Title: Does connected wedge imply distillable entanglement?

Speakers: Takato Mori

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Subject: Quantum Gravity, Quantum Information

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Abstract:

In holography, when two boundary subsystems have large mutual information, they are connected by their entanglement wedge. However, it remains mysterious whether these subsystems are EPR-like entangled. In this talk, I resolve this problem by finding bulk duals of one-shot distillable entanglement. Namely, I show that in one-shot scenarios: i) there is no distillable entanglement only by local operations at leading order in G_N, suggesting the absence of bipartite entanglement in a holographic mixed state, and ii) one-way LOCC-distillable entanglement is related to the entanglement wedge cross section, which is further dual to entanglement of formation. By demonstrating an explicit distillation protocol by holographic measurements, I conclude that a connected wedge does not necessarily imply finite distillable entanglement even when one-way LOCC is allowed. This talk is based on arXiv:2411.03426 [hep-th] and 2502.04437 [quant-ph].

Does connected wedge imply distillable entanglement?

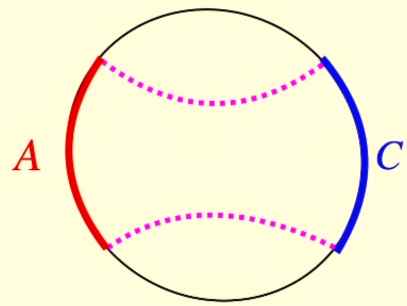
Takato Mori (Rikkyo U)

Based on 2411.03426 with Beni Yoshida (Perimeter)
See also 2502.04437v2 with BY and Zhi Li (NRC)
as well as 2506.02131

QIQG 2025 at Perimeter Institute for Theoretical Physics, Waterloo on June 27, 2025

Ryu-Takayanagi formula

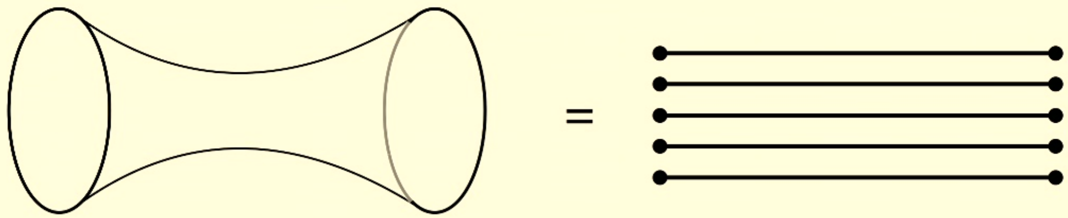
$$S_A = \min_{\gamma_A} \frac{\text{Area}(\gamma_A)}{4G_N} + O(1)$$



Connected wedge implies $O(1/G_N)$ correlation but how?

Bipartite vs non-bipartite

- 2001, 2013 ER=EPR [Maldacena; Maldacena-Susskind]



Spatial connectivity = EPR?

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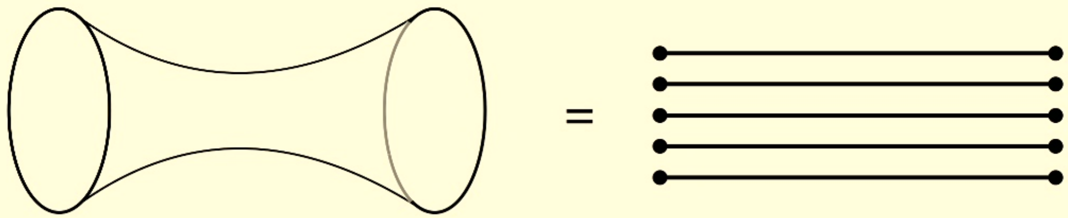
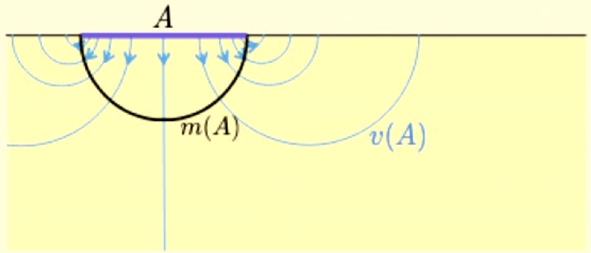
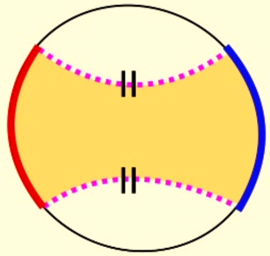


Spatial connectivity = EPR?

- 2016-2018 Bit thread [Freedman-Headrick; Cui-Hayden-He-Headrick-Stoica-Walter]

$$S_A = \max_v \int_A v$$





= wormhole

Connected wedge = a bunch of EPR pairs (up to local operations (LO))?

Bipartite vs non-bipartite

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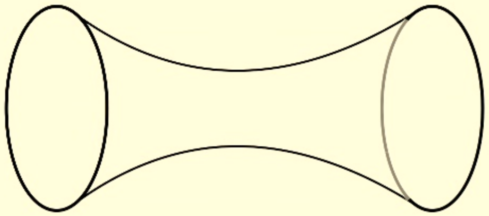

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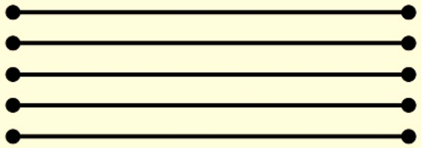

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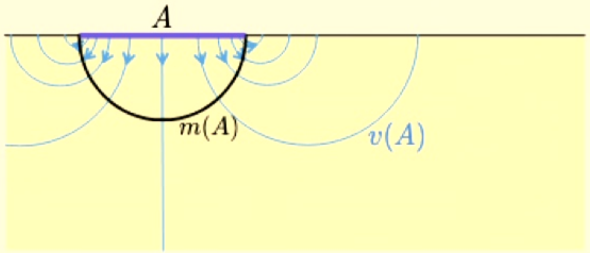
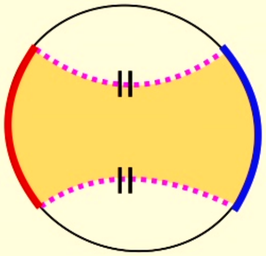




= wormhole

Connected wedge = a bunch of EPR pairs (up to local operations (LO))?

- 2019, 2021 Not mostly bipartite [Akers-Rath; Hayden-Parrikar-Sorce] (based on the Markov gap)

Still not clear if it implies mostly non-bipartite.

It is quantified by distillable entanglement.

$E_D^{[\text{operation}]}(A:C)$ = (max # EPR pairs one can get from ρ_{AC} via given set of operations)

Formally, it is defined up to errors

$$E_D^{[\text{operations}]}(A:C) = \sup_r \left\{ r \middle| \inf_{\Lambda \in \text{operations}} d\left(\Lambda(\rho_{AC}), \Phi_{\text{EPR}}^{\otimes r}\right) \leq \epsilon \right\}$$

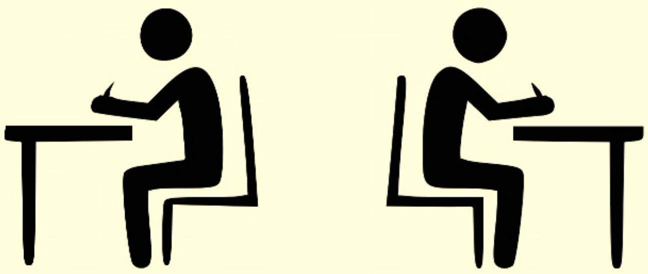
$$|\text{EPR}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\Phi_{\text{EPR}} = |\text{EPR}\rangle\langle\text{EPR}|$$

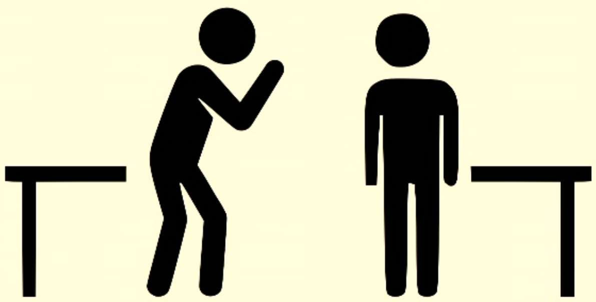
where $d(\rho, \sigma)$ is some distance measure between two states ρ, σ

LO vs. LOCC

We are interested in how to distill and how many EPRs can be distilled via



Local Operations (LO)

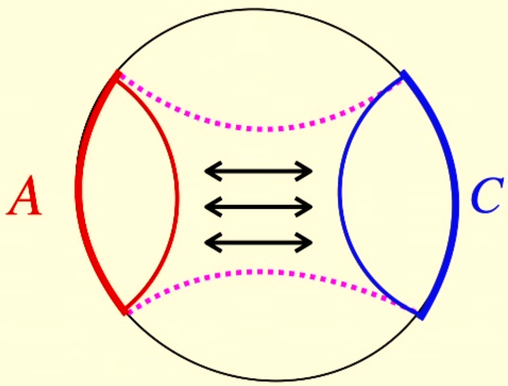


LO and Classical Communication (LOCC)

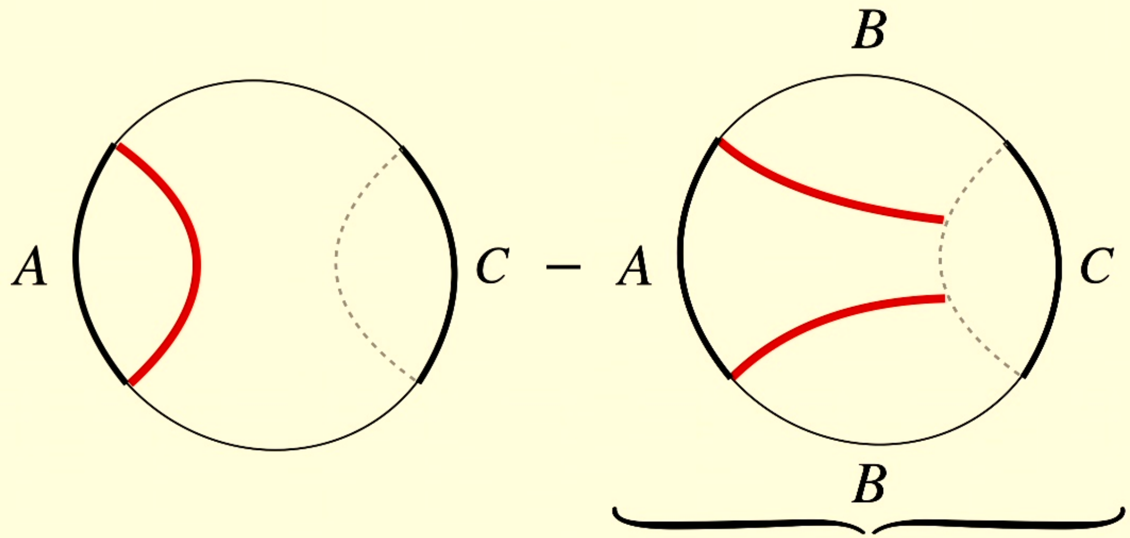
image credit: ChatGPT 4o

Main Results

$$E_D^{[\mathrm{LO}]}(A:C) = 0$$



$$E_D^{[\mathrm{1WAY\ LOCC}]}(A \leftarrow C) =$$



$$=: E^W(A:B)$$

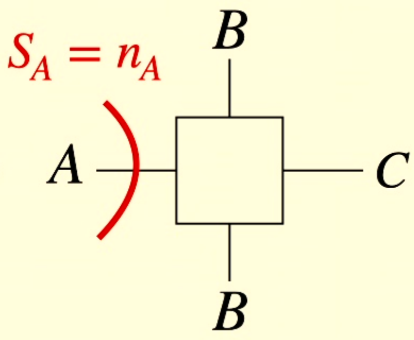
[Takayanagi-Umemoto]

All results are up to $o(1/G_N)$ corrections

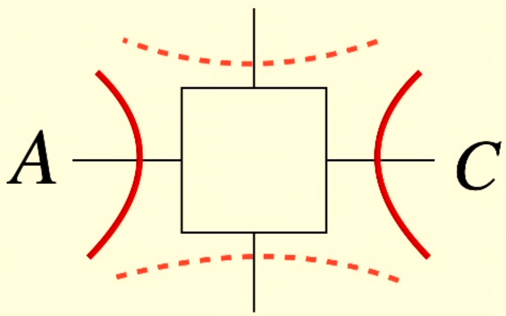
LO distillable entanglement in Haar random states

Haar random states (randomly sampled pure states) are the simplest toy model of AdS/CFT

$$U_{\text{Haar}}|0\rangle_{ABC} =$$

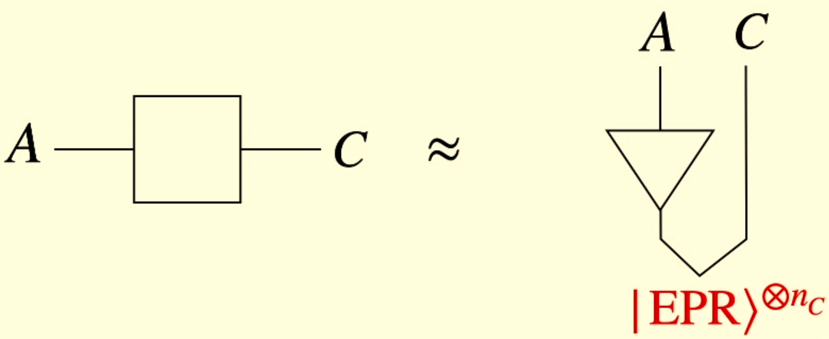


$$E_D^{[\text{LO}]}(A:C) \stackrel{?}{=} 0$$



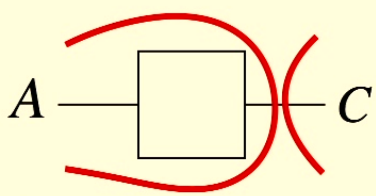
Haar — bipartite case

Bipartite pure state is LO distillable due to Page's theorem



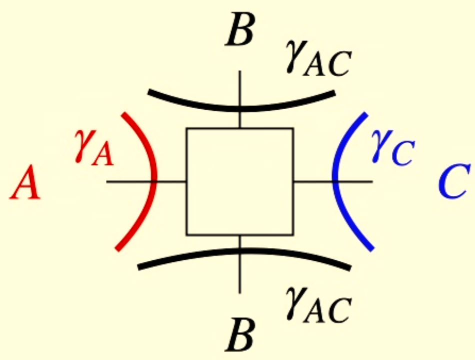
$(n_A > n_C)$

Diagrammatically understood as a consequence of overlapping RT surfaces



Haar — tripartite case

Tripartite state has connected wedge



Does connected wedge ($I(A:C) = O(n)$) imply LO distillable entanglement?

Takato Mori, Beni Yoshida, "Does connected wedge imply distillable entanglement?" arXiv:2411.03426

No LO distillable entanglement in Haar random states

No! We proved [Li-**TM**-Yoshida]

$$\text{Prob}\left(E_D^{[\text{LO}]}(A:C) \geq 1\right) \lesssim 2^{-2^n}$$

n : # qubits

More formally, we rigorously proved based on the measure concentration:

Theorem 2. *If* $\delta \overset{\text{def}}{=} h^2 - 2^{-m} > 0$, *then for an arbitrary constant* $0 < c < 1$, *we have*

$$\log \mathbb{P}\left(\text{ED}_h^{[\text{LO}]}(A:B) \geq m\right) \leq -c\delta^2 d + O(2^{2m}(d_A^2 + d_B^2)\log\frac{1}{\delta}).$$

$$\text{ED}_h^{[\text{LO}]}(A:B) \equiv \sup_{m\in\mathbb{N}} \sup_{\Lambda\in\text{LO}} \left\{m \middle| \text{Tr}\left(\Lambda(\rho_{AB})\Pi_{R_A R_B}^{[\text{EPR}]}\right) \geq h^2\right\}$$

Holography — Bound from Petz map

The measure concentration technique does not work for holographic states.

Instead, Petz map $\mathscr{D}_{C\to A'}$ = pretty good decoder/distiller gives a looser (but more general bound):

$$E_D^{[\mathrm{LO}]}(A:C) \leq \frac{1}{2} I(A:A')$$

RHS can be 0 even when $I(A:C) > 0$ (connected wedge)!

($I(A:A')$ itself is discussed in earlier literature on reflected entropy [Akers-Faulkner-Lin-Rath])

Summary: LO distillation

There exists a regime for Haar random states and holographic states such that

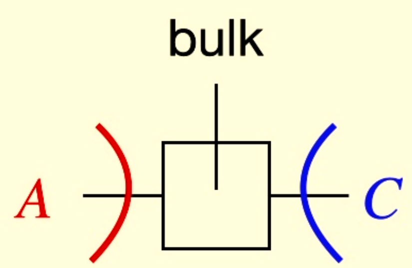
$$E_D^{[\mathrm{LO}]} = O(1) \ll \frac{I(A:C)}{2}$$

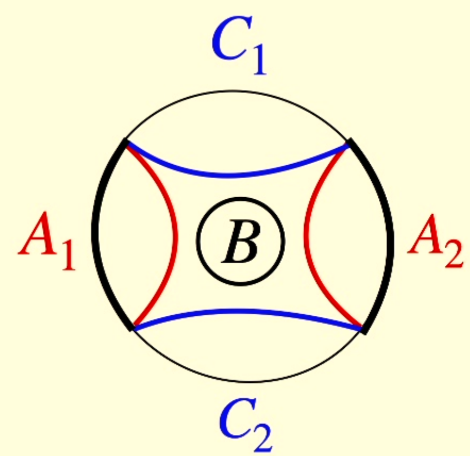
Based on Haar results, we expect this is also true for holographic states with any A, C unless ρ_{AC} is pure.

Takato Mori, Beni Yoshida, "Does connected wedge imply distillable entanglement?" arXiv:2411.03426

Implication: Shadow of EW reconstruction

When bulk matter carries $O(1/G_N)$ entropy, bulk reconstruction is possible neither from A nor C





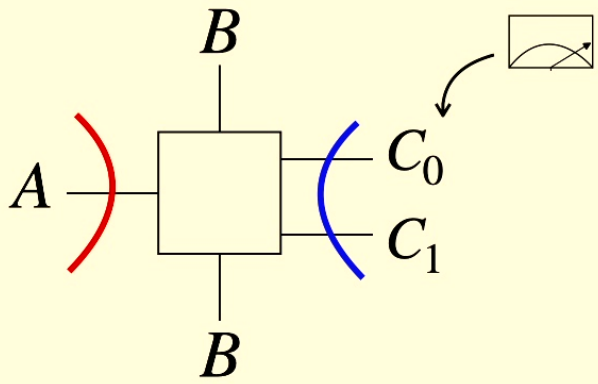
[Akers-Penington]

Does CC assist distillation, outperforming LO?

— Yes, because a measurement induces EW transition

LOCC distillation protocol for Haar random states

Measurement induces the EW transition, leading to overlapping minimal surfaces.



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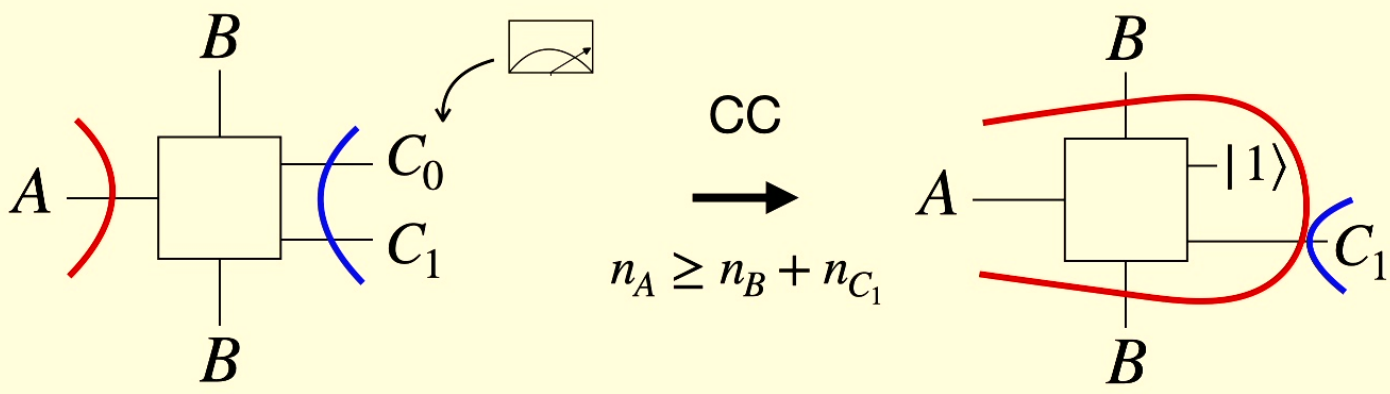

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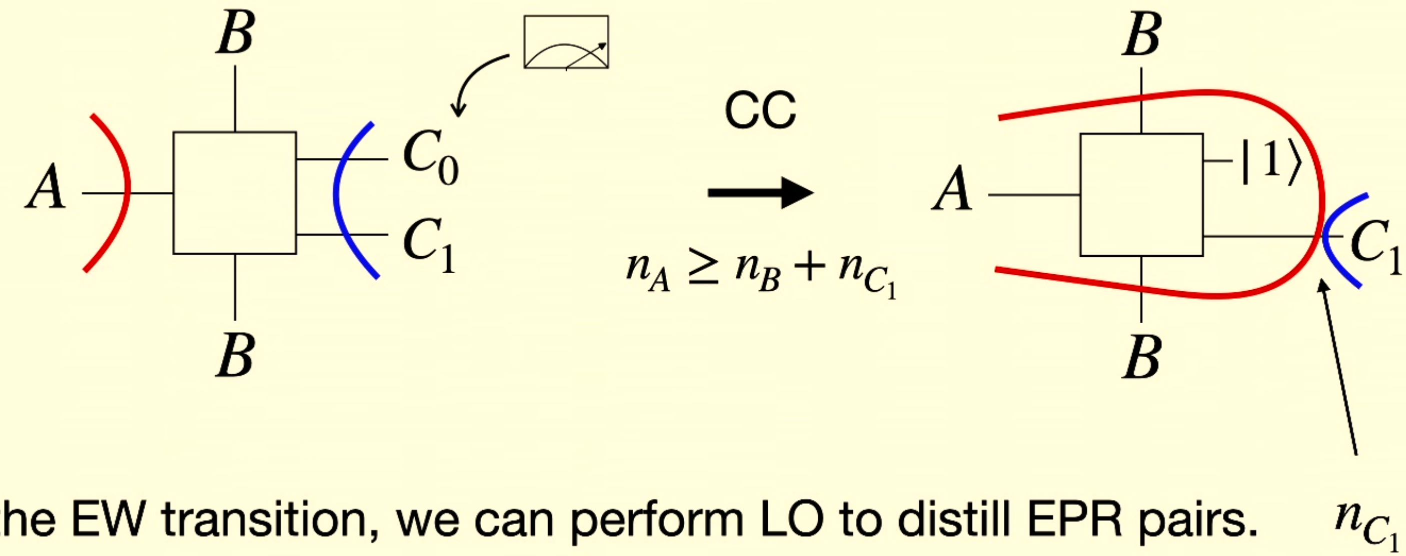


After the EW transition, we can perform LO to distill EPR pairs.

LOCC distillable entanglement for Haar random states

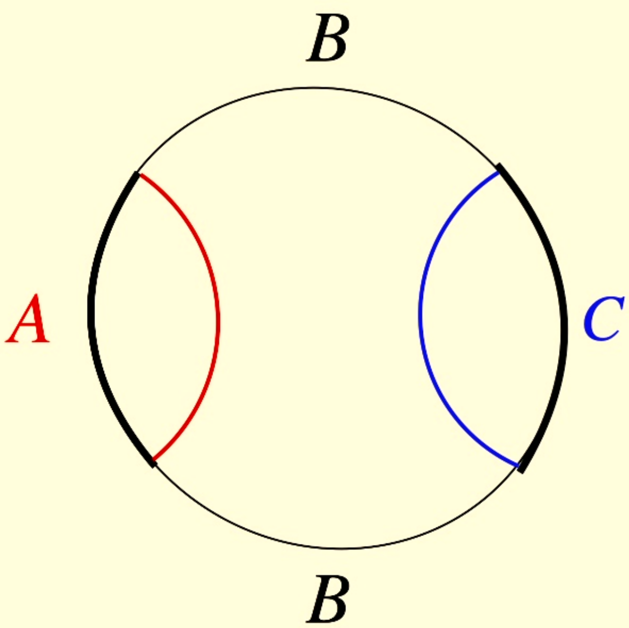
We conclude that

$$E_D^{[1\text{WAY LOCC}]}(A \leftarrow C) = \max(0, n_A - n_B)$$

[Hayden-Leung-Winter; **TM**-Yoshida]

It can be rigorously shown by the measure concentration.

LOCC distillation protocol for holographic states



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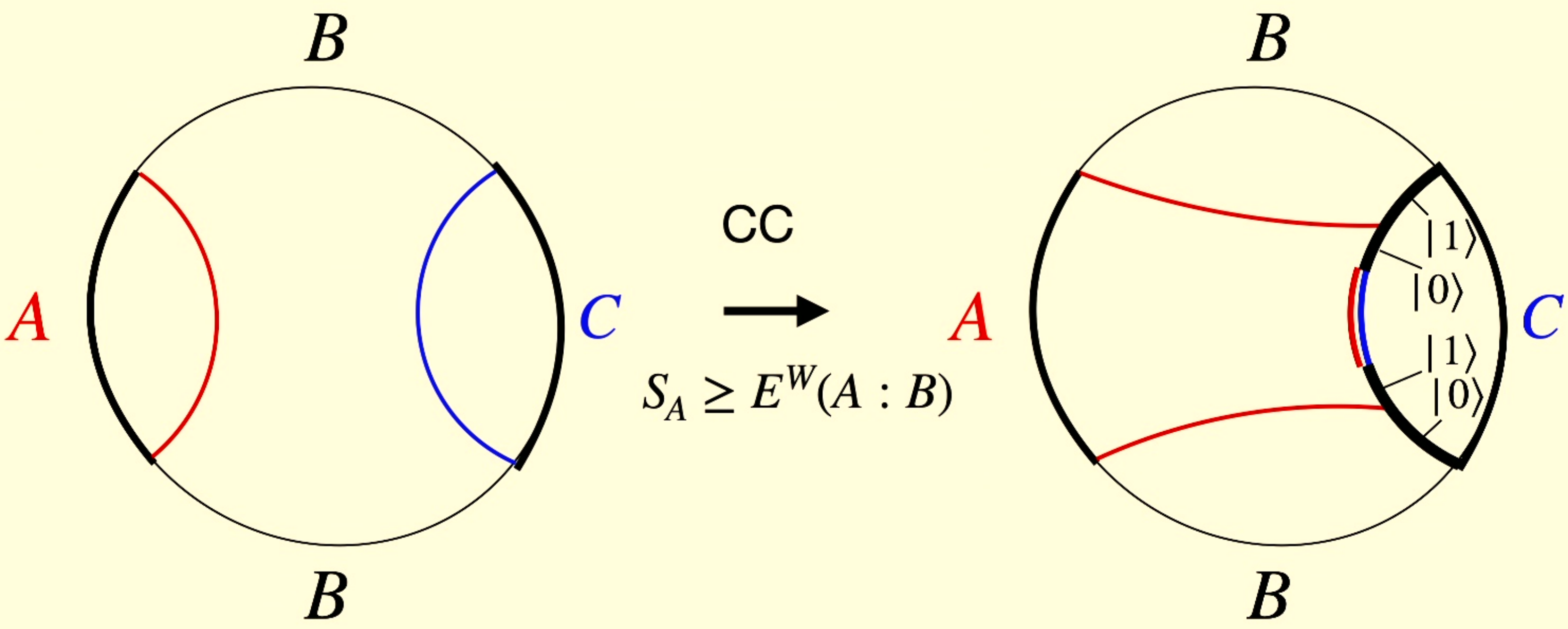

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LOCC distillable entanglement for holographic states

After the EW transition, we can perform LO to distill EPR pairs.

We propose that this is an optimal distillation protocol. Namely,

$$E_D^{[\text{1WAY LOCC}]}(A \leftarrow C) = S_A - E^W(A:B)$$

LOCC distillable entanglement for holographic states

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∃ Several supporting evidence (holographic optimization, generalized entropy, bulk causality) … ask me later

Corollary: EW=EoF

We also find $E_D^{[1\text{WAY LOCC}]}(A \leftarrow C) = S_A - E^W(A:B)$ implies via Koashi-Winter relation that

$$E^W(A:B) = E_F(A:B)$$

where

$$E_F(A:C) = \min_{\rho=\sum_i p_i |\psi_i\rangle\langle\psi_i|} \sum_i p_i S_A(|\psi_i\rangle)$$

is called the entanglement of formation ≈ # EPR required to form the state.

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Equivalently, it is a manifestation of monogamy of entanglement

$$E_D^{[1\text{WAY LOCC}]}(A:C) + E_F(A:B) = S_A$$

Summary

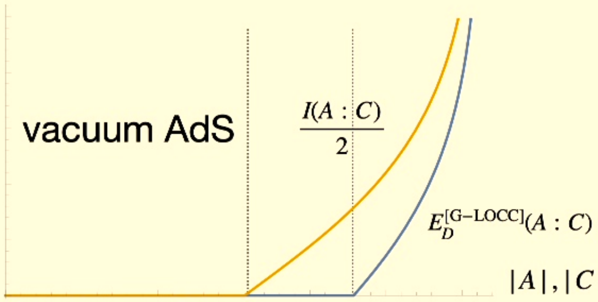
We find the **connected wedge does NOT imply distillable entanglement**.

$E_D^{[\mathrm{LO}]} \approx 0$

Stronger theorem for random tensor networks? General holographic states?

$E_D^{[\mathrm{1WAY\ LOCC}]}(A \leftarrow C) \approx S_A - E^W(A:B)$

Can we exclude fine-tuned measurement basis? 2WAY LOCC?



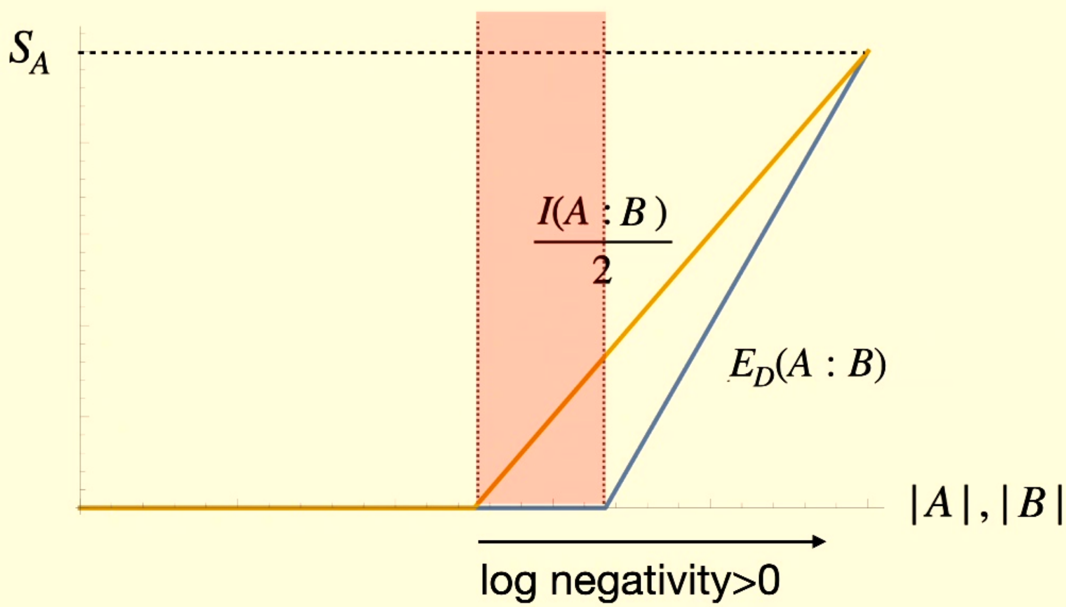
There are many relevant (exciting) results:

NPT bound entanglement, traversable wormholes, holographic quantum tasks, …

NPT bound entanglement in large D algebra

If our conjecture (geometric optimization is optimal) is true, based on our formula, there exists a regime where the holographic state is NPT bound entangled.

Namely, the state has entanglement that is not distillable to EPR pairs.



In fact, the existence of NPT bound entangled states is an unsolved problem in QI over 25 years!

Neglected subleading corrections? They vanish in the strictly large D limit for Haar random states.

In strictly large D, we solved the long-standing question? What about holographic states?

Takato Mori, Beni Yoshida, "Does connected wedge imply distillable entanglement?" arXiv:2411.03426

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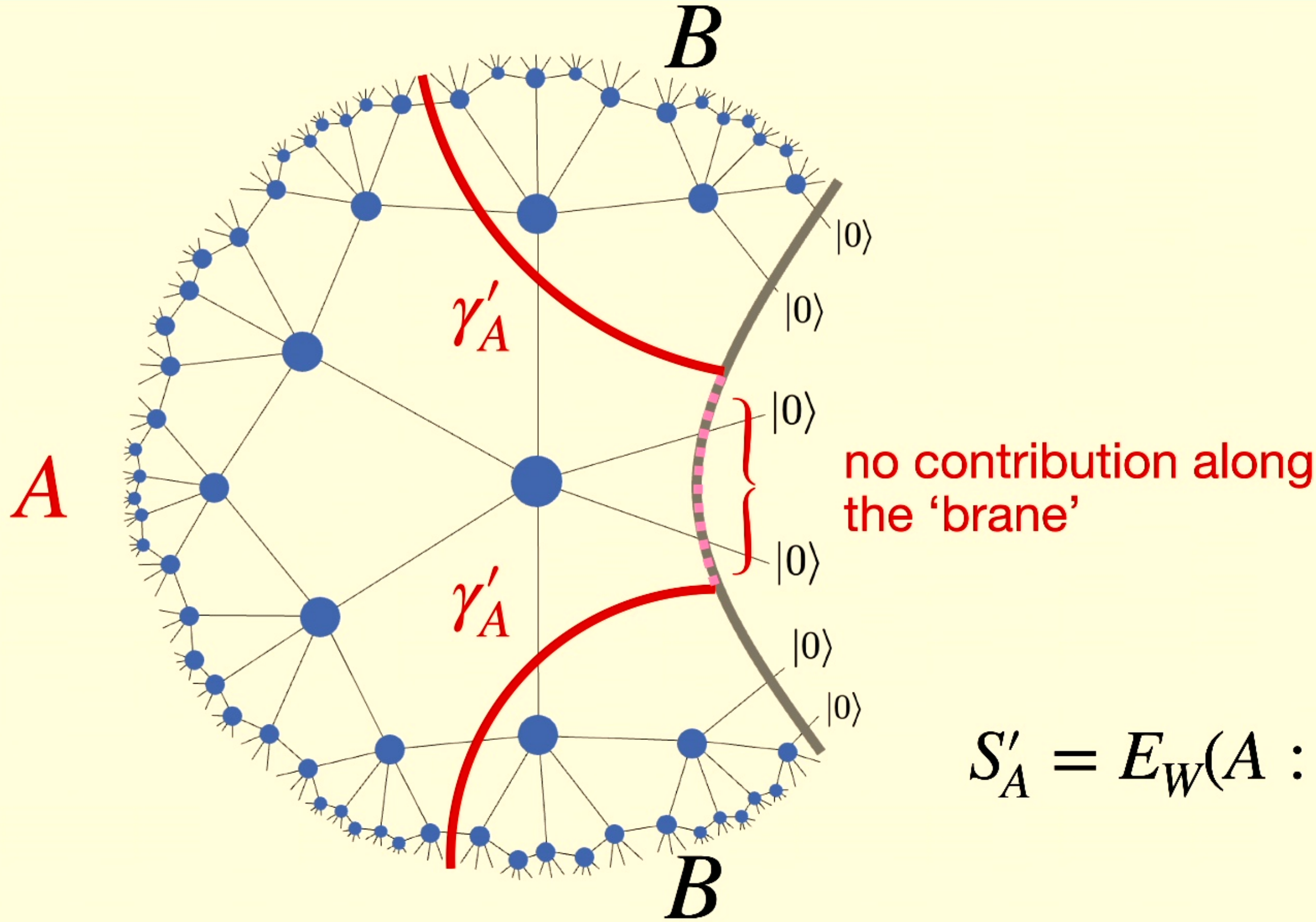
$$E_D^{[\text{1WAY LOCC}]}(A:C) + E_F(A:B) = S_A$$

Measurements reduce entanglement; enhance EW



$$S'_A = E_W(A : B)$$

Takato Mori, Beni Yoshida, "Does connected wedge imply distillable entanglement?" arXiv:2411.03426