Title: Quantum Field Theory on the edge

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Abstract:

In this talk I will report on recent developments concerning quantization of field theories on manifolds with boundary. In perturbative AQFT this can be realized using a version of BV-BFV formalism, following the program outlined by Mnev, Cattaneo and Reshetikin. I will also argue that in pAQFT one always implicitly introduces boundaries and that there are good conceptual reasons to do so. This is joint work with Michele Schiavina.

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Quantum Field Theory on the edge

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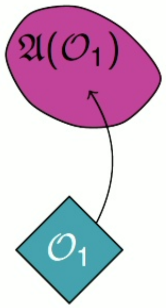
[1]based on joint works with Michele Schiavina

Outline of the talk

1. Living on the edge
2. pAQFT

Algebraic quantum field theory

- A convenient framework to investigate conceptual problems in QFT is Algebraic Quantum Field Theory.
- Axiomatic framework of Haag-Kastler: a model is defined by associating to each region $\mathcal{O}$ of Minkowski spacetime $\mathbb{M}$ an algebra $\mathfrak{A}(\mathcal{O})$ of observables that can be measured in $\mathcal{O}$.



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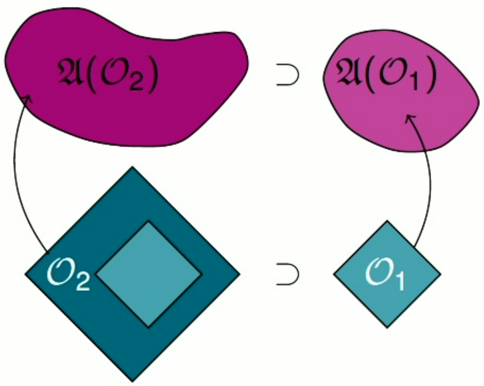
- The physical notion of subsystems is realized by the condition of isotony, i.e.: $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$. We obtain a net of algebras.



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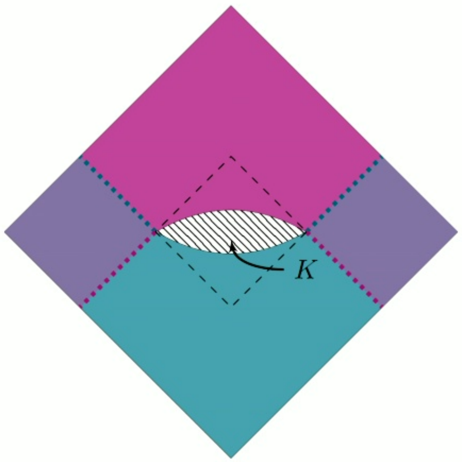
- This generalizes to curved spacetimes.

On the road to QG...

- In GR, individual points in manifolds have no physical meaning.
- In (perturbative) QG this can be taken into account by considering diffeomorphism-invariant observables.
- As an example, consider relational observables.
- Analogously, if we consider a background spacetime with a given metric, points have meeing up to transformations by isometries rather than arbitrary diffeomorphisms.
- To fix this ambiguity we need a reference frame.
- And now we make it all quantum!

. . . we meet QI

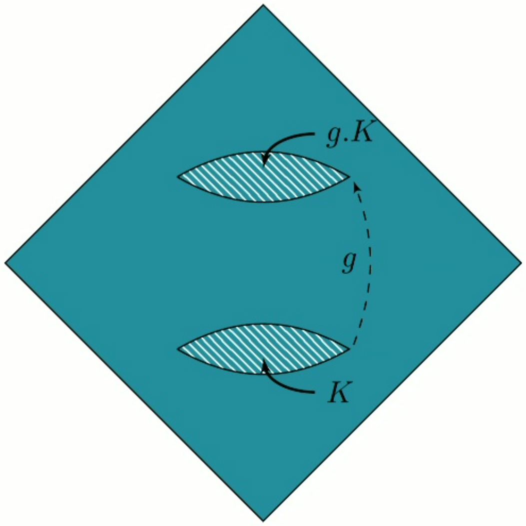
How to perform a measurement for some QFT $\mathfrak{A}$ on a spacetime M? [Fewster and Verch 2020].



- Couple $\mathfrak{A}$ to some probe QFT $\mathfrak{B}$ in compact coupling zone $K \subset M$.
- Probe field is prepared in some initial state uncorrelated to $\mathfrak{A}$ state in pre-coupling region.
- Probe observable B measured in post-coupling region.
- Expectation value of B corresponds to expectation value of induced system observable A localisable in region containing K.
- Such scheme is interpreted as measurement of A.

Measurement and background symmetry

- Given G a symmetry group of the background spacetime, assume $\mathfrak{A}$ and $\mathfrak{B}$ respect this symmetry (they are G-covariant). However, the coupling in K typically breaks the symmetry.
- Action of $g \in G$ transforms coupled theory to 'translated' coupled theory with coupling zone $g.K$.
- A transformation of the measurement scheme leads to covariant transformation of induced system observable $A \mapsto \alpha(g)A$ with $\alpha : G \to \mathrm{Aut}(\mathfrak{A}(M))$.



What is the point?

- The coupled theory can only be distinguished from the g-transformed one by specifying a physical system (that transforms under G) to act as a reference frame.
- For a fully quantum description we need a QRF, so measurement of quantum fields in spacetimes with symmetry requires the specification of a QRF, with a G-covariant observable.
- Physical observables should be elements of the combined observable algebra of the system and reference frames that are invariant under the combined actions of G.
- What can provide a QRF?
  - Other dynamical quantum fields (relational observables in QG).
  - Another QM system, e.g. a single particle, harmonic oscillator, etc. (recently linked to type II von Neumann algebra arising in this conext)
  - In the presence of boundary: degrees of freedom living on the boundary or corners.

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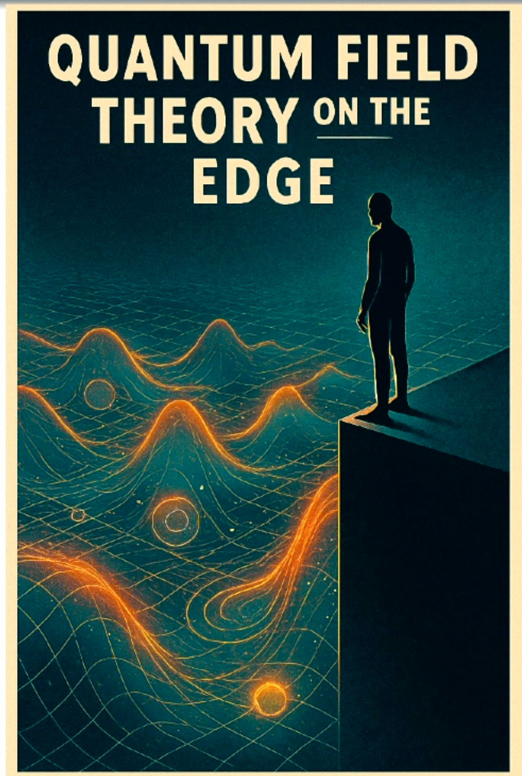
  - In the presence of boundary: degrees of freedom living on the boundary or corners.
  - Asymptotic degrees of freedom.

Beyond locality



- In order to consider QFT together with QRFs, we need new concepts that go beyond locality.
- Question: What is the natural extension of algebraic QFT axioms (or something similar in spirit) to the situation with boundary and corners (semi-local quantum physics?).
- Hint: look at the BV-BFV framework, [Cattaneo, Mnev, Reshetikhin, CMP 2011, CMP 2015].
- To avoid things being too singular, combine this with perturbative AQFT (pAQFT), which allows to construct interacting QFT models in 4D using Epstein-Glaser renormalisation.
- *Perturbative algebraic quantum field theory. An introduction for mathematicians*, KR, Springer 2016.
- In fact, boundaries were present in pAQFT all along!

Physical input

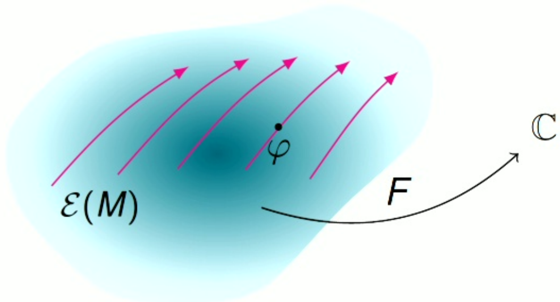
- A globally hyperbolic spacetime M.
- Configuration space $\mathcal{E}(M)$: choice of objects we want to study in our theory (scalars, vectors, tensors,...).
- Typically $\mathcal{E}(M)$ is a space of smooth sections of some vector bundle $E \xrightarrow{\pi} M$ over M.
- Classical observables are functionals $F \in \mathcal{C}^\infty(\mathcal{E}(M), \mathbb{R})$, whose derivatives satisfy appropriate regularity conditions.
- Dynamics: we use a covariant modification of the Lagrangian formalism. Since M is non-compact, the action S is not of the form $S = \int \mathcal{L}(\varphi)$ for some Lagrangian density, but a function $\mathcal{C}_c^\infty(M) \ni f \mapsto \int f\mathcal{L}(\varphi)$ that assigns a functional to each cutoff f.
- From S we obtain a 1-form dS on configuration space that gives the equations of motion: $dS(\varphi) = 0$.

Symmetries

- In the BV framework, symmetries are identified with vector fields (directions) on $\mathcal{E}$.
- We denote vector fields that are multilocal and compactly supported by $\mathcal{V}$. They act on $\mathcal{F}$ as derivations:

$$\partial_X F(\varphi) := \langle F^{(1)}(\varphi), X(\varphi)\rangle$$

- For $X \in \mathcal{V}$ and action S, denote $\langle dS(\varphi), X(\varphi)\rangle$.
- A symmetry of S is a direction in $\mathcal{E}$ in which the action is constant, i.e. it is a vector field $X \in \mathcal{V}$ such that: $\forall \varphi \in \mathcal{E}$: $\langle dS, X\rangle \equiv 0$.
- EOMs and symmetries are encoded in the cohomology of the BV differential s.

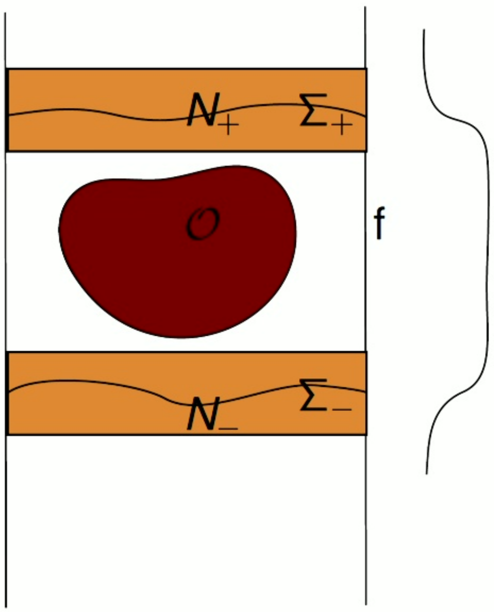


Taking the test function seriously

Why nobody takes me seriously?

- Terms with the support governed by df can be interpreted as smoothed boundary.
- Normally we ignore them, but maybe we should take them seriously?
- In the joint project with Schiavina, we are developing a version of the CMR framework, where we specify a compactly supported test function f and M is replaced with $\text{supp}\, f$ and ∂M with $\text{supp}\, df$.

pAQFT bistro



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- The limit where the thickness of $N_{\pm}$ goes to zero corresponds to the limit where f becomes a sum of two step functions.
- We call it flat-bread limit.

Thank you very much for your attention!