Title: Chaos and the Emergence of the Cosmological Horizon

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Abstract:

We construct algebras of diff-invariant observables in a global de Sitter universe with two observers and a free scalar QFT in two dimensions. In the limit when the observers have infinite mass and are localized along geodesics at the North and South poles, it was shown in previous work (CLPW) that their algebras are mutually commuting type II_1 factors. Away from this limit, we show that the algebras fail to commute and that they are type I non-factors. Physically, this is because the observers' trajectories are uncertain and state-dependent, and they may come into causal contact. We compute out-of-time-ordered correlators along an observer's worldline, and observe a Lyapunov exponent given by 4πβ_dS, as a result of observer recoil and de Sitter expansion. This should be contrasted with results from AdS gravity, and exceeds the chaos bound associated with the de Sitter temperature by a factor of two. We also discuss how the cosmological horizon emerges in the large mass limit and comment on implications for de Sitter holography.

Chaos and the Emergence of the Cosmological Horizon

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Based on 2411.08090 with Hong Liu

QIQG 2025
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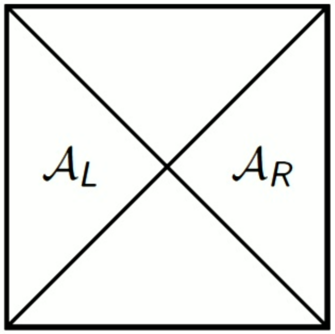
Introduction and Summary

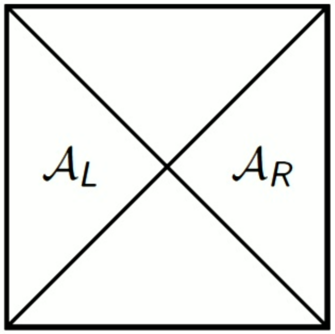
- At this point, the holographic dictionary in AdS/CFT is well-developed on multiple fronts (correlation functions, entanglement, probes of chaos, complexity, ...)
- How can the lessons we have learned apply to de Sitter space?
- This talk is based on the conjecture that the worldline of an observer is a holographic screen [Anninos,Hartnoll,Hofman '11] [Witten '23] [Narovlansky-Verlinde '23] [Harlow-Usatyuk-Zhao '25] [Abdalla-Antonini-Iliesiu-Levine '25] [Akers et al '25] [Tietto-Verlinde '25] [Narovlansky '25]
- [CLPW '22] associated an entropy to the algebra of operators accessible to an observer, which agrees with the generalized entropy of the observer's cosmological horizon.
- This suggests that the observer-centric hologram should have a dual description of the entropy of the observer's cosmological horizon.

Introduction and Summary



- [CLPW '22] considered global de Sitter space with two observers at the North and South Poles, with algebras $\mathcal{A}_L$ and $\mathcal{A}_R$.
- They showed that the algebras factorize: $\mathcal{A}_L' = \mathcal{A}_R$, and $\mathcal{A}_L \cap \mathcal{A}_R = \mathbb{C}$ (compare with $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$).
- Although the Hilbert space does not factorize, one can define a trace on $\mathcal{A}_L$ and $\mathcal{A}_R$, and thus compute $-\mathrm{tr}\rho \log \rho$, with $\rho \in \mathcal{A}_R$ or $\mathcal{A}_L$. The trace is unique up to a state-independent normalization.

Introduction and Summary



- In AdS/CFT, $\mathcal{A}_R$ and $\mathcal{A}_L$ represent the exteriors of a two-sided black hole. Algebraic factorization has been observed semiclassically [Witten '21, CPW '22], and also exactly in JT gravity [Penington-Witten '23, DK '23]. It is inherited from Hilbert space factorization of two copies of the CFT.
- In this talk, I will show that the de Sitter algebraic factorization fails due to chaotic observer dynamics.
- The presence of two observers does not imply two independent holographic screens.

Introduction and Summary

- CLPW worked in the following limits:
  1. $G_N \to 0$.
  2. Observer has infinite entropy.
  3. $\ell_{dS}^{-1} \ll M_{obs}$.
- We relax assumption 3. We do not find a satisfactory generalization of the type II trace of CLPW. The algebra is a type I non-factor. There is still a sensible definition of the Hartle-Hawking state $|\mathrm{HH}\rangle$.
- The Hartle-Hawking state furnishes a probability distribution for the observer's mass. From CLPW,

$$\langle \mathrm{HH}|f(H_R)|\mathrm{HH}\rangle = \int_{E_0}^{\infty} dE\, f(E) e^{-\beta_{dS} E}.$$

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- In two dimensions, we compute corrections to this distribution for cosmic scale masses ($\beta_{dS} = 2\pi$).

$$\langle \mathrm{HH}|f(H_R)|\mathrm{HH}\rangle = \int_0^{\infty} ds\, \frac{s \tanh \pi s}{\cosh^2(\pi s)} f\left(\sqrt{s^2 + \frac{1}{4}}\right).$$

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- We compute an OTOC of operators on the observer's worldline. The scrambling time is logarithmic in the observer's mass, and the Lyapunov exponent is $\frac{4\pi}{\beta_{\mathrm{dS}}}$.
- These calculations narrow the search for the putative holographic dual.

Quantization of a scalar field

- We work in two dimensions,

$$ds^2 = -dt^2 + d\theta^2 \cosh^2 t.$$

- Let $\varphi^s(t,\theta)$ be a scalar field of mass $m = \sqrt{\frac{1}{4} + s^2}$, where $s > 0$. We expand it as

$$\varphi^s(t,\theta) = \sum_{n\in\mathbb{Z}} \Psi_n^s(t,\theta) a_n + (\Psi_n^s(t,\theta))^* a_n^\dagger$$

$$[a_{n_1}, a_{n_2}^\dagger] = \delta_{n_1 n_2}.$$

Morally, $\Psi_n^s(t,\theta)$ is "$e^{in\theta - i\omega_n t}$."

- The Fock vacuum is also known as the Bunch-Davies state, or Hartle-Hawking vacuum.

Quantization of an observer

- The Hilbert space of an observer is the one-particle subspace of the Fock Hilbert space, but with the mass promoted from a parameter to a quantum number.
- Note that

$$\int dt d\theta \sqrt{-g} \left(\Psi^{s_1}_{n_1}(t,\theta)\right)^* \Psi^{s_2}_{n_2}(t,\theta) = \delta_{n_1 n_2}\delta(s_1 - s_2) = \langle s_1\, n_1 | s_2\, n_2 \rangle \,.$$

- The Hilbert space of an observer $\mathcal{H}_{obs}$ is defined to be the span of $\Psi^s_n(t,\theta) \leftrightarrow |s\, n\rangle$ for $n \in \mathbb{Z}$ and $s > 0$, with the inner product given above.

Setup

- We consider a dS_2 universe with two observers and a free scalar QFT. The physical Hilbert space is

$$\mathcal{H} := \frac{\mathcal{H}_{obs,L} \otimes \mathcal{H}_{obs,R} \otimes \mathcal{H}_{QFT}}{SO(2,1)}.$$

- This is the subspace of singlets of the isometry group. There is a well-defined inner product on the space of singlets (related to the work of [Higuchi '91, Marolf-Morrison '08, ...]).
- The subspace of $\mathcal{H}$ in which the QFT is in the Bunch-Davies state is called the vacuum sector.
- In the classical limit, the vacuum sector describes configurations in which the observers are located at antipodal points. The observers must have the same mass.

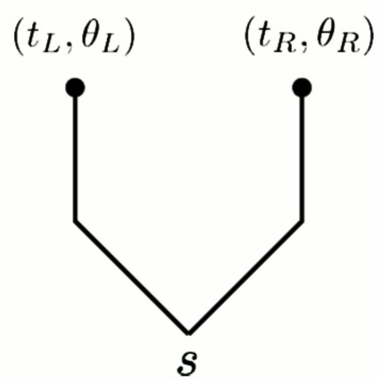
Vacuum sector

- States in the vacuum sector are labeled by $s > 0$,

$$|s\rangle = \sum_{n\in\mathbb{Z}} |s\, n\rangle_L \, |s, -n\rangle_R \, (-1)^n \leftrightarrow \sum_{n\in\mathbb{Z}} \Psi^s_n(t_L, \theta_L)\Psi^s_{-n}(t_R, \theta_R)(-1)^n,$$
$$:= \Psi^s_{LR}(t_L, \theta_L, t_R, \theta_R)$$

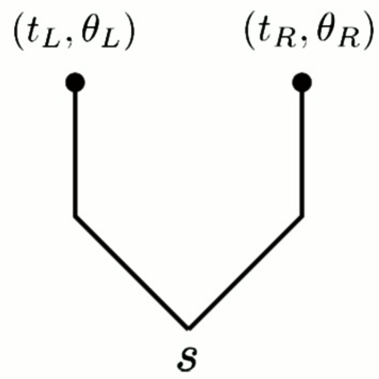
and they are normalized so that

$$\langle s_1 | s_2 \rangle = s_1 \tanh(\pi s_1)\, \delta(s_1 - s_2).$$

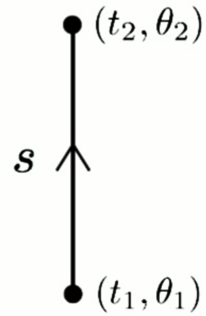


$$= \Psi^s_{LR}(t_L, \theta_L, t_R, \theta_R)$$

Diagrammatic rules

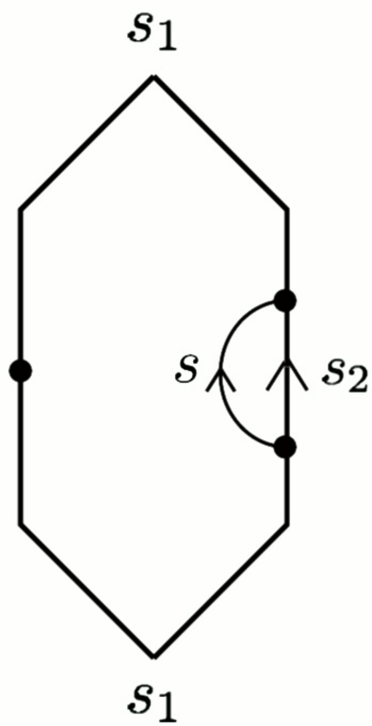


$$= \Psi^{s}_{LR}(t_L, \theta_L, t_R, \theta_R)$$



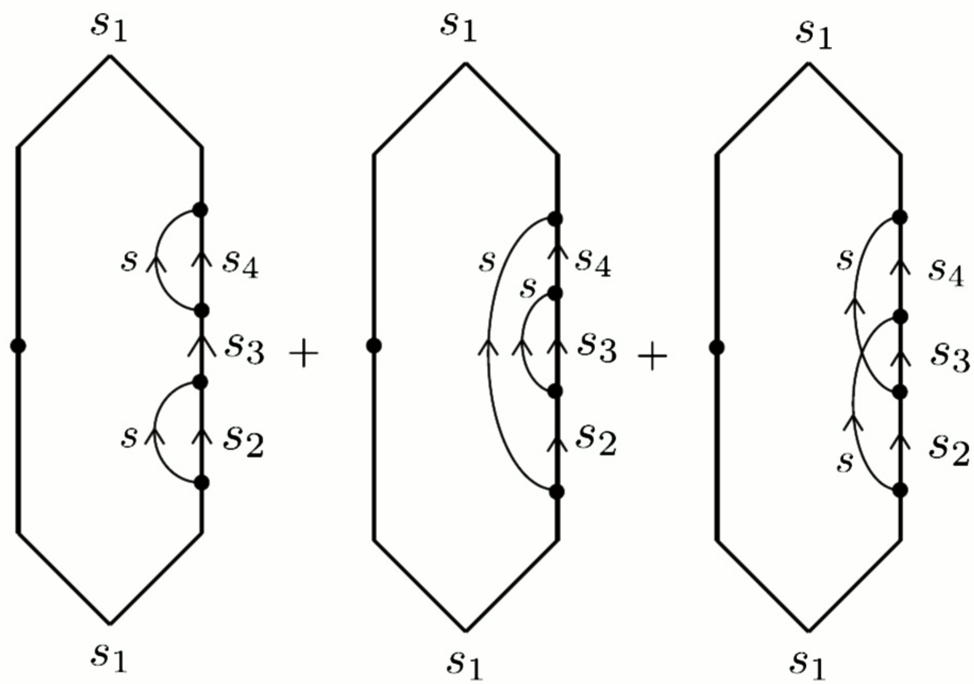
$$= \sum_{n \in \mathbb{Z}} \Psi^{s}_{n}(t_2, \theta_2) \Psi^{s\,*}_{n}(t_1, \theta_1)$$

Examples



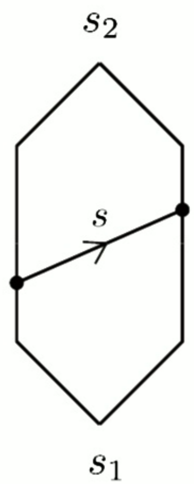
$$= \langle s_1 | \phi_R P_{s_2,R} \phi_R | s_1 \rangle$$

Examples

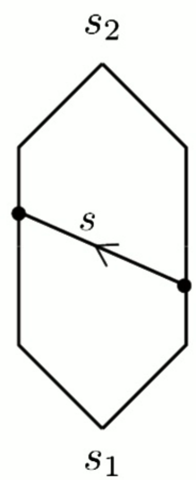


$$= \langle s_1 | \phi_R \, P_{s_4,R} \, \phi_R \, P_{s_3,R} \, \phi_R \, P_{s_2,R} \, \phi_R | s_1 \rangle$$

Examples

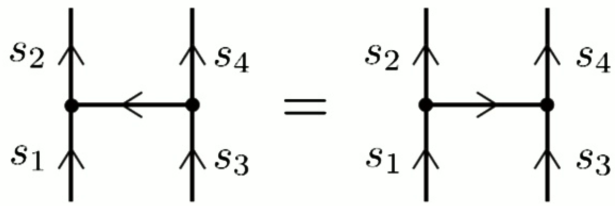


$$= \langle s_2 | \phi_R \phi_L | s_1 \rangle,$$

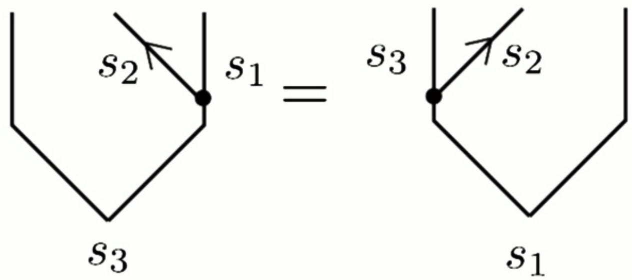


$$= \langle s_2 | \phi_L \phi_R | s_1 \rangle$$

Matter operators

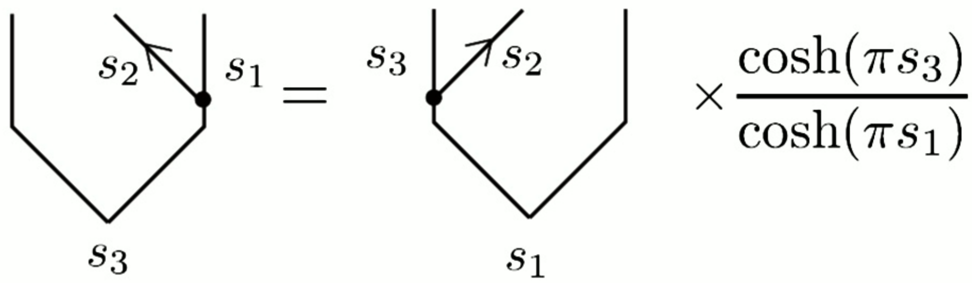


$$\times \frac{\cosh(\pi s_2)\cosh(\pi s_3)}{\cosh(\pi s_1)\cosh(\pi s_4)}$$



$$\times \frac{\cosh(\pi s_3)}{\cosh(\pi s_1)}$$

Hartle-Hawking state



- The Hartle-Hawking state $|\mathrm{HH}\rangle$ is the unique state (up to normalization) that obeys

$$\phi_L \, |\mathrm{HH}\rangle = \phi_R \, |\mathrm{HH}\rangle \, .$$

Hartle-Hawking state

$$|\mathrm{HH}\rangle := \int_0^\infty ds\, \frac{|s\rangle}{\cosh \pi s},$$

- The probability distribution on the observer's mass is determined by

$$\langle \mathrm{HH}|f(H_R)|\mathrm{HH}\rangle = \int_0^\infty ds\, \frac{s \tanh \pi s}{\cosh^2 \pi s}\, f\left(\sqrt{\frac{1}{4} + s^2}\right),$$

and becomes $m\, e^{-2\pi m}$ at large m.

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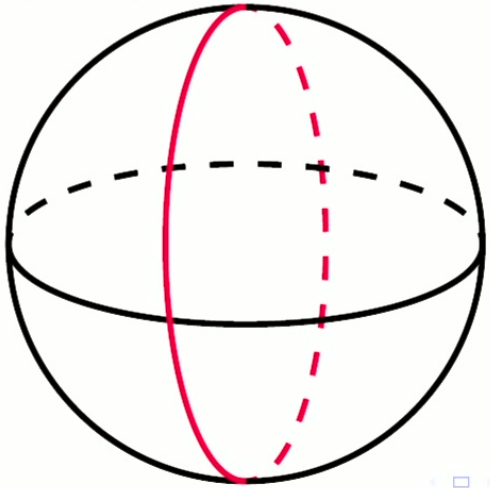
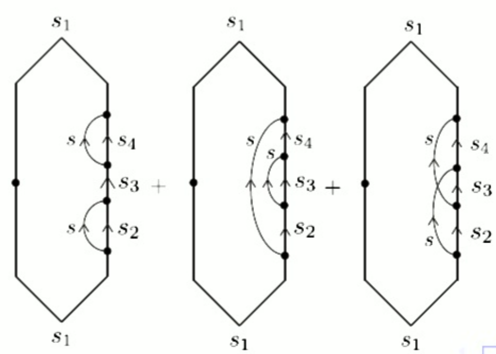
  and becomes $m\, e^{-2\pi m}$ at large m.
- When the geodesic approximation is valid, this agrees with the Euclidean path integral up to a phase [Maldacena '24].



The Observer's algebra

- The right observer's algebra $\mathcal{A}_R$ is generated by H_R, ϕ_R, ... The left observer's algebra $\mathcal{A}_L$ is defined analogously.
- Unlike CLPW, we find that the right and left algebras do not commute. For example, $[\phi_L, \phi_R]$ annihilates the vacuum sector, but is not identically zero.
- We also find that the HH state does not furnish a trace on $\mathcal{A}_R$. In particular,

$$\langle \mathrm{HH}| P_{s_1,R}\phi_R P_{s_2,R}\phi_R P_{s_3,R}\phi_R P_{s_4,R}\phi_R |\mathrm{HH}\rangle$$
$$\neq \langle \mathrm{HH}| P_{s_2,R}\phi_R P_{s_3,R}\phi_R P_{s_4,R}\phi_R P_{s_1,R}\phi_R |\mathrm{HH}\rangle$$



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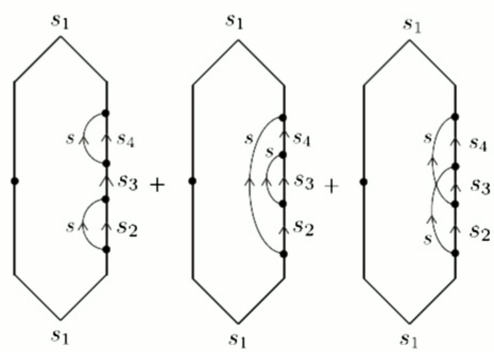


- The semiclassical limit is the limit of large s_i with fixed $s_i - s_j$. In this limit, the HH state becomes tracial.

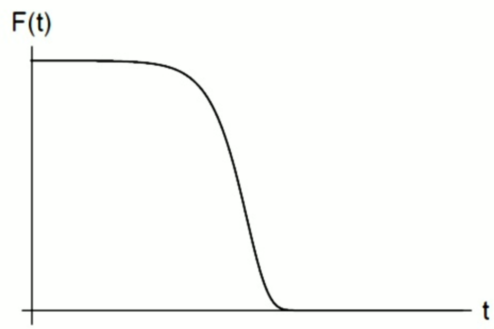
The Observer's algebra

- Other than H_L, there are no obvious candidates for operators in $\mathcal{A}'_R$. We conjecture that $\mathcal{A}'_R$ is generated by functions of H_L.
- Because $\mathcal{A}_R = (\mathcal{A}'_R)'$, $\mathcal{A}_R$ is a direct integral of type I factors. It only becomes type II in the semiclassical limit.
- When the observer is fully quantized and dynamical, we have not found an algebraic way to generalize the notion of their cosmological horizon. This contrasts an analogous study in JT gravity [Penington-Witten '23, DK '23] .

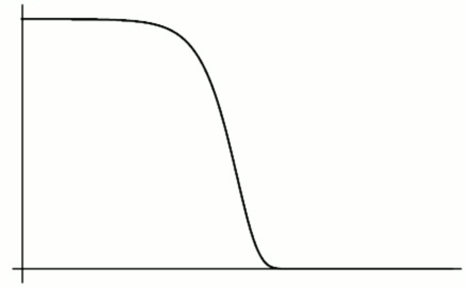
Out-of-time-ordered correlator

- Let V and W be simple Hermitian operators in a large N chaotic quantum system. An example of an OTOC at inverse temperature β is

$$F_\beta(t) := \mathrm{Tr}\ e^{-\frac{\beta}{4}H} W(t) e^{-\frac{\beta}{4}H} V(0) e^{-\frac{\beta}{4}H} W(t) e^{-\frac{\beta}{4}H} V(0)$$



Out-of-time-ordered correlator



- Typically,

$$\frac{F_\beta(t)}{F_\beta(0)} \sim 1 - \frac{1}{N} e^{\lambda_L t} + \cdots ,$$

where N is the number of qubits, and λ_L is the Lyapunov exponent, which cannot exceed $\frac{2\pi}{\beta}$ [Maldacena-Shenker-Stanford '15] .

- The scrambling time is

$$t_{scr} := \lambda_L^{-1} \log N.$$

Out-of-time-ordered correlator

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- The scrambling time is

$$t_{scr} := \lambda_L^{-1} \log N.$$

- In this work, we compute an OTOC of four operators along the observer's worldline. We find a scrambling phenomenon associated with the exponential expansion of de Sitter space.

OTOC along Observer's worldline

- Consider the OTOC

$$\frac{\langle \mathrm{HH}|\phi_R^B(\frac{\tau}{2})\phi_R^A(-\frac{\tau}{2})\phi_R^B(\frac{\tau}{2}+u_2)\phi_R^A(-\frac{\tau}{2}+u_1)\Pi_\Lambda|\mathrm{HH}\rangle}{\langle \mathrm{HH}|\Pi_\Lambda|\mathrm{HH}\rangle},$$

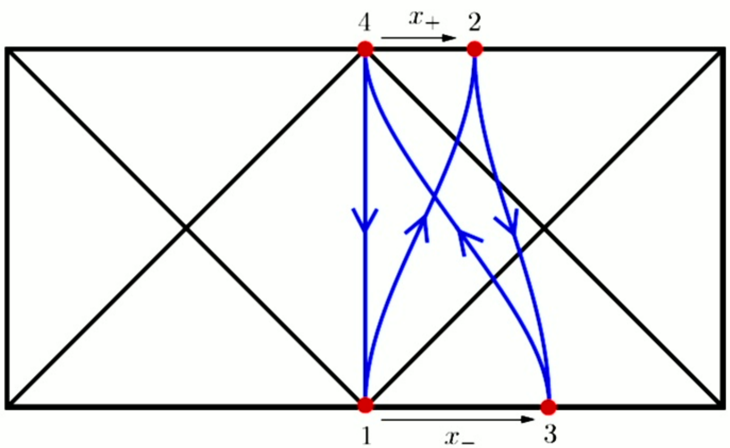
where

$$\phi_R(\tau) := e^{-iH_R\tau}\phi_R e^{iH_R\tau}, \qquad \Pi_\Lambda := \int_\Lambda^\infty ds\, P_{s,R}.$$

- We take large Λ and large τ, holding $e^T := \frac{e^\tau}{2\Lambda}$ fixed.

OTOC along Observer's worldline

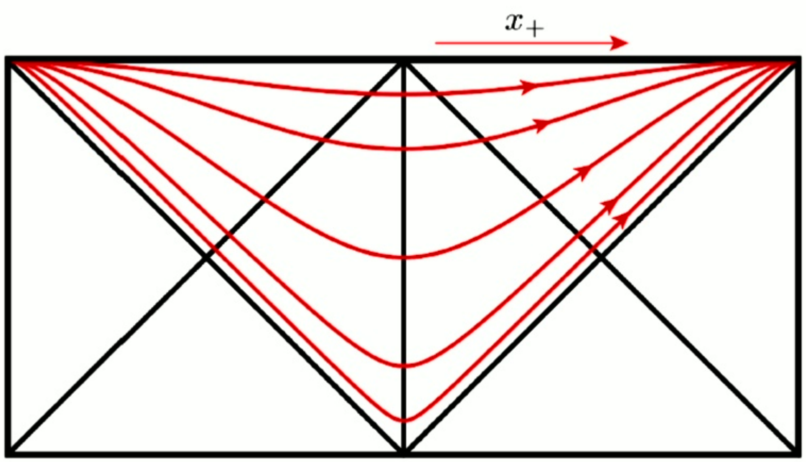
$$\langle\Omega|\overbrace{\varphi_R^B\left(\frac{\tau}{2}\right)}^{4}\overbrace{\varphi_R^A\left(-\frac{\tau}{2}\right)}^{3}\overbrace{\varphi_R^B\left(\frac{\tau}{2}+u_2\right)}^{2}\overbrace{\varphi_R^A\left(-\frac{\tau}{2}+u_1\right)}^{1}|\Omega\rangle$$



OTOC along Observer's worldline

$$\approx \int_{-\infty}^{\infty} dx_+ dx_- \, e^{2i\Lambda x_+ x_-} \, \langle\Omega|\varphi_R^B\left(\frac{\tau}{2}\right) e^{ix_+\hat{P}^+} \varphi_R^B\left(\frac{\tau}{2}+u_2\right)|\Omega\rangle$$

$$\times \langle\Omega|\varphi_R^A\left(-\frac{\tau}{2}\right) e^{ix_-\hat{P}^-} \varphi_R^A\left(-\frac{\tau}{2}+u_1\right)|\Omega\rangle$$



OTOC along Observer's worldline

- Rescaling P^+ and P^-, this becomes

$$\int_{-\infty}^{\infty} dP^- dP^+ \, e^{-i\frac{P^+P^-e^\tau}{2\Lambda}} \langle\Omega|\varphi_R^B(0)|P^+\rangle \langle P^+|\varphi_R^B(u_2)|\Omega\rangle$$
$$\times \langle\Omega|\varphi_R^A(0)|P^-\rangle \langle P^-|\varphi_R^A(u_1)|\Omega\rangle$$

- The leading correction to the OTOC is of order $\left(\frac{e^\tau}{2\Lambda}\right)^2$. The Lyapunov exponent is $2 = \frac{4\pi}{\beta_{\text{dS}}}$ and the scrambling time is $\frac{\beta_{dS}}{2\pi}\log(2\Lambda)$.

Summary and Conclusion

- We quantized a system of two observers and a scalar QFT in dS_2.
- The right observer's algebra $\mathcal{A}_R$ consists of all local quantum fields dressed to the observer, and the observer's Hamiltonian.
- There is no algebraic analogue of the observer's horizon away from the large mass limit.
- The Hartle-Hawking state may be defined algebraically, and furnishes a probability distribution for an observer's mass.
- The OTOC naively indicates that chaos spreads faster than the maximum rate allowed for large N chaotic systems.
- Future directions: incorporate gravitational corrections, make contact with top-down proposals involving DSSYK.