

Title: Operator Algebras and Third Quantization

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Collection/Series: QIQG 2025


Subject: Quantum Gravity, Quantum Information

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Abstract:

In quantum gravity, the gravitational path integral involves a sum over topologies, representing the joining and splitting of multiple universes. To account for topology change, one is led to allow the creation and annihilation of both closed and open universes in a framework often called third quantization or universe field theory. We argue that since topology change in gravity is a rare event, its contribution to late-time physics should be universally governed by a Poisson distribution. In the Fock space of closed baby universes, this Poisson distribution corresponds to the statistics of the number operator in a coherent state, whereas allowing for the creation of asymptotic open universes calls for a non-commutative generalization of a Poisson process. We propose such an operator algebraic framework, called Poissonization, which takes as input the observable algebra and a (unnormalized) state of a quantum system and outputs a von Neumann algebra represented on its symmetric Fock space. Physically, our construction is a generalization of the coherent state vacua of bipartite quantum systems.



Swing Surfaces in AdS/CFT

QIQG 2025

Sabrina Pasterski, Perimeter

Goal

- Explore a proposal for holographic entanglement entropy in asymptotically flat spacetimes by Wei Song and collaborators.
- Uplift their construction to AdS/CFT to understand its consequences.

based on work in progress with J. Caminiti & R.C. Myers
1410.4089 [A. Bagchi, R. Basu, D. Grumiller, M. Riegler]
1706.07552 [H. Jiang, W. Song, Q. Wen]

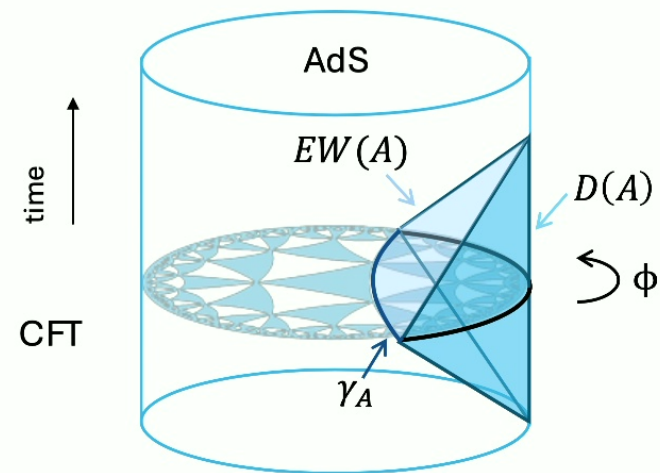
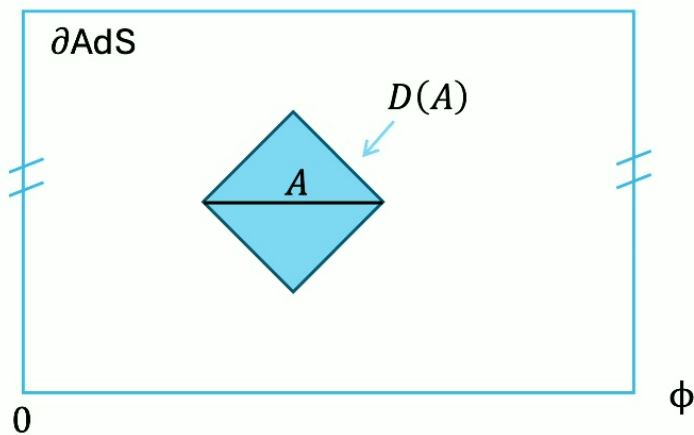


Road Map

1. Motivation
2. Taking the Flat Limit
3. Lifting the Swing Proposal
4. Discussion

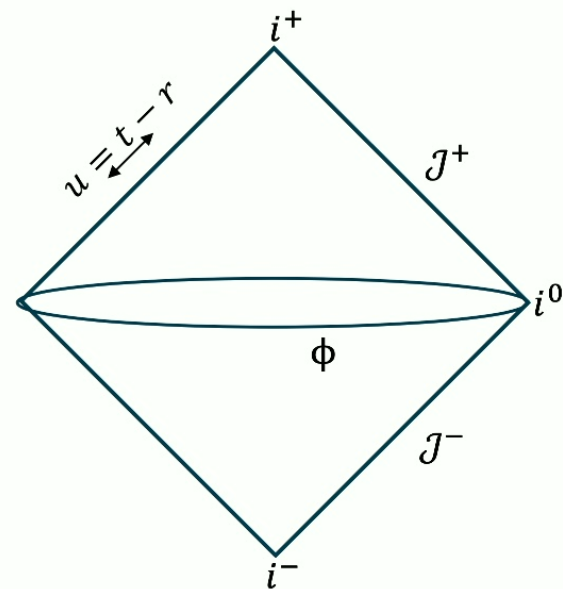
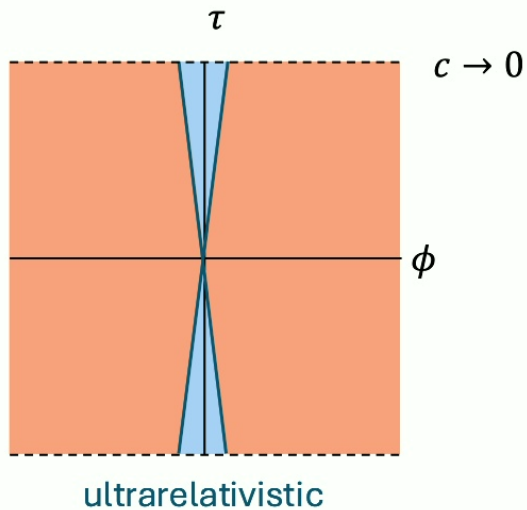
Motivation

In AdS/CFT we have seen that we can learn about deep connections between QI and GR by studying how the bulk geometry is encoded in the boundary.



Motivation

We don't understand flat holography very well, however in 3D there has been progress in understanding how the bulk could be dual to a 2D BMSFT at \mathcal{J}^+ .



Entanglement Entropy in BMSFT

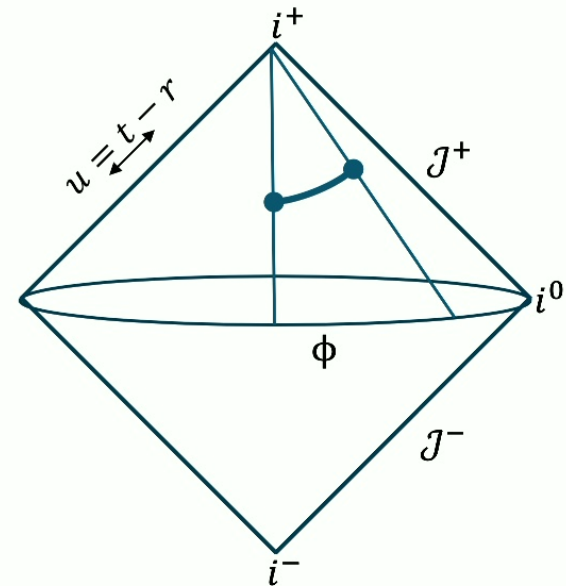
[Bagchi et al '14] examined entanglement entropies in 2D BMSFT. For an interval of size $(\Delta u, \Delta\phi)$ at \mathcal{J}^+ we have:

$$S_{EE}^{BMSFT}(A) = \frac{c_L}{6} \ln \left(2 \sin \frac{\Delta\phi}{2} \right) + \frac{c_M}{12} \Delta u \cot \frac{\Delta\phi}{2}$$

\uparrow
 $c_L = \frac{3}{\mu G}$

\uparrow
 $c_M = \frac{3}{G}$

(TMG)



Swing Proposal

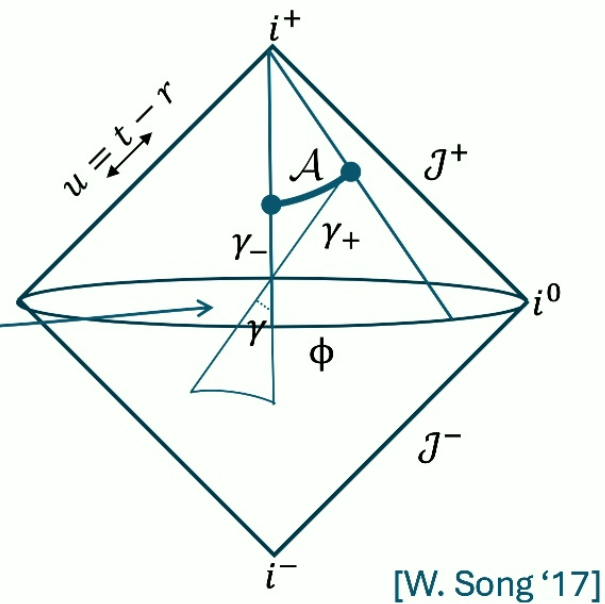
[Jiang et al '17] found a geometric quantity that matches this answer. We'll describe it for $c_L = 0$.

$$S_{EE}^{BMSFT}(A) = \frac{c_L}{6} \ln \left(2 \sin \frac{\Delta\phi}{2} \right) + \frac{c_M}{12} \Delta u \cot \frac{\Delta\phi}{2}$$

(The first term is crossed out with a blue line in the original image, and a blue arrow points from the second term to the diagram.)

For interval A on \mathcal{J}^+ :

- shoot radial null geodesics γ_{\pm} from ∂A
- identify extremal spacelike geodesic γ spanning γ_{\pm}
- $\text{len}(\gamma)$ matches the BMSFT EE





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Let's start with Global AdS

Meanwhile from the boundary point of view, this is a Carrollian limit. Namely sending $\ell \rightarrow \infty$ is like sending $c \rightarrow 0$.

$$ds^2 = -\left(\frac{r^2}{\ell^2} + 1\right)du^2 - 2dudr + r^2d\phi^2$$

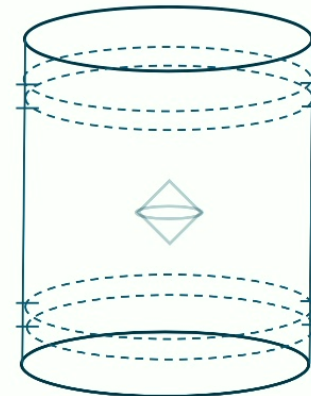
$$\xrightarrow{\ell \rightarrow \infty} -du^2 - 2dudr + r^2d\phi^2$$

$$\xrightarrow{r \rightarrow \infty} r^2\left(-\frac{1}{\ell^2}du^2 + d\phi^2\right)$$

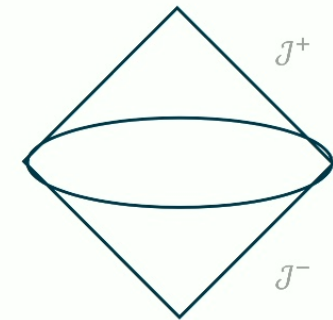
$$\uparrow \\ c = \frac{1}{\ell}$$

$$\tau = \frac{\pi}{2} + \frac{u}{\ell}$$

$$\tau = -\frac{\pi}{2} + \frac{v}{\ell}$$



$$\Lambda < 0$$



$$\Lambda = 0$$

Let's start with Global AdS

With an appropriate change of coordinates, we see that future null infinity maps to a small window near $\tau = \ell^{-1}t = \frac{\pi}{2}$.

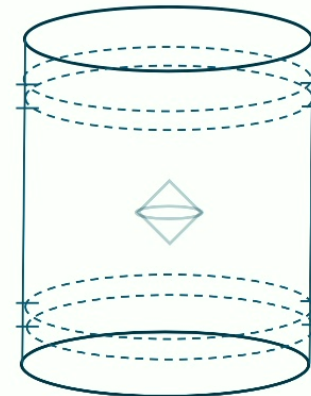
$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \frac{1}{1 + \frac{r^2}{\ell^2}} dr^2 + r^2 d\phi^2$$

$$\rightarrow = -\left(\frac{r^2}{\ell^2} + 1\right) du^2 - 2du dr + r^2 d\phi^2$$

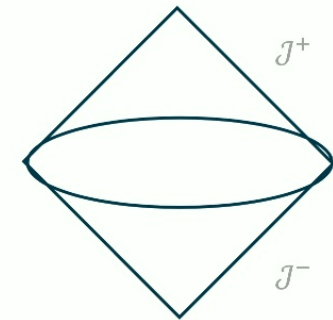
$$t = u + \ell \arctan \frac{r}{\ell}$$

$$\tau = \frac{\pi}{2} + \frac{u}{\ell}$$

$$\tau = -\frac{\pi}{2} + \frac{v}{\ell}$$



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$$\Lambda = 0$$

This extends beyond the global symmetries

For Topologically Massive Gravity (TMG) in AdS_3

$$S_{\text{TMG}} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) + \frac{1}{32\pi G \mu} \int d^3x \sqrt{-g} \epsilon^{\lambda\mu\nu} \Gamma_{\lambda\sigma}^{\rho} \left(\partial_{\mu} \Gamma_{\rho\nu}^{\sigma} + \frac{2}{3} \Gamma_{\mu\tau}^{\sigma} \Gamma_{\nu\rho}^{\tau} \right)$$

we have

$$\begin{aligned} [\mathcal{L}_n^{\pm}, \mathcal{L}_m^{\pm}] &= (n - m) \mathcal{L}_{n+m}^{\pm} + \frac{c^{\pm}}{12} n (n^2 - 1) \delta_{n+m,0} \\ [\mathcal{L}_n^{+}, \mathcal{L}_m^{-}] &= 0 \end{aligned} \quad \text{where} \quad c^{\pm} = \frac{3\ell}{2G} \left(1 \pm \frac{1}{\mu\ell} \right)$$

This extends beyond the global symmetries

Starting from the Virasoro generators in $\text{AdS}_3/\text{CFT}_2$ we can form the linear combinations

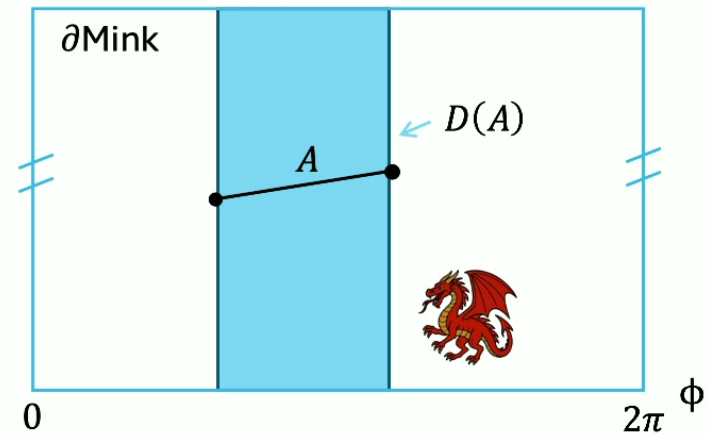
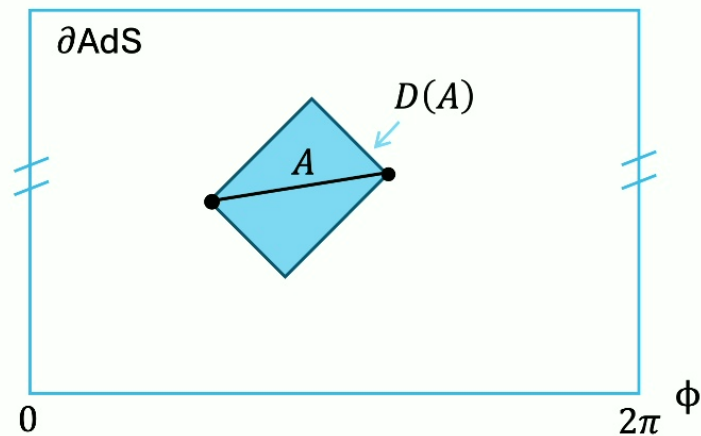
$$\begin{aligned} \text{superrotations} \longrightarrow \mathcal{L}_n &= \mathcal{L}_n^+ - \mathcal{L}_{-n}^-, \quad \mathcal{M}_n = \frac{1}{\ell} (\mathcal{L}_n^+ + \mathcal{L}_{-n}^-) \longleftarrow \text{supertranslations} \\ c_L &= c^+ - c^-, \quad c_M = \frac{1}{\ell} (c^+ + c^-) \end{aligned}$$

which limit to the BMS algebra when we take ℓ large

$$\begin{aligned} [\mathcal{L}_n, \mathcal{L}_m] &= (n-m)\mathcal{L}_{n+m} + \frac{c_L}{12} n(n^2-1) \delta_{n+m,0} \longleftarrow c_L = \frac{3}{\mu G} \\ [\mathcal{L}_n, \mathcal{M}_m] &= (n-m)\mathcal{M}_{n+m} + \frac{c_M}{12} n(n^2-1) \delta_{n+m,0} \longleftarrow c_M = \frac{3}{G} \\ [\mathcal{M}_n, \mathcal{M}_m] &= 0 \end{aligned} \quad \text{(dimensionful)}$$

An AFS/BMSFT Correspondence?

It thus seems natural that the holographic dual of asymptotically flat 3D gravity could be a BMS field theory living on null infinity.



ultralocal, different entanglement structure?

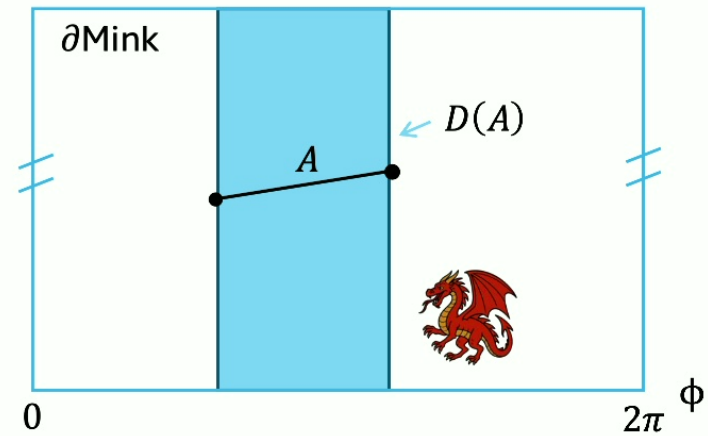
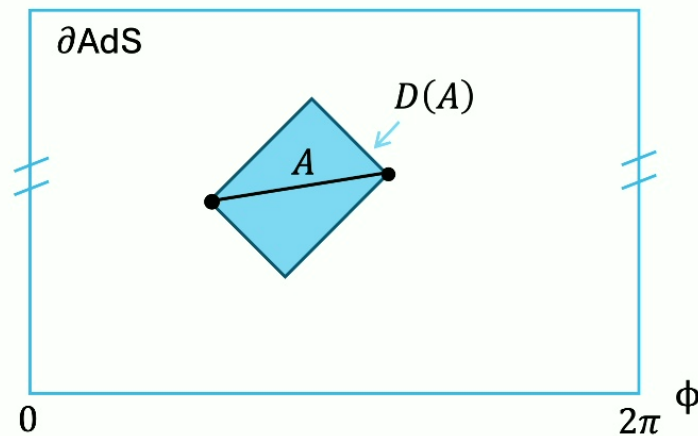


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superrotations \longrightarrow $\mathcal{L}_n = \mathcal{L}_n^+ - \mathcal{L}_{-n}^-$ $\mathcal{M}_n = \frac{1}{\ell} (\mathcal{L}_n^+ + \mathcal{L}_{-n}^-)$ \longleftarrow supertranslations

$c_L = c^+ - c^-$, $c_M = \frac{1}{\ell} (c^+ + c^-)$

here be dragons 

which limit to the BMS algebra when we take ℓ large

$$[\mathcal{L}_n, \mathcal{L}_m] = (n - m) \mathcal{L}_{n+m} + \frac{c_L}{12} n (n^2 - 1) \delta_{n+m,0} \quad \longleftarrow \quad c_L = \frac{3}{\mu G}$$

$$[\mathcal{L}_n, \mathcal{M}_m] = (n - m) \mathcal{M}_{n+m} + \frac{c_M}{12} n (n^2 - 1) \delta_{n+m,0} \quad \longleftarrow \quad c_M = \frac{3}{G}$$

$$[\mathcal{M}_n, \mathcal{M}_m] = 0$$



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Looking back at the Swing Proposal

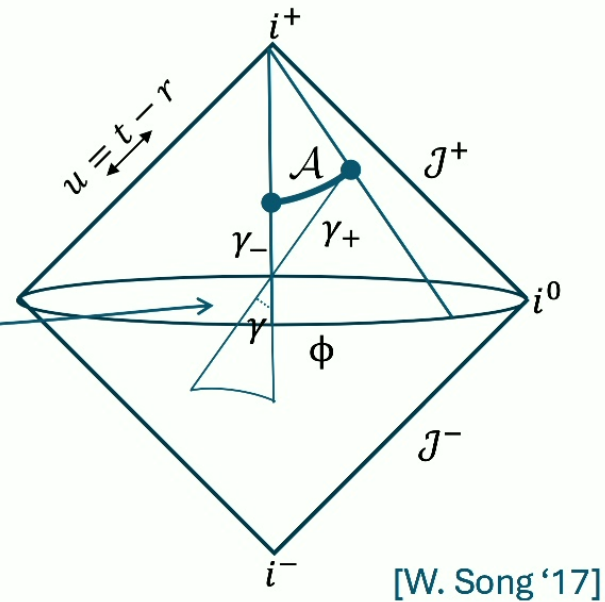
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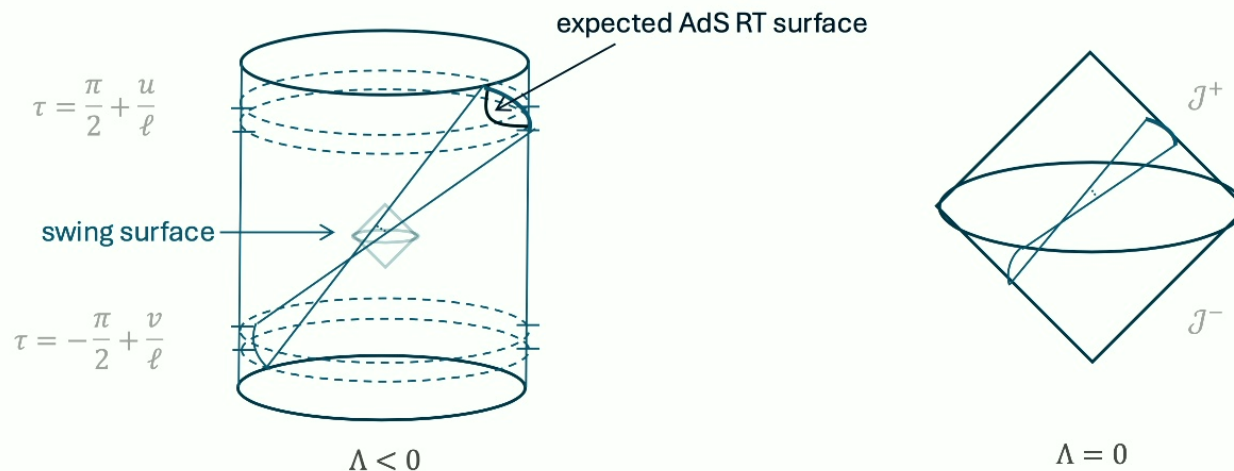
For interval A on \mathcal{J}^+ :

- shoot radial null geodesics γ_{\pm} from ∂A
- identify extremal spacelike geodesic γ spanning γ_{\pm}
- $\text{len}(\gamma)$ matches the BMSFT EE



We can now try to lift it to AdS

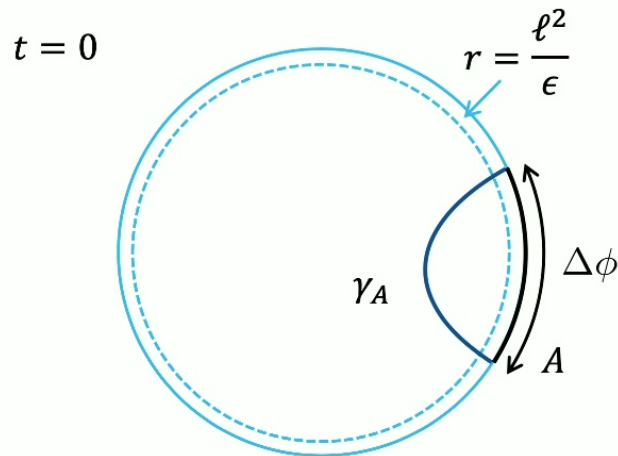
We saw that future null infinity maps to a small window near $\tau = \frac{\pi}{2}$.
We can still send in radial geodesics and construct the swing.



RT proposal in AdS/CFT

For the usual RT proposal we have $S(A) = \underset{V \sim A}{\text{ext}} \frac{A_V}{4G_N}$.

And indeed the (regulated) length of the geodesic between ∂A ...

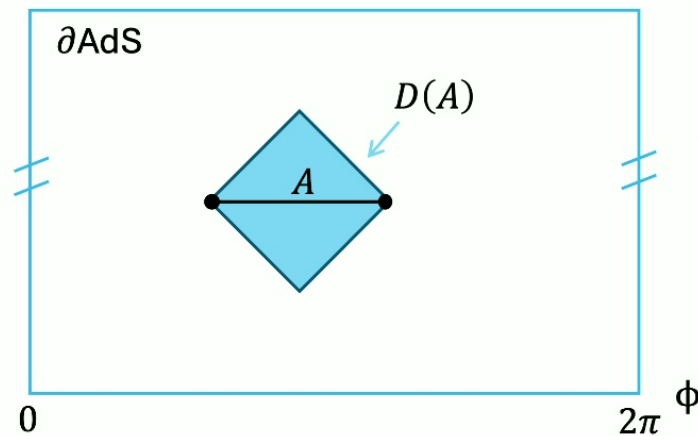


$$ds^2 = - \left(1 + \frac{r^2}{\ell^2} \right) dt^2 + \frac{1}{1 + \frac{r^2}{\ell^2}} dr^2 + r^2 d\phi^2$$

$$\text{len}(\gamma_A) = 2\ell \ln \left(\frac{2\ell}{\epsilon} \sin \frac{\Delta\phi}{2} \right)$$

RT proposal in AdS/CFT

... matches the entanglement entropy on the boundary.



$$S_A = - \lim_{n \rightarrow 1} \frac{\partial}{\partial n} \text{Tr} \rho_A^n$$

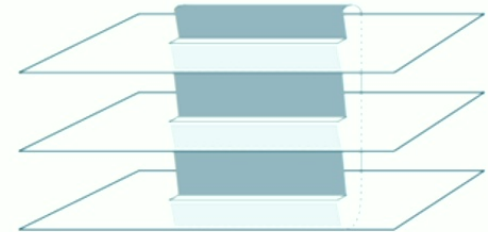
$$= \frac{c}{6} \ln \left(\frac{2\ell}{\epsilon} \sin \frac{\Delta\phi}{2} \right)$$

$$\uparrow = \frac{\text{len}(\gamma_A)}{4G} \text{ if } c = \frac{3\ell}{2G}$$

[Brown, Henneaux '86]

$$\langle T(w) \rangle_{\mathcal{R}_n} = \frac{\langle T(w) \sigma_n(\phi_1) \bar{\sigma}_n(\phi_2) \rangle_{\mathbb{C}}}{\langle \sigma_n(\phi_1) \bar{\sigma}_n(\phi_2) \rangle_{\mathbb{C}}}$$

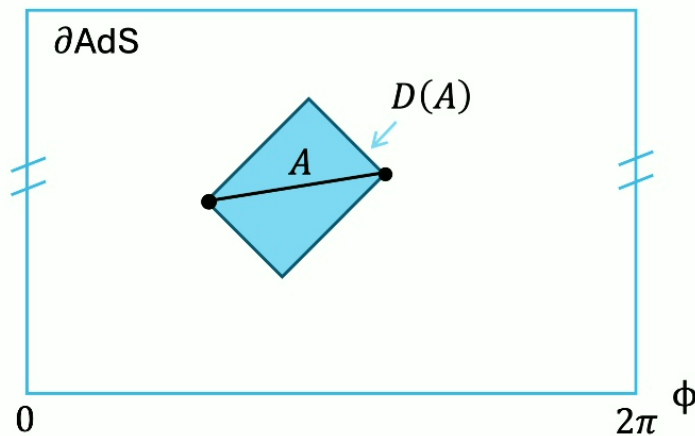
$$\Delta_{\sigma_n} = \frac{c}{12} \left(n - \frac{1}{n} \right)$$



[Calabrese, Cardy '09]

RT proposal in AdS/CFT

In the case of TMG, we would find that the entanglement entropy for a general interval on the cylinder would be:



$$S_A = S_A^+ + S_A^-$$

$$S_A^\pm = \frac{c^\pm}{6} \ln \left(\frac{2\ell}{\epsilon} \sin \frac{\Delta x^\pm}{2} \right)$$

$$\Delta x^\pm = \Delta\phi \pm \Delta\tau$$

Where did BMSFT EE come from?

Under the Carrollian contraction, Virasoro primaries are different than BMS primaries

$$\begin{array}{l} L_{n>0} |h, \bar{h}\rangle = 0 \\ \bar{L}_{n>0} |h, \bar{h}\rangle = 0 \end{array} \not\Rightarrow \begin{array}{l} \mathcal{L}_{n>0} |h_M, h_L\rangle = 0 \\ \mathcal{M}_{n>0} |h_M, h_L\rangle = 0 \end{array}$$

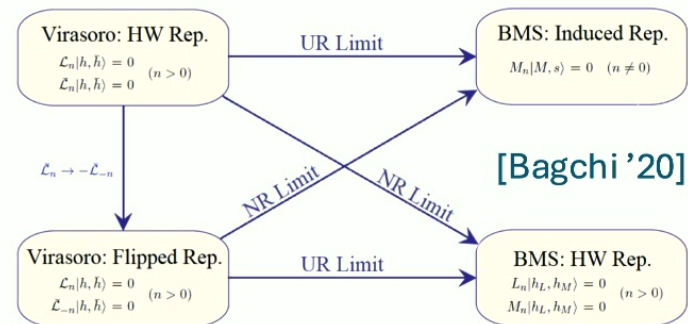
while

$$\begin{array}{l} L_{n>0} |h, \bar{h}\rangle = 0 \\ \bar{L}_{n<0} |h, \bar{h}\rangle = 0 \end{array} \Rightarrow \begin{array}{l} \mathcal{L}_{n>0} |h_M, h_L\rangle = 0 \\ \mathcal{M}_{n>0} |h_M, h_L\rangle = 0 \end{array}$$

Where did the BMSFT EE come from?

To get BMS primaries from the Carrollian contraction of Virasoro primaries one could use the “automorphism”

$$\bar{L}_n \rightarrow -\bar{L}_{-n}, \quad c^- \rightarrow -c^-$$



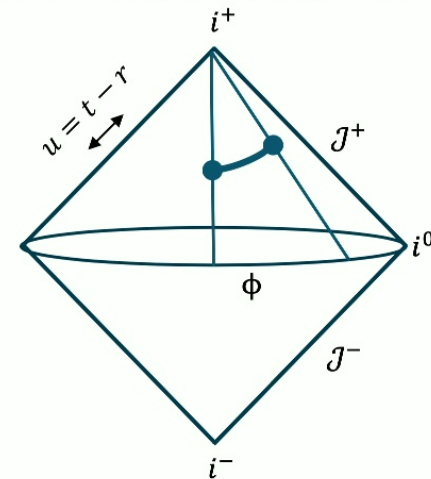
which exchanges highest and lowest weight conditions for the \bar{L}_n .

Entanglement Entropy in BMSFT

[Bagchi et al '14] were using this relation between contractions to get their entanglement entropies.

$$S_{EE}^{BMSFT}(A) = \frac{c_L}{6} \ln \left(2 \sin \frac{\Delta\phi}{2} \right) + \frac{c_M}{12} \Delta u \cot \frac{\Delta\phi}{2}$$

$$c_L = c^+ - c^- = 0 \quad c_M = \frac{c^+ + c^-}{\ell} = \frac{3}{G}$$



They demand their twist operators are primaries under BMS algebra we get from a Carrollian contraction with $h_L = 0$, $h_M = \frac{c_M}{24} \left(n - \frac{1}{n} \right)$

Be careful with the change of frame!

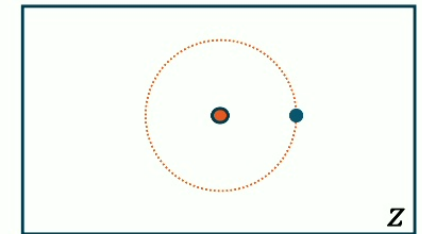
To prepare our primary states we want to put our operators at $z=0$.

$$L_{n>0} |h, \bar{h}\rangle = 0 \quad \bar{L}_{n>0} |h, \bar{h}\rangle = 0$$

From the exponential map

$$\bar{z} = e^{\bar{w}} = e^{-iy^-}, \quad y^- = \phi - \frac{u}{\ell}, \quad \bar{L}_n = -\bar{z}^{n+1} \partial_{\bar{z}}$$

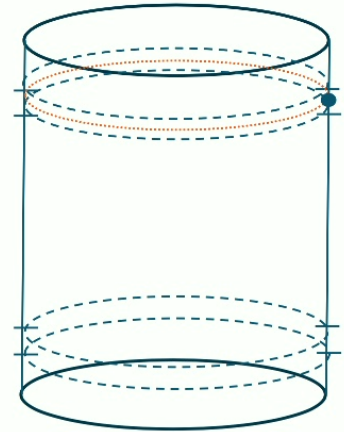
we see that inverting $\bar{z} \mapsto \bar{z}^{-1}$ corresponds to exchanging $u \leftrightarrow \phi$.



Be careful with the change of frame!

An operator at $(u, \phi) = (0, 0)$ is at $(z, \bar{z}) = (1, 1)$

$$Z = \frac{z - 1}{z + 1} : \quad \begin{aligned} Z(1) &= 0 \\ \ell_n &= -Z^{n+1} \partial_Z = W L_n W^{-1} \end{aligned}$$



The change of frame that puts our usual twist operators there conjugates us to a different BMS subalgebra.

$$\left\{ \begin{aligned} l_n &= \ell_n + (-1)^n \bar{\ell}_n & c_L &= \frac{3\ell}{G}, c_M = 0 \\ m_n &= \frac{1}{\ell} (\ell_n - (-1)^n \bar{\ell}_n) \end{aligned} \right. \quad \text{have a finite flat limit}$$

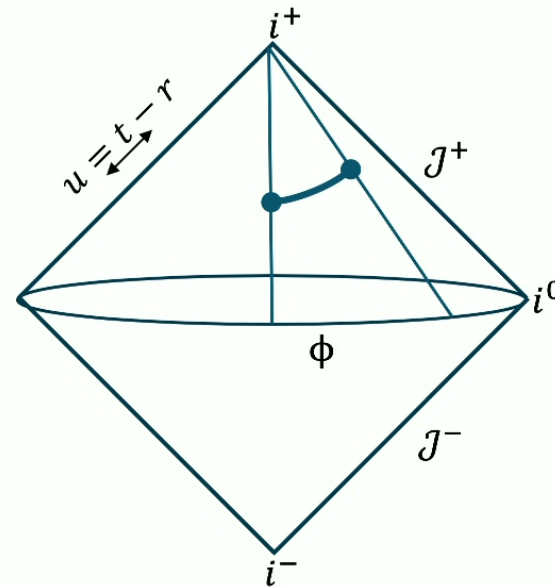
Entanglement Entropy at \mathcal{J}^+

Following the same steps in [Bagchi et al '14] with this contraction gives the usual (albeit divergent) RT formula as we send $\ell \rightarrow \infty$.

$$S_{EE}^{BMSFT}(A) = \frac{c_L}{6} \ln \left(2 \sin \frac{\Delta\phi}{2} \right) + \frac{c_M}{12} \Delta u \cot \frac{\Delta\phi}{2}$$

$$c_L = c^+ + c^- = \frac{3\ell}{G} \quad c_M = \frac{c^+ - c^-}{\ell} = 0$$

Under this BMS subalgebra the twist operators have weights $h_l = \frac{c}{12} \left(n - \frac{1}{n} \right)$, $h_m = 0$



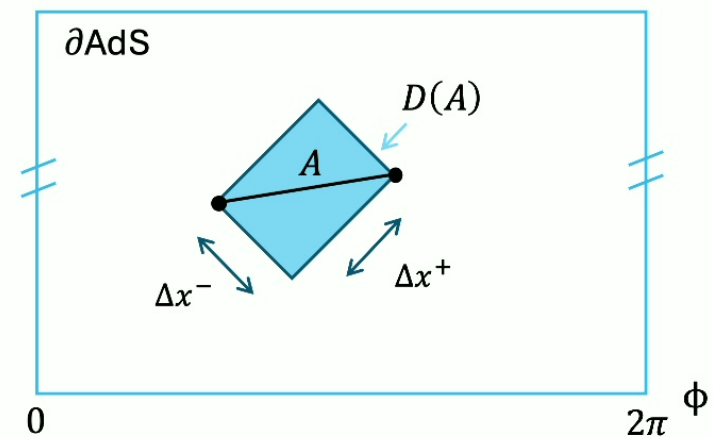
What do we have?

Rather than the usual EE given by

$$S_A = S_A^+ + S_A^- \quad S_A^\pm = \frac{c^\pm}{6} \ln \left(\frac{2\ell}{\epsilon} \sin \frac{\Delta x^\pm}{2} \right)$$

the “BMSFT EE” is given by

$$S_{BMS} = S_A^+ - S_A^-$$



What do we have?

Rather than the usual EE given by

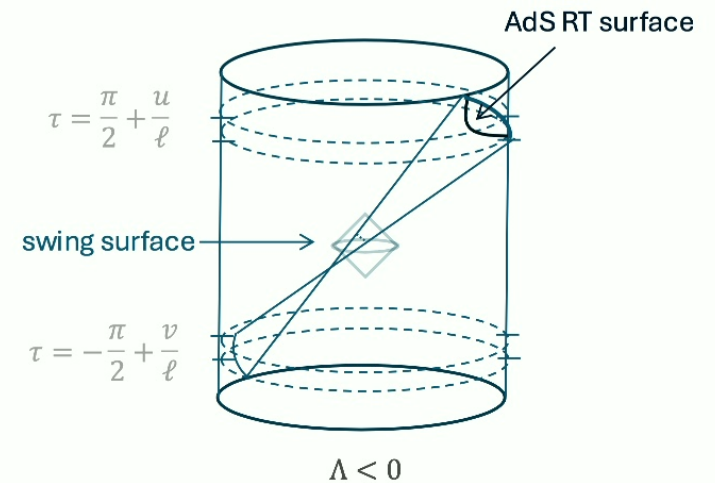
$$S_A = S_A^+ + S_A^- \quad S_A^\pm = \frac{c^\pm}{6} \ln \left(\frac{2\ell}{\epsilon} \sin \frac{\Delta x^\pm}{2} \right)$$

the “BMSFT EE” is given by

$$S_{BMS} = S_A^+ - S_A^-$$

Meanwhile the swing bench is given by

$$\frac{A(\text{swing})}{4G} = |S_A^+ - S_A^-|$$



We would like to know...

- What is this BMSFT EE?
- Why does it match the Swing bench?
- What can we learn from these flat space observations about AdS?

What is this Quantity?

We saw that the “BMSFT EE” amounted to flipping the relative sign between left and right moving contributions

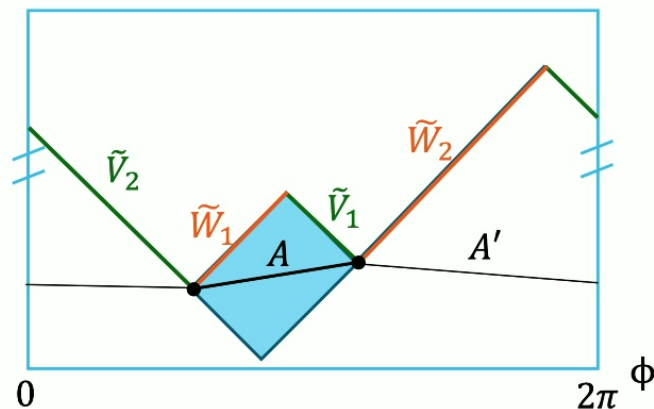
$$S_{EE}(A) = -\text{tr } \rho_A \log \rho_A = \langle K \rangle, \quad \rho_A \propto e^{-K}, \quad K = K_L + K_R$$

$$S_{EE}^{BMSFT} = \lim_{\ell \rightarrow \infty} S_{EE}^+(A) - S_{EE}^-(A) = \langle K_R - K_L \rangle$$

Which we can write as the expectation value of a spacelike modular flow.

What is this Quantity?

For a single interval in global AdS it can be interpreted as a mutual information.



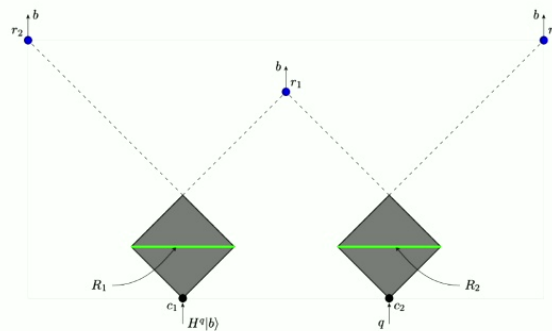
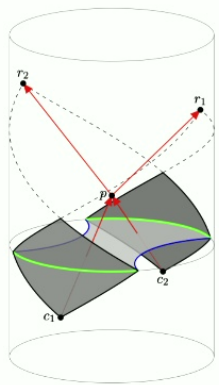
$$S_{EE}^+(A) = S_{EE}(\tilde{V}_1) = \frac{1}{2}(S_{\tilde{V}_1} + S_{\tilde{V}_2}),$$

$$S_{EE}^-(A) = S_{EE}(\tilde{W}_1) = \frac{1}{2}(S_{\tilde{W}_1} + S_{\tilde{W}_2})$$

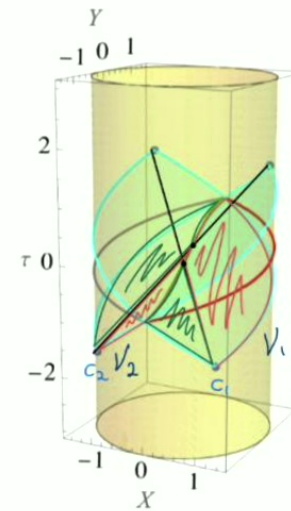
$$|S_{EE}^-(A) - S_{EE}^+(A)| = \frac{1}{2} |S_{\tilde{W}_1} + S_{\tilde{W}_2} - S_{\tilde{V}_1} - S_{\tilde{V}_2}| = \frac{1}{2} \max \left\{ I(\tilde{V}_1 : \tilde{V}_2), I(\tilde{W}_1 : \tilde{W}_2) \right\}$$

What is this Quantity?

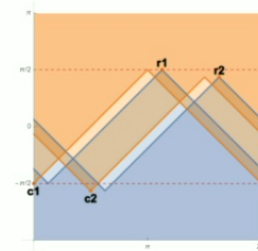
Which we are led to by looking at a very collinear limit of an AdS scattering problem...



boundary entanglement
mediates bulk scattering

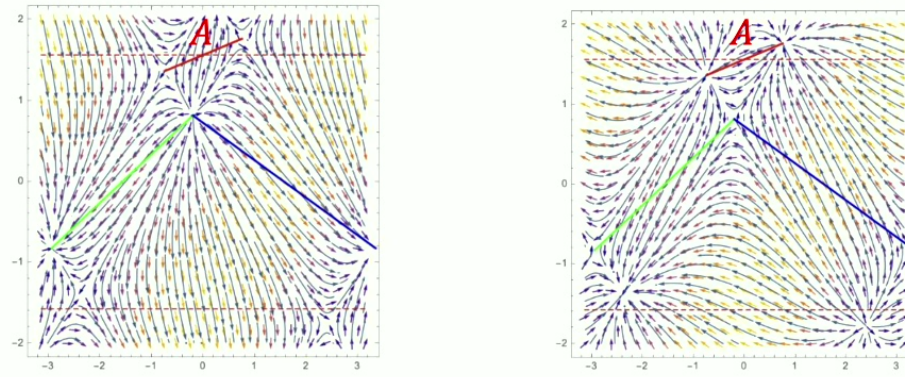


swing = ridge in proof
from [May et al. '19]



Why Does the Swing Match?

These spacelike modular flows were what was used by Wei Song and collaborators to motivate the swing proposal.



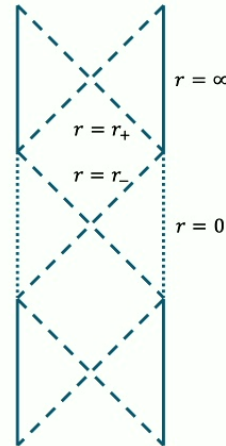
Following an analog of the CHM method, the swing bench maps to the segment of the *inner horizon* of a topological black hole.

Lessons from Flat?

Indeed, another encouraging result from AFS/BMSFT investigations is that the Flat Space Cosmology (FSC) entropy matches the flat limit of the inner horizon.

$$ds_{\text{BTZ}}^2 = \left(8GM - \frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{-8GM + \frac{r^2}{\ell^2} + \frac{16G^2 J^2}{r^2}} - 8GJ dt d\phi + r^2 d\phi^2$$

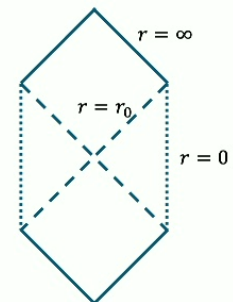
$$M = \frac{r_+^2 + r_-^2}{8G\ell^2}, \quad J = \frac{r_+ r_-}{4G\ell}$$



spinning BTZ

$$ds^2 = \Theta(\psi) du^2 - 2dr du + 2 \left[\Xi(\psi) + \frac{u}{2} \Theta' \right] d\psi du + r^2 d\psi^2$$

$$\Theta = 8GM \text{ and } \Xi = 4GJ$$



FSC

Lessons from Flat?

Curiously, these spacelike modular flows also appear as subleading saddles in the Cardy formula.

$$D(h, \bar{h}) = \int d\tau d\bar{\tau} e^{2\pi i \left(-h\tau + \frac{\tau c}{24} + \frac{c}{24\tau} \right)} e^{2\pi i \left(\bar{h}\bar{\tau} - \frac{\bar{\tau} \bar{c}}{24} - \frac{\bar{c}}{24\bar{\tau}} \right)} Z(-1/\tau, -1/\bar{\tau})$$

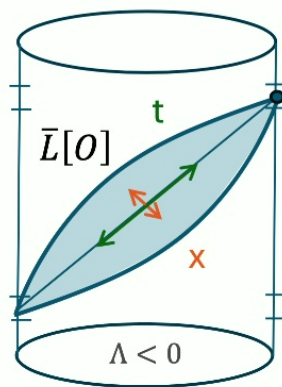
stationary at

$$\begin{cases} (\tau, \bar{\tau}) = \left(\pm i \sqrt{\frac{c}{24h}}, \pm i \sqrt{\frac{\bar{c}}{24\bar{h}}} \right) \\ (\tau, \bar{\tau}) = \left(\pm i \sqrt{\frac{c}{24h}}, \mp i \sqrt{\frac{\bar{c}}{24\bar{h}}} \right) \end{cases}$$

The matching of the FSC entropy to the flat limit of the inner horizon area works by the fact that the flat limit eliminates the leading saddle.

Lessons from Flat?

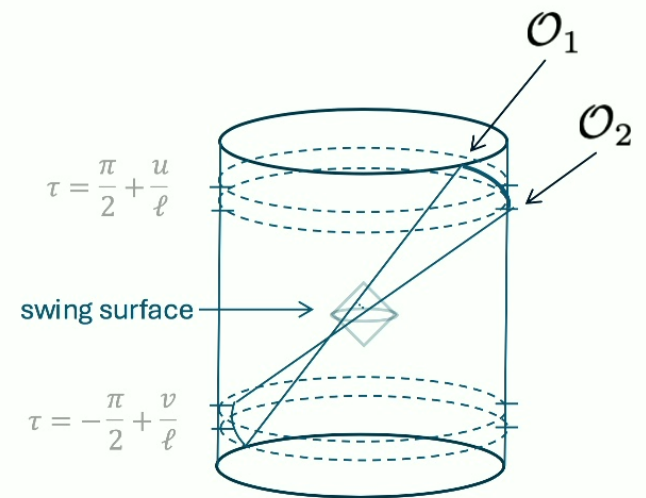
Meanwhile we can recast the operators preparing the swing ropes in terms of light ray operators on the conformal boundary.



from symmetries

$$\mathcal{O}(0,0) = \int dt \Phi \left(\ell \cosh x, \frac{t}{2}, \frac{t}{2}, \ell \sinh x \right)$$

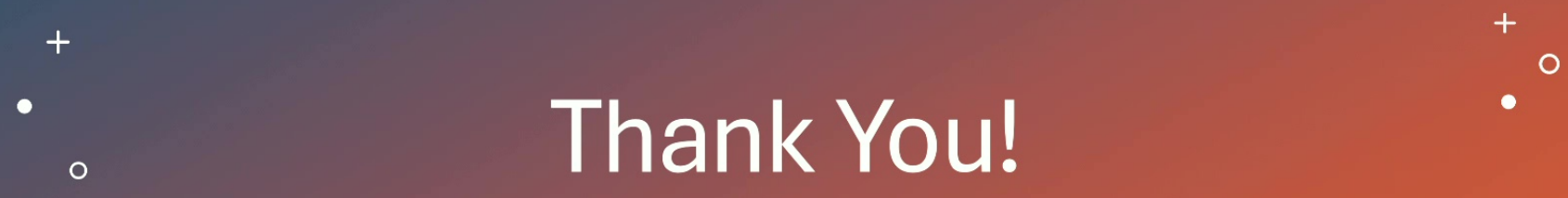
$$\stackrel{?}{=} c_1 e^{-x(\Delta-1)} \bar{L}[O] + c_2 e^{x(\Delta-1)} L[O]$$



Do these help us probe sub-AdS scale physics?

Takeaways Next Steps

1. How do we generalize these interpretations beyond simple examples?
2. How much of this generalizes to higher dimensions?
3. Are there other geometric probes of entanglement relevant to scattering?
4. How do we probe sub-AdS scale physics in our boundary dual?
5. How can we probe inner horizon thermodynamics from the AdS boundary?



Thank You!

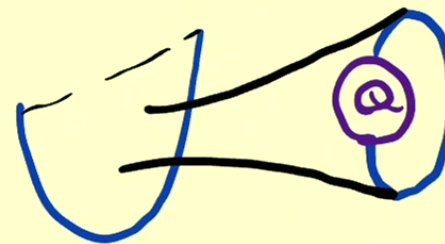
QIQG 2025

Operator Algebras & Third Quantization

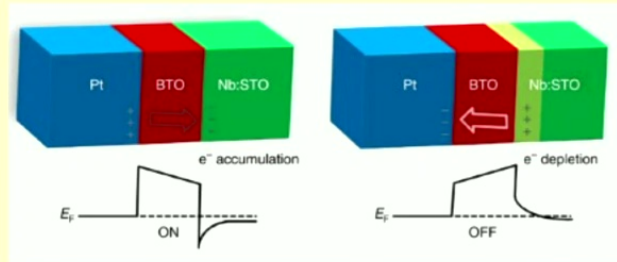


Nima Lashkari
Purdue University

with Yidong Chen, Marius Junge 2506. xxxxx

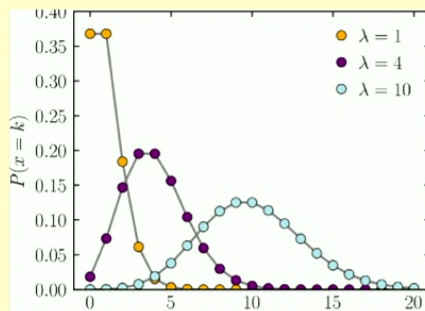


Shot noise in tunnel junctions



Electrons tunneling through on potential barrier is a **rare & discrete** process

Probability of K electrons tunneling (K events) is a **Poisson distribution**



$$P_{\lambda}(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Shot noise has **Poisson distribution**
Thermal noise has **Gaussian distribution**

What I'll say today

- In QG topology fluctuation is a **rare** event
it contributes a **universal term** to
late-time physics controlled by a **Poisson distribution**
- In QG, it's natural to allow for creation/annihilation
of **closed baby universes** as well as
asymptotic open universes.

(3rd quantization/ Universe field theory)

its **universal** contribution **late-time** physics
is a "**non-Commutative Poisson process**"
(Poissonization)

What I can tell you offline

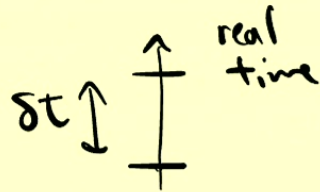
- Poissonization is a generalization of bipartite quantum coherent states

- Examples:

1. Marolf/Maxfield \rightarrow EoW branes
2. 2d open/closed TQFT
3. Quantum Chaotic theories

Law of rare events & Universal Poisson distribution

single rare event:



e. g. a collection of radioactive atoms
emitting radiation in some time interval
has very small probability

e.g. alpha decay

$$P \sim \exp(-2S_E/\hbar)$$

quantum tunneling

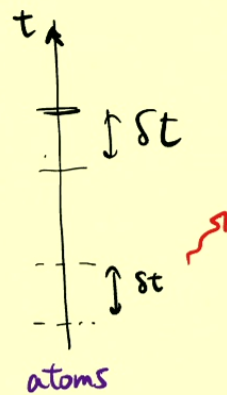
it's rare so we can ignore occurrence
of multiple events in δt

Poisson limit theorem

Multiple rare events:

If emissions at different times δt_k are **independent** at late times:

$T = e^{-\lambda \delta t / \hbar} \delta t$ the prob. of emission becomes order one.



$$P(k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

Poisson limit theorem: the total # of emissions at late time is **universally** described by a **Poisson distribution**

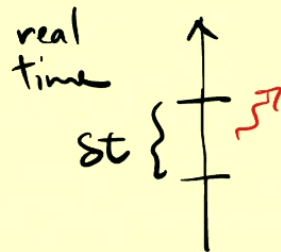
Assumptions

Rare, independent
discrete events

Universality

weak time-dependence
weak correlations in time

Rare quantum events & universal coherent



single rare event

→ Single particle
Hilbert space

$$\mathcal{H}_{\text{atoms}} \otimes \mathcal{H}_{\text{radiation}}$$

small prob. of emission

$$H_{\text{int}} = g(\mathbb{0} \otimes \sigma_x)$$

$$\begin{aligned} & \xleftarrow{\mathcal{H}_{\text{atoms}}} \langle E', 1 | e^{-i(H_{\text{parent}} + H_{\text{int}})\Delta t} | E, 0 \rangle \\ & \quad \downarrow \quad \quad \quad \downarrow \\ & \quad 1 \text{ photon} \quad \quad \quad \text{no photon} \\ & = g\Delta t e^{-iE\Delta t} \langle E' | 0 | E \rangle + \dots \\ & \quad \downarrow \\ & g\Delta t \sim e^{-S} \ll 1 \end{aligned}$$

Multiple events → Symmetric Fock

Multiple events \rightarrow Symmetric Fock space of photons

$\mathcal{H}_{\text{atoms}} \otimes \left(\bigotimes_k \mathcal{H}_k \right)$
 \hookrightarrow independent events

Fock space of photons
 $\mathcal{F}_{\text{photons}} = \bigoplus_n \mathcal{H}^{\otimes n, \text{sym}}$
 \nwarrow indistinguishable

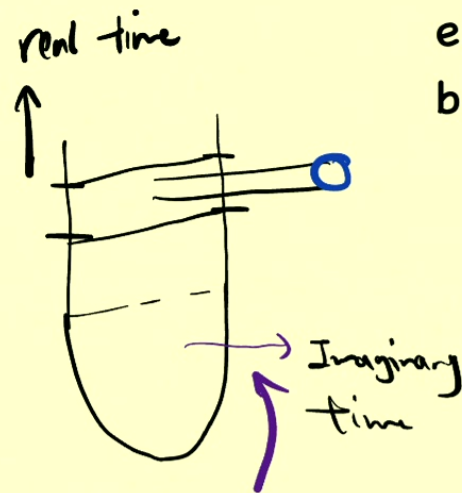
assume $\Omega = 1$ then at times
 $T \sim e^{2S_0}$ the state is **universally** described by



Coherent state $|W(\sqrt{T}e^{-S_0})\rangle$

$|\langle n | W(\sqrt{T}e^{-S_0}) \rangle|^2 = p_{Te^{-2S_0}}^{(n)} \rightarrow$ Poisson distribution

Topology fluctuations are rare quantum events



An entangled pair of
CFTs in
|TFD> state

emission/absorption of a closed
baby universe



Rare

prob. $\exp(-2S_E/G_N)$

Discrete

Topology

dilute gas approx.

Independent of instantons

At late time

$$t \sim \exp(2S_E/G_N)$$

we expect a **universal Poisson**
contribution due to baby universes

Comments

Comments

- This Poisson contribution is often overwhelmed with **Gaussian/dynamical** noise.

That's why we focus on **topological** theories to isolate it.

- In a system with interactions the **Universal Poisson** term is the physics of the diagonal term.

$$\left\langle \sum_{E \neq E'} e^{i(E-E')t} \right\rangle = \underbrace{\sum_E 1}_{\text{diagonal term: Poisson stat.}} + \left\langle \sum_{E \neq E'} e^{i(E-E')t} \right\rangle$$

diagonal term: Poisson stat.

at late time this term is irrelevant

term is irrelevant

(Plateau in spectral form factor.)

simple \rightarrow $\langle E' | O | E \rangle \approx \frac{f(O)}{E} \delta(E-E')$ \leftarrow only the diagonal term

long-time average $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt O(t)$

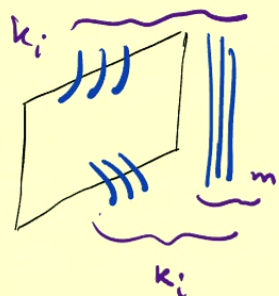
How large should T be?

depends on typical eigenvalue gap

Poisson limit theorem: It's not longer

than $\exp(2S/\hbar)$

Summing over instantons & baby universes

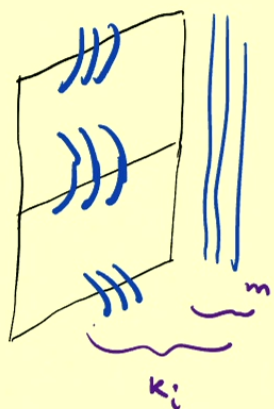


parent universe
probe limit

(Coleman; Giddings, Strominger;...]

$$\langle k_f | e^{-HT} | k_i \rangle_m = \frac{\sqrt{k_i! k_f!}}{m!} \frac{z^{k_i + k_f - m}}{(k_i - m)! (k_f - m)!}$$

$$z = e^{-S_E} \sqrt{V T} \equiv \sqrt{\lambda}$$

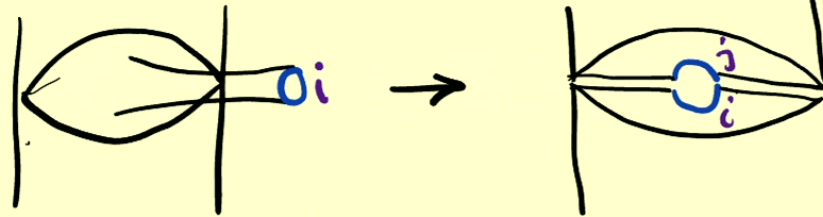


$$|\langle k_f | e^{-HT} | k_i \rangle_m|^2 = \binom{k_i}{m} \binom{k_f}{m} \underbrace{P_\lambda(k_f - m)}_{\text{Poisson distributions}} \underbrace{P_\lambda(k_i - m)}_{\text{Poisson distributions}}$$

Poisson distributions

Emission/Absorption of open universes

real time
↑



1st order process
emission/absorption
of a **closed baby**
universe

$$a_i^+$$

2nd order process
emission followed by
the reabsorption
of **open universes**

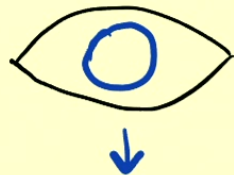
$$a_j a_i^+$$

Marolf-Maxfield example

Beyond probe limit

Third quantization

Universe field theory

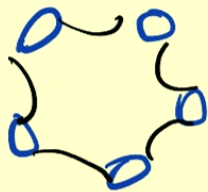


Baby universes have no labels
the Hartle-Hawking vacuum is

$$a^\dagger a = \hat{N}$$

the coherent state

$$|w(\lambda)\rangle = e^{i\lambda(a+a^\dagger)}|0\rangle$$



$$\hat{N}|w(\lambda)\rangle$$

Creates a closed boundary

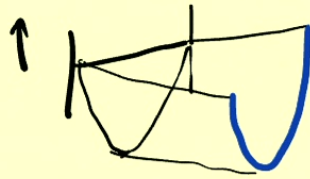
$$= \langle w(\lambda) | \hat{N}^p | w(\lambda) \rangle_{\text{Conn}} = \lambda$$

Poisson
distribution

independent of "p".

Emission/Absorption of asymptotic open universes

real time




Each open universe is an entangled pair of boundary CFTs labelled by

$$\boxed{i \times k} \cup = i \cup k$$

we need creation/annihilation operators

$$[a_{ik}, a_{jk'}^+] = \delta_{ij} \delta_{kh'}$$

Third quantization of open universes



$$\rightarrow \lambda(Q) = \sum_{ijk} Q_{ij} a_{ik}^+ a_{jk}$$

Second order process
involves creation & reabsorption
of asymptotic open Universes




$$[\lambda(Q_1), \lambda(Q_2)] = \lambda([Q_1, Q_2])$$

Non-commutative algebra

Non-commutative Poisson process

Hartle-Hawking state

$$|W(\omega^{\frac{1}{2}})\rangle = e^{\sum_i \omega_i^{\frac{1}{2}} a_i^\dagger a_i} |\Omega\rangle$$

$\lambda(Q) |W(\omega^{\frac{1}{2}})\rangle$ creates a universe with 

$$\langle W(\omega^{\frac{1}{2}}) | \lambda(Q_1) \dots \lambda(Q_p) | W(\omega^{\frac{1}{2}}) \rangle_{\text{Com}} = \omega(Q_1 \dots Q_p)$$

If we choose all Q_i to be the same we are back to the Abelian example & a multi-mode coherent state

Mathematics of Decoionization

Mathematics of Poissonization

Non-Commutative generalization of Poisson Process

Single event	Input:	von Neumann algebra	$\mathcal{Q} \in \mathcal{A}$
		weight	ω
single-particle Hilbert space	represented on	$\mathcal{H} \leftarrow$ e.g. $K \otimes K^*$	
Multiple events:	Output:	the algebra generated by	
Fock space	$\lambda(\mathcal{Q})$	$\mathcal{F}_{\mathcal{H}} = \bigoplus_n \mathcal{H}^{\otimes n \text{ sym}}$	

the analog of Hartle-Hawking state is the **coherent state**

$$|W(\omega^k)\rangle$$

Key formula:

Fock space
correlator

$$\langle W(\omega^k) | \lambda(\Phi_1) \dots \lambda(\Phi_p) | W(\omega^k) \rangle_{\text{conn}}$$

$$= \omega(\Phi_1 \dots \Phi_p)$$

1- particle correlator

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