz.

The problem is even worse if we consider an observer in a closed universe.

For simplicity, consider a 2D closed universe solution with a negative cosmological constant that describes a big-bang/big-crunch universe.

$$ds^2 = -d\rho^2 + b^2 \cos^2 \rho \, dx^2$$



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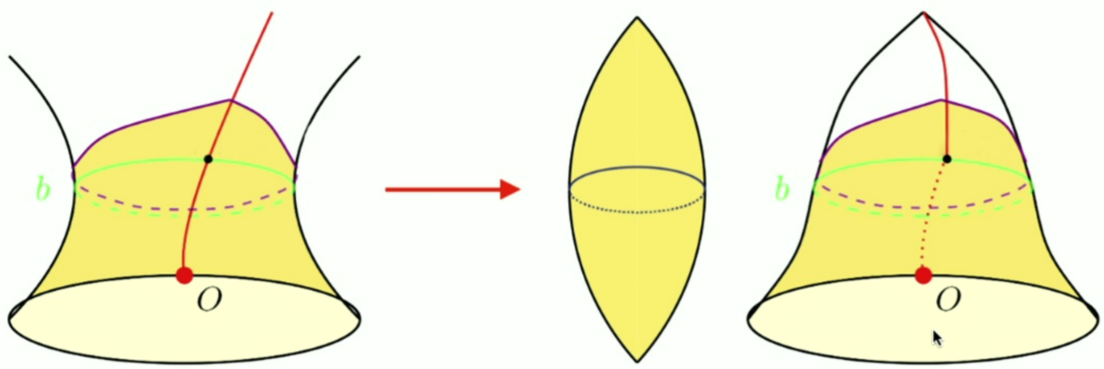
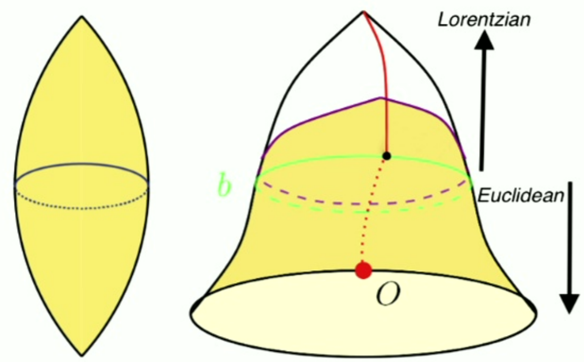
For simplicity, consider a 2D closed universe solution with a negative cosmological constant that describes a big-bang/big-crunch universe.

$$ds^2 = -d\rho^2 + b^2 \cos^2 \rho \, dx^2 \to d\sigma^2 + b^2 \cosh^2 \sigma \, dx^2$$

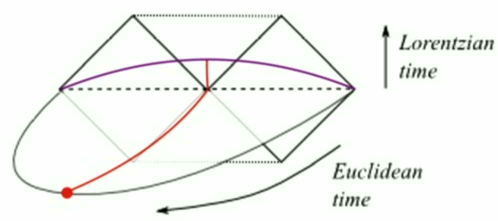


[remember Mark's talk]

Case study: The AdS$_2$ closed universe



is analogous to the analytic continuation of the two-sided BH,



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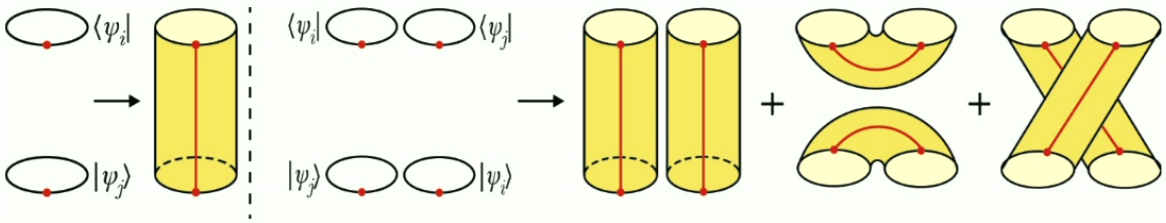
Consider an inner-product between any two states in the closed universe:



These are the same wormhole geometries that we encountered before.

However, their contribution to inner-products is now different: the variance in the inner-product of *any* two states is now enormously large. In fact, we will shortly see that this is signaling that

$$\dim \mathcal{H}_{\substack{\text{Closed}\\ \text{universe}}} = 1.$$

[see Marolf, Maxfield; Usatyuk, Zhao]

By itself, this is a massive problem that needs a resolution

$$\text{e.g.,} \qquad \langle\psi|\,\Pi\,|\psi\rangle = 1, \qquad [O_1, O_2] = 0.$$

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Summary of the problem

- Thus, with the standard rules of the GPI, there can be large overlaps between states with and without an observer: observers or subsystems in the bulk are not well-defined.
- Therefore, with these standard rules, we cannot compute the probability of a given observer's experience or compute any observables dressed to their worldline
- What is the issue? How come observers are seemingly ill-defined?

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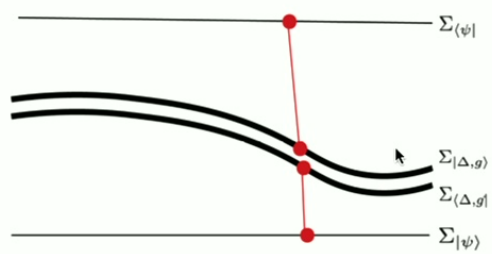
The GPI from an observer's point of view

Suppose we are interested in computing the probability that the observer and gravitational system transition from $|\psi\rangle$ to a given state. This is given by the expectation value of a projector Π.

Suppose, $\Pi = \sum_{g,g'} \Pi(g,g') \, (|\Delta\rangle \otimes \langle\Delta|) \otimes (|g\rangle_\Delta \langle g'|_\Delta)$, where $|\Delta\rangle \in \mathcal{H}_{\text{obs}}$ and $|g\rangle_\Delta \in \mathcal{H}^{\text{rel}}$.

$$\langle\psi|\Pi|\psi\rangle = \sum_{g,g'} \Pi(g,g') \;\times$$



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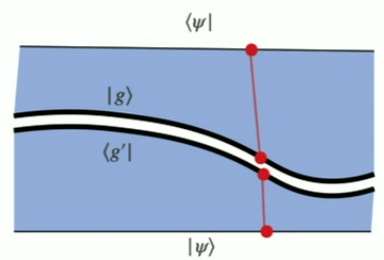

$+ \ldots$

Typically leading order

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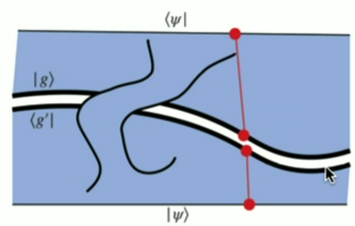
$$\langle\psi|\Pi|\psi\rangle \supset \sum_{g,g'} \Pi(g,g') \times$$



Should contribute!

The GPI from an observer's point of view

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This is given by the expectation value of a projector Π.

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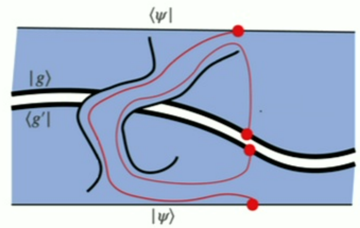
$$\langle\psi|\Pi|\psi\rangle \supset \sum_{g,g'} \Pi(g,g') \times$$



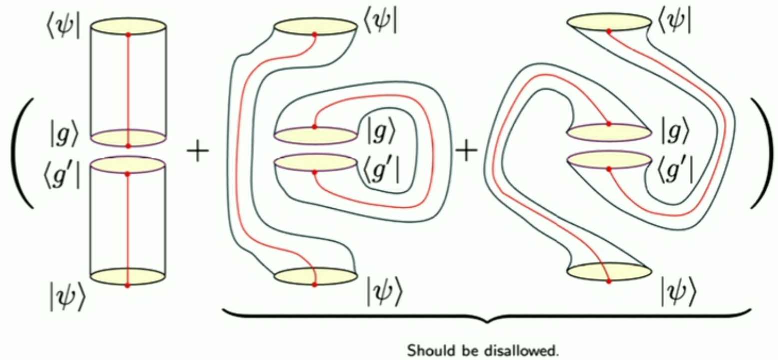
Should not contribute!

The subsystem never even intersects the slice on which the projection is made. The projector does not measure the state of the subsystem that we are interested in.

Projections in a closed universe

Geometries are clearly problematic for closed universes:

$$\langle\psi|\Pi|\psi\rangle \supset \sum_{g,g'} \Pi(g,g') \times$$



$+ \quad \ldots$

We need to define a new inner product which computes $\langle\psi|\,\Pi\,|\psi\rangle$ correctly.

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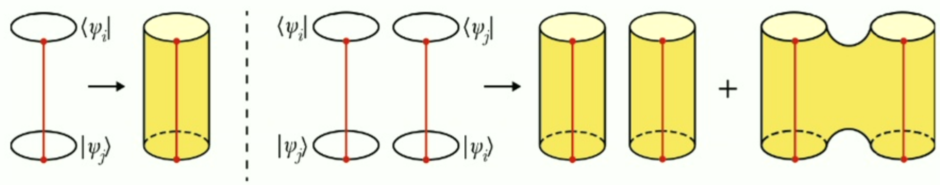
Our proposal

Thus, we propose that in the worldline approximation, an observer's (subsystem's) worldline must connect a ket with the corresponding bra.



Main result: The rule eliminates the non-perturbative contributions that lead to the issues above and yields a well-defined inner-product and a well-defined Hilbert space:

$$\mathcal{H}_{\text{bulk}}^{\text{obs POV}} = \underbrace{\mathcal{H}_{\text{obs}}}_{\substack{\text{Hilbert space}\\\text{of subsystem}}} \otimes \underbrace{\mathcal{H}^{\text{rel}}}_{\substack{\text{Hilbert space}\\\text{that the observer}\\\text{is entangled with}}}$$

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Summary of results

The new inner product that defines $\mathcal{H}_{\text{bulk}}^{\text{obs POV}}$ has the following properties:

- For a closed universe, $\dim(\mathcal{H}^{\text{rel}}_{\text{non-pert}}) \neq 1$. Instead, $\dim(\mathcal{H}^{\text{rel}}_{\text{non-pert}}) \sim e^{2S_0}$.
- For a two-sided black hole, $\dim(\mathcal{H}^{\text{rel}}_{\text{non-pert}}) \sim e^{4S_0} \sim (\dim \mathcal{H}_{\text{bdy}})^2$. Hilbert space from observer's POV is completely different than $\mathcal{H}_{\text{bdy}}$. [see also recent code by Akers et al.].
- Observables along the observer's worldline in a closed universe are non-trivial (i.e. $[O_1, O_2] \neq 0$).
- Observer falling into an exponentially old black hole sees GR break down. [see also Stanford-Yang, LVI-Levine-Lin-Maxfield-Mezei, Blommaert-Chen-Nomura]
- Results inspire generalization in higher-dimensional examples of AdS/CFT.

A simplified model: observer action and relational Hilbert space

How do we model an observer? Minimal model for observer:

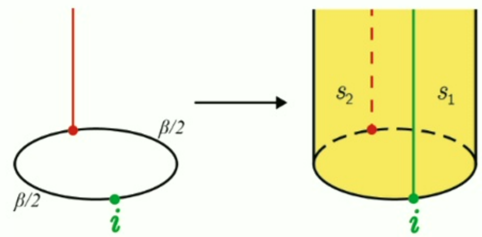
$$I_{\text{observer}} = \int d\tau \left[P \partial_\tau Q - Q\sqrt{g_{\tau\tau}}\right]$$

Hamiltonian: $Q\,|\Delta_O^{(i)}\rangle = E_\Delta^{(i)}\,|\Delta_O^{(i)}\rangle, \qquad |\Delta_O^{(i)}\rangle \in \mathcal{H}_{\text{obs}}.$

Dimension of $\mathcal{H}^{\text{rel}}_{\text{non-pert}}$ for closed universe

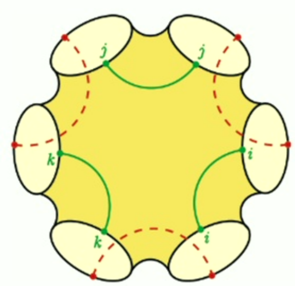
Now consider the new rules in JT gravity:

- Since our observer is now created by a special subset of all matter fields, we can fix observer state ($\Delta_O^{(i)} = \Delta_O$), $\Delta_m^{(j)} = \Delta_m$, and state i of all other matter fields. A state $|\psi_i\rangle$ is thus prepared by,



- With our rules for the GPI, and for $K, e^{2S_0} \to \infty$, $K/e^{2S_0} = O(1)$, correction to n-th moment of overlap is from connected pinwheel:

$$\overline{\langle\psi_i|\psi_k\rangle\langle\psi_k|\psi_l\rangle\ldots\langle\psi_m|\psi_i\rangle}_{conn} \approx Z_n =$$



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Dimension of $\mathcal{H}^{\text{rel}}_{\text{non-pert}}$ for closed universe

From moments: dimension $\dim(\mathcal{H}_{\text{non-pert}}(K) = \text{span } |\psi_i\rangle)$ is computed using the resolvent of the matrix of overlaps $M_{ij} = \langle\psi_i|\psi_j\rangle$

[similar to Penington, Shenker, Stanford, Yang; Boruch, LVI, Lin, Yan]

$$\Downarrow$$

$$\dim\left(\mathcal{H}^{\text{rel}}_{\text{non-pert}}(K)\right) = \begin{cases} K & K < d^2 \\ d^2 & K > d^2 \end{cases}$$

$$d^2 = e^{2S_0}\int ds_1 ds_2 \rho_0(s_1)\rho_0(s_2)$$

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Dimension of $\mathcal{H}^{\text{rel}}_{\text{non-pert}}$ for closed universe

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The Hilbert space $\mathcal{H}^{\text{obs}} \otimes \mathcal{H}^{\text{rel}}_{\text{non-pert}}$ relevant for the description of an observer's experience in a closed universe is non-trivial and has dimension $\sim d_{\text{obs}} e^{\#/G_N}$.

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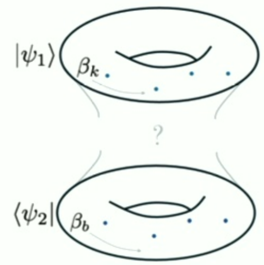
How do we generalize this result in higher-dimensions?

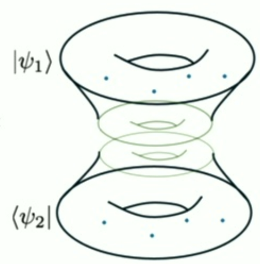
Can we recover such a Hilbert space from boundary calculations in AdS/CFT? Focus on a closed universe. Let's first understand old rules:

$\langle \psi_1 | \psi_2 \rangle_{\text{old rules}} =$

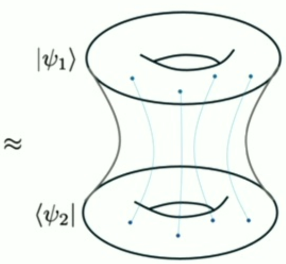


$$= \underbrace{\mathrm{Tr}\left(O_1^b \dots O_n^b e^{-\beta^b H^b}\right)}_{\text{Bra}} \underbrace{\mathrm{Tr}\left(O_1^k \dots O_{n'}^k e^{-\beta^k H^k}\right)}_{\text{Ket}}$$

$=$



$\approx$



$+$ lots of noise

The noise is large because the square has a large variance!
$\dim \mathcal{H} = 1$ because overlap matrix is simply $M_{ij} = v_i^* v_j$ with rank 1.

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How do we generalize this result in higher-dimensions?

Let's focus on an "observer": let $\mathcal{O}, \tilde{\mathcal{O}} \in \mathcal{A}_{\text{obs}}$ be simple bdy. operators associated to excitations that create an observer in the bulk.

$$\langle \psi_1 | \psi_2 \rangle_{\text{old rules}} = \text{Tr}_{\mathcal{H}_{\text{bra}} \otimes \mathcal{H}_{\text{ket}}} \left(\mathcal{O}^b \tilde{\mathcal{O}}^k O_1^b \ldots O_n^b e^{-\beta^b H^b} O_1^k \ldots O_{n'}^k e^{-\beta^k H^k} \right)$$

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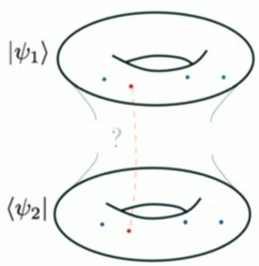
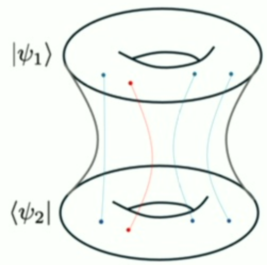
New rules: coarse-graining over the matrix elements of $\mathcal{O}$ and $\mathcal{O}'$:

$$\langle\psi_1|\psi_2\rangle_{\text{new rules}} = \text{Tr}_{\mathcal{H}_{\text{bra}}\otimes\mathcal{H}_{\text{ket}}}\left(\overline{\mathcal{O}\tilde{\mathcal{O}}} O_1^b \dots O_n^b e^{-\beta^b H^b} O_1^k \dots O_{n'}^k e^{-\beta^k H^k}\right)$$

$$= \sum_{E_1,E_2,E_1',E_2'} \underbrace{\overline{\langle E_1|\mathcal{O}|E_2\rangle\langle E_1'|\tilde{\mathcal{O}}|E_2'\rangle}}_{\text{Coarse-graining over OPE coefficients}}$$

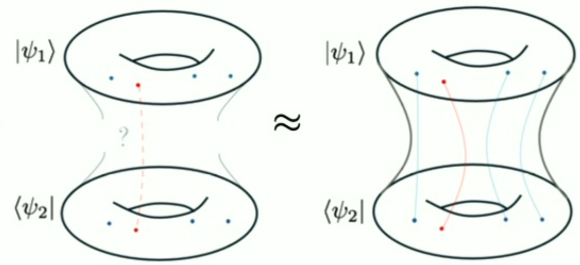
$$\times \langle E_2|O_1^b \dots O_n^b e^{-\beta^b E_1}|E_1\rangle\langle E_2'|O_1^k \dots O_n^k e^{-\beta^k E_1}|E_1'\rangle$$

$=$  $\approx$ 

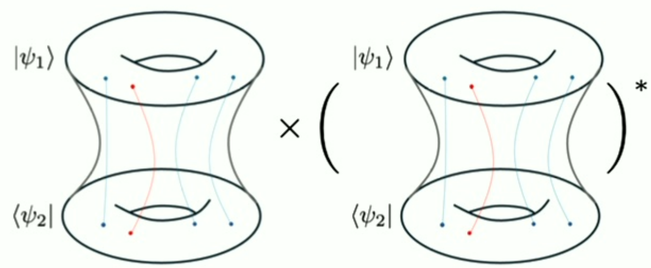
[similar Mark's talk - different motivation for coarse-graining]

How do we generalize this result in higher dimensions?

$$\langle\psi_1|\psi_2\rangle_{\text{new rules}} =$$



$$|\langle\psi_1|\psi_2\rangle_{\text{new rules}}|^2 =$$



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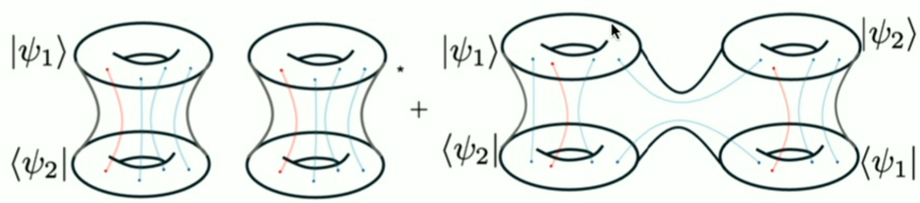
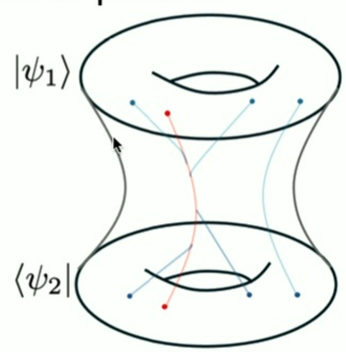


$+\dots$

Consequences

- Coarse-graining over $\mathcal{A}_{\text{obs}}$ is responsible for connecting bra and ket to get a connected closed universe & recover sensible $\langle\psi|\,\Pi\,|\psi\rangle$.
- From matrix of overlaps, dimension of the Hilbert space is now: $\dim(\mathcal{H}_{\text{bulk}}^{\text{obs POV}}) = \#_{\mathcal{A}_{\text{obs}}} \times \#^2_{\text{Energies}}$ when coarse-graining.
- HUZ: Average over operators in $\mathcal{A}_{\text{obs}}$.
  Here: Average over OPE coefficients $\overline{\langle E_1|\mathcal{O}|E_2\rangle\,\langle E_1'|\tilde{\mathcal{O}}|E_2'\rangle}$.
- In conventional AdS/CFT: coarse-graining for black holes, $\mathcal{H}_{\text{bulk}}^{\text{obs POV}} \neq \mathcal{H}_{CFT}$, while for vacuum AdS, $\mathcal{H}_{\text{bulk}}^{\text{obs POV}} = \mathcal{H}_{CFT}$.
- If $\mathcal{O} = \tilde{\mathcal{O}} = \text{Id}$ (no obs.), we find standard CFT inner-product.
- Beyond worldline approximation:

$$\overline{\langle E_1|\mathcal{O}|E_2\rangle\,\langle E_1'|\tilde{\mathcal{O}}|E_2'\rangle} \neq 0 \qquad \text{for} \qquad \mathcal{O} \neq \tilde{\mathcal{O}}:$$



Recap

- We proposed a modification of the rules of the GPI relevant when describing the experience of a gravitating observer
- With this new inner product, non-perturbative Hilbert space is much larger: non-trivial for closed universe and doubled for 2-sided BH
- Rules can be implemented by coarse-graining the bdy. CFT

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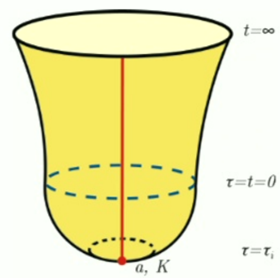
Future directions: de Sitter

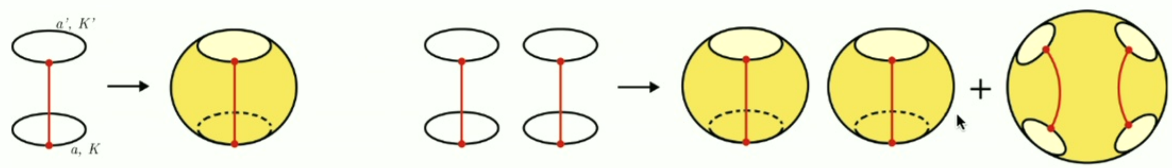
Can we provide an approximate state count for de Sitter?

- A generic state in de Sitter is specified by the length and extrinsic curvature of an initial Euclidean-dS_2, and the observer's energy



$\leftrightarrow |a, K, \Delta_O^{(i)}\rangle$

  - With our prescription, variance of overlap is suppressed by e^{-2S_0}: Hilbert space is large and not 1-dimensional



(a) (b)

  - Precise calculation? Observables? Even more generic spacetimes?