Title: Absolute Entropy and the Observer's No-Boundary State

Speakers: Jonah Kudler-Flam

Collection/Series: QIQG 2025

Subject: Quantum Gravity, Quantum Information

Date: June 24, 2025 - 12:00 PM

URL: https://pirsa.org/25060010

Abstract:

I will discuss the algebra of observables accessible to an observer in a generic closed universe and the role of the no-boundary density matrix in defining entropy.

Absolute Entropy and the Observer's No-Boundary State

Everything you always wanted to know about the identity matrix (but were afraid to ask)

Jonah Kudler-Flam

QIQG: June 24, 2025

arXiv:2505.14771 w/ A. Blommaert, E. Y. Urbach

Quantum Gravity in Closed Spacetimes

- AdS/CFT has led to tremendous progress in understanding quantum gravity in open spacetimes
- Much less is known about quantum gravity in closed spacetimes - without a boundary, defining meaningful quasi-local observables is challenging - it has been discussed for many years that one needs to include a clock [Page-Wootters...]
- CLPW [2206.10780] took the clock/observer seriously in closed universes (de Sitter) to define quasi-local observables commuting with the Hamiltonian constraint
- The structure of this algebra led to better understanding of entropy in quantum gravity
- For semi-classical states, von Neumann entropy of the gravitationally dressed algebra equals generalized entropy of static patch (up to state-independent constant):

$$S_{\mathrm{vN}}(\rho) = \underbrace{\frac{\delta A_{\mathrm{dS}}}{4G} + \delta S_{\mathrm{QFT}}}_{O(1)} + C \tag{1}$$

Background Dependence Problem

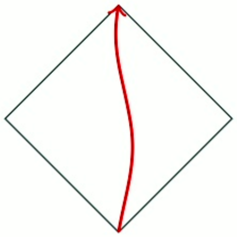
- The Hartle-Hawking state was found to be of maximum entropy in the perturbative Hilbert space, consistent with previous literature [Bousso, Maeda, Koike, Narita, Ishibashi, Banks, Fischler, Dong, Silverstein, Torroba...]
- This means that expectation values in the HH state have the property of a trace

$$\mathrm{Tr}(ab) \equiv \langle HH|\, ab\, |HH\rangle = \langle HH|\, ba\, |HH\rangle \,, \quad a, b \in \mathcal{A} \tag{2}$$

- This progress in understanding entropy came at the price of background dependence - only small fluctuations about dS
- Entropies cannot be compared between different spacetime backgrounds
- Different backgrounds have different additive constants not fixed by the algebraic structure

Subregions in Quantum Gravity

- Difficult to define subregions in quantum gravity in a diffeomorphism invariant manner
- Two approaches we understand:
  1. Region bounded by a codimension-2 extremal surface
  2. Causal diamond of an immortal observer
- We will focus on option 2 today which is universal - can define algebra of observables along worldline in any background
- Witten [2308.03663] used this to construct a "background independent algebra." Once choosing a representation (background), the algebra can be completed and one can ask what "type" it is



Maximum Entropy in Finite Dimensions

- In finite dimensional quantum mechanics, the state of maximum entropy is proportional to the identity matrix $\frac{\mathbb{1}}{\dim(\mathcal{H})}$
- Expectation values in this state is a trace

$$\langle \quad \rangle_{\mathbb{1}} \quad \Leftrightarrow$$

- The relative entropy of any state with respect to the unnormalized maximally mixed state is

$$S(\rho||\mathbb{1}) \equiv \mathrm{Tr}\, \rho \log \rho - \mathrm{Tr}\, \rho \log \mathbb{1} = -S_{vN}(\rho) \tag{3}$$

- We thus wish to understand the form of the maximally mixed state for observers in QG in closed universes, where there is normally no $\mathcal{H}$ exists for region associated to observer

5

Witten's Proposal: No-Boundary Trace

- Witten proposed that expectation values in Hartle and Hawking's no-boundary state was of maximum entropy so could be used as a trace for any closed spacetime [See also Jacobson 2 hours ago]

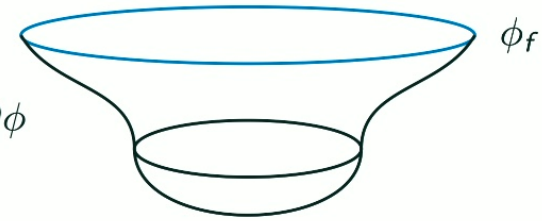
$$\mathrm{Tr}(a) \equiv \langle \mathrm{NB}|a|\mathrm{NB}\rangle \tag{4}$$

- This enables one to define the entropy as $S_{\mathrm{vN}}(\rho) \equiv -S_{\mathrm{rel}}(\rho||\rho_{\mathrm{NB}})$ where ρ_{NB} is the unnormalized no-boundary state. This is a proposal for an absolute entropy, no arbitrary additive constant
- This definition led to sensible results in the restricted case where the only spacetimes considered were de Sitter vacua with different values of the cosmological constant. It was pointed out that so little is known about the no-boundary state, that it is difficult to disprove this conjecture.
- We will make some modest progress by providing a path integral argument for this, and then explore the consequences for entropy in closed spacetimes with an observer

No-Boundary Proposal

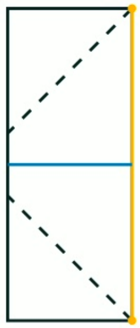
- The wavefunction of the universe is given by a path integral over compact geometries with boundary conditions ϕ on the boundary (ϕ_f is all fields including the metric)
- Provides a natural quantum state for closed cosmological models

$$\Psi[\phi_f] = \int \mathcal{D}\phi$$



Observers without horizons

- Naively, the proposal that $\rho_{NB} = \mathbb{1}$ is puzzling when an observer has causal access to the whole universe because the no-boundary state is pure while $\mathbb{1}$ is maximally mixed. Two options:
  1. This is a feature not a bug because the Hilbert space of quantum gravity in closed universes is one-dimensional [Almheiri, Mahajan, Maldacena, Zhao, Penington, Shenker, Stanford, Yang...] $\rho_{NB} = |NB\rangle\langle NB| = \mathbb{1}$
  2. We were wrong in saying that the no-boundary state is a pure state (or even close to pure)

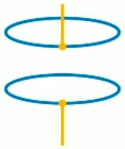


- We will first explain why $\rho_{NB} = \mathbb{1} \neq 1$ when the observer sees the whole universe
- We will then generalize this spacetimes where the observer experiences a horizon
- An entropy formula for observers bounded by generically non-extremal surfaces: area $\neq$ entropy?

Bra-Ket Wormholes

- For the observer's no-boundary state, there are no standard Hartle-Hawking configurations where spacetime caps off in the past - the observer worldline has no place to end

- Must consider the density matrix rather than pure states. In a density matrix, we specify boundary field values for both bra and ket:

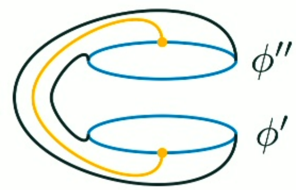


- Observer worldline can now stretch from bra to ket. Connection implies the no-boundary state is mixed

Path Integral Definition

- Observer's no-boundary state computed by path integral over spacetimes with wormhole topology $M \times I$ (can include higher topologies)

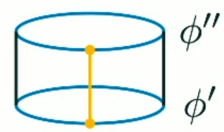
$$\langle \phi'' | \, \rho_{NB} \, | \phi' \rangle = \int \mathcal{D}\phi_{\text{bulk}} \quad \phi_{\text{bulk}}$$



(5)

- Relies on the conservation of observerons, relating to the rules for treating observers in the gravitational path integral to be nontrivial of [Abdalla, Antonini, Iliesiu, Levine, Harlow, Usatyuk, Zhao...]
- This is the same path integral for the inner product and so $\rho_{NB} = \mathbb{1}$

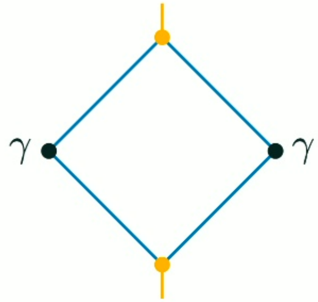
$$\langle \phi'' | \, \phi' \rangle = \int \mathcal{D}\phi_{\text{bulk}}$$



(6)

Observers with Causal Horizons (think dS)

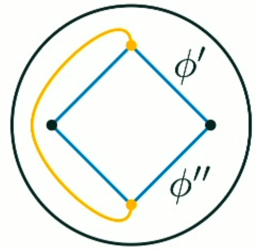
- Consider density matrix for subregion when observer has causal horizons (e.g., de Sitter)
- Recall: Two ways to define invariant subregions
  1. Region bounded by a codimension-2 extremal surface
  2. Causal diamond of an immortal observer
- We use null surfaces instead of spatial surfaces for boundary conditions to explicitly incorporate that observer only sees portion of universe
- Causal diamond defined by intersection γ of boundaries of observer's future and past light cones



Observers with Causal Horizons (think dS)

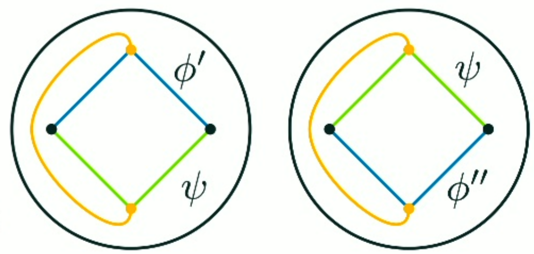
- Matrix elements of ρ_{NB}

$$\langle \phi'' |\, \rho_{\text{NB}} \,| \phi' \rangle = \int \mathcal{D}\phi_{\text{bulk}} \quad \phi_{\text{bulk}} \tag{7}$$



- Matrix elements of ρ_{NB}^2 computed as:

$$\langle \phi'' |\, \rho_{\text{NB}}^2 \,| \phi' \rangle = \int \mathcal{D}\phi_{\text{bulk}} \int_{\substack{\text{boundary conditions} \\ \text{modulo diffeomorphisms}}} \mathcal{D}\psi \tag{8}$$



- Therefore: $\rho_{NB}^2 = \rho_{NB} \Rightarrow \rho_{NB} = \mathbb{1}$ for this subregion

12

Absolute Entropy from the No-Boundary State

- Unlike replica trick in AdS/CFT where conical excess arises at asymptotic boundary, matrix multiplication does not result in conical excess at γ
- When dominated by saddle point, the normalization is $\approx e^{-I}$ with I the on-shell action
- Von Neumann entropy of normalized state $\tilde{\rho}_{NB} = (\mathrm{Tr}\rho_{NB})^{-1}\rho_{NB}$:

$$S_{vN}(\tilde{\rho}_{NB}) = -S_{rel}(\tilde{\rho}_{NB}||\rho_{NB}) = -I \tag{9}$$

- For any normalized state ρ:

$$S_{vN}(\rho) = -S_{rel}(\rho||\tilde{\rho}_{NB}) - I \tag{10}$$

- No arbitrary additive constant
- This makes sense as an asymptotic expansion in G, which is where ρ_{NB} is best understood
- In dS sphere path integral equals area of cosmological horizon to leading order in G
- entropy related to action rather than area

Comparison to some Previous Results

- Our analysis reproduces and extends CLPW results for de Sitter space and known constructions for (asymptotic) Killing horizons from JKF-Leutheusser-Satishchandran [2309.15897,2406.01669]
- Consistent with algebraic approach of Chen-Penington [2406.02116] for general regions bounded by extremal surfaces, for which they identified the relevant gravitational mode. In slow-roll inflation example, they explicitly invoked semiclassical limit of no-boundary state
- In the case of regions bounded by extremal surfaces, consistent with Jensen, Sorce, Speranza [2306.01837], but gives different predictions for more generic subregions.
- An interesting recent example where action $\neq$ area is in the discussion of Ivo-Li-Maldacena for slow-roll inflation [2409.14218]. There, the action was also proposed to be the entropy (the area was much larger)

Wormhole Solutions

- Consider the 3D FLRW with closed slices [Aguilar-Gutierrez, Hertog, Tielemans, van der Schaar, Van Riet 2306.13951]

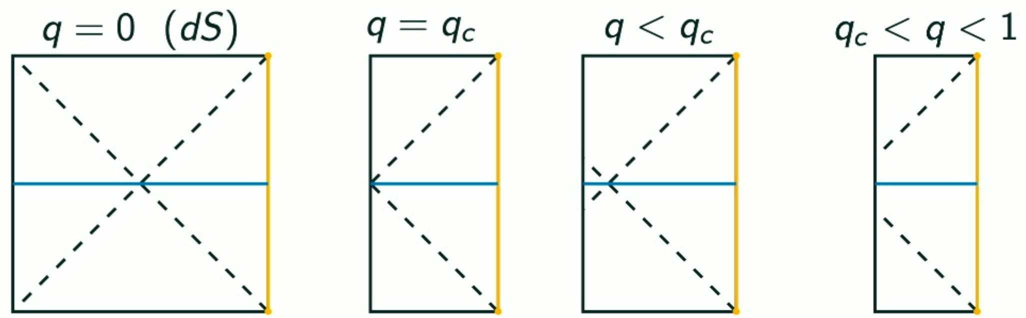
  (A similar solution exists in 4D supported by an SU(2) instanton)

$$ds^2 = -dt^2 + a(t)^2 d\Omega_2, \quad d\Omega_2 = d\theta^2 + \sin^2(\theta) d\phi^2 \tag{11}$$

- With axion flux density $Q = qH^{-1}(16\pi G)^{-1/2}$: The solution is asymptotically de Sitter for $q < 1$

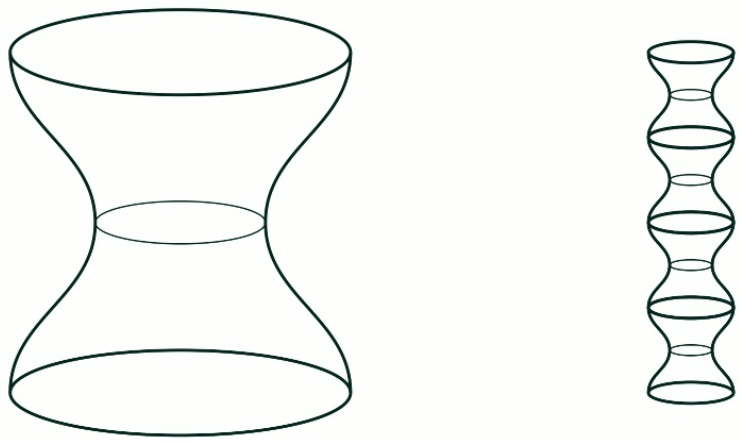
$$a(t) = \left[\frac{1 + \sqrt{1 - q^2} \cosh(2Ht)}{2H^2} \right]^{1/2} \tag{12}$$

- There is a "type transition" as a function of q



Euclidean Wormhole

- Solution can be extended to have $n \in \mathbb{Z}^+$ periods by extending the domain of τ. Do these all contribute to ρ_{NB}?



16

Von Neumann Algebra and Entropy

- The on-shell action of the $(n = 1)$ wormhole:

$$I_1 = -\frac{\pi}{2GH}(1 - q) \tag{13}$$

- Vanishes for $q = 1$ (Einstein static case), consistent with pure states having zero entropy (type I)
- Distinct from the area of the (non-extremal) causal horizon

$$A_h = 2\pi H^{-1}(1 + \sqrt{1 - q^2})\sin(\theta_h)^2, \quad \theta_h = \frac{\pi}{2}\frac{1+\sqrt{1-q^2}}{K\left(\frac{1-\sqrt{1-q^2}}{1+\sqrt{1-q^2}}\right)} \tag{14}$$

- Puzzle: the classical action for the (purely real Euclidean) periodic solutions are unbounded from below, $I_n = nI_1$

Conclusions

Today:

- Provided a path integral derivation of the conjecture that the no-boundary state is a universal maximum entropy state (i.e. a trace)
- Consistent with previous literature and also for closed universes in JT gravity with $\Lambda < 0$
- Entropy $=$ Action for observers in closed universes

Vision:

- Are there more tractable/interesting examples where we can explicitly determine the observer's no-boundary state/the corresponding algebra/entropy?
- How to resolve entropic puzzles? Lorentzian path integral [Jacobson 2 hours ago/Marolf 4pm today?]?
- Is there an analogous gravitational mode to that of Chen-Penington but for non-extremal surfaces?

18