Title: Theoretical physics at ELI ERIC

Speakers: Sergey Bulanov

Collection/Series: Special Seminars

Subject: Other

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Abstract:

As part of a visit to Perimeter of a delegation from the ELI Beamlines laser facility in the Czech Republic, Dr. Bulanov will speak about potential topics for collaboration between Perimeter Institute and ELI theorists on topics related to high energy laser physics. To highlight the interplay between theory and experiment, Dr. Bulanov will briefly mention two experiments where this was realized and is currently planned at the ELI Beamlines facility.

Theoretical Physics @ ELI-ERIC: Strong Field QED Limits

Sergei Bulanov

Extreme Light Infrastructure ERIC (ELI ERIC),
ELI Beamlines Facility

29 May 2025

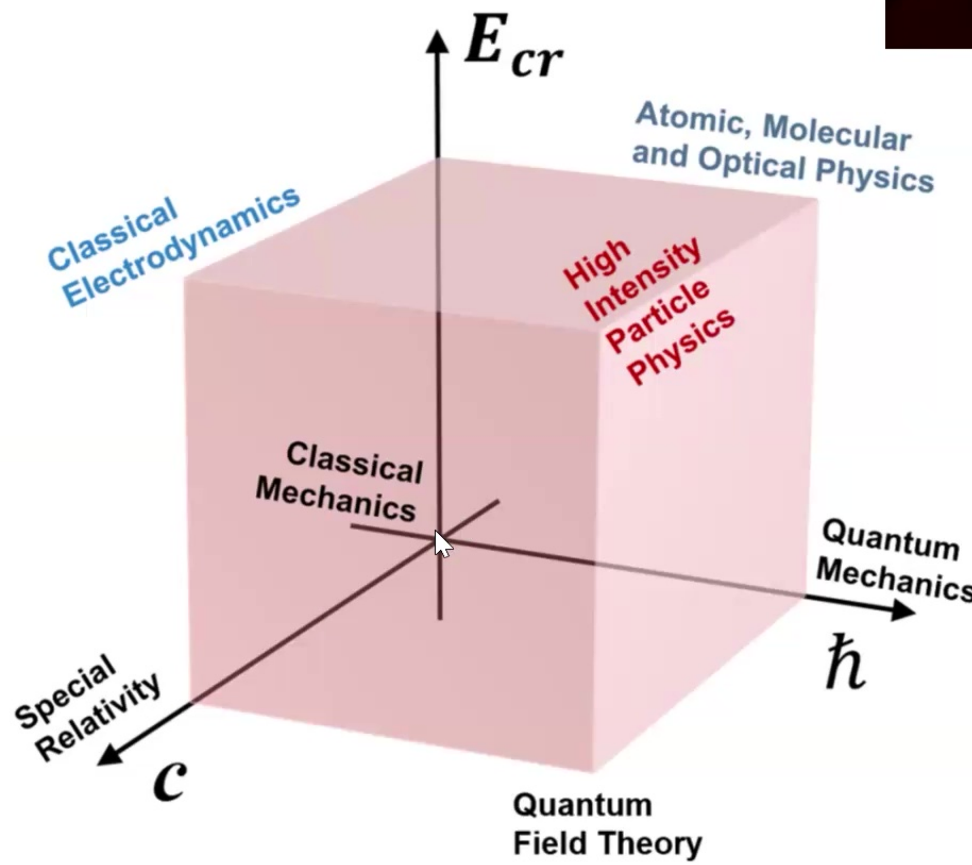
Czech Republic to Canada

Cube of Theories (SF QED)

STRONG FIELD
QUANTUM ELECTRODYNAMICS
(SF QED)

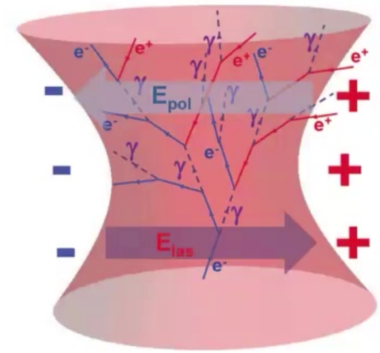
QUANTUM ELECTRODYNAMIC =
QUANTUM MECHANICS + SPECIAL RELATIVITY

SF QED IS AT THE VERTEX OF THE THEORY CUBE



A. Gonoskov, T. G. Blackburn, M. Marklund, and S. S. Bulanov, Charged particle motion and radiation in strong electromagnetic fields, *Rev. Mod. Phys.* 94 045001 (2022)

Critical Field of QED: Sauter-Schwinger Field



$$E_S = \frac{m_e^2 c^3}{e\hbar}$$ Sauter-Schwinger (Bohr) electric field

$$eE_S \lambda\!\!\!^{-}_C = m_e c^2$$ It produces a work equal to $m_e c^2$ over the distance of $\lambda\!\!\!^{-}_C$

$$\lambda\!\!\!^{-}_C = \frac{\hbar}{m_e c}$$ Compton scattering wavelength

$$I_S = 10^{29}\,\frac{\mathrm{W}}{\mathrm{cm}^2} \Rightarrow \mathcal{P} = 10^{21}\left(\frac{\lambda}{1\mu\mathrm{m}}\right)^2 \mathrm{W} \Rightarrow \mathcal{E}_{las} = 3\times10^{6}\left(\frac{\lambda}{1\mu\mathrm{m}}\right)^3 \mathrm{J}$$

Intensity — Power — Energy of EM wave in λ^3 volume

Yoon, J. W., Kim, Y. G., Choi, I. W., et al., Realization of laser intensity over 10^{23} W/cm^2, *Optica*, 8, 631 (2021)

Parameters: Extreme Field Limits (SFQED)

Since quantum electrodynamics is a marriage of quantum mechanics and special theory of rela be described in terms of the Lorentz invariant parameters. For example, the electric field, electron momentum (energy), photon energy depend on the frame of reference.

Invariants:

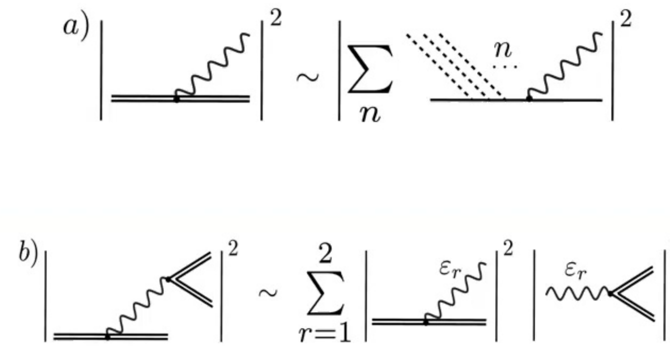
$a_0 = \dfrac{eE}{m_e \omega c}$ (normalized amplitude of the wave)

$\omega_1 \omega_2$ (for the EM waves, which are not copropagating)

$\chi_e = \dfrac{e\hbar\sqrt{-\left(F^{\mu\nu} p_\mu\right)^2}}{m_e^3 c^4} \approx \dfrac{a_0}{a_S}\dfrac{p_e}{m_e c}$ (multi-photon Compton scattering here $a_S = \dfrac{eE_S}{m_e \omega c} = \dfrac{m_e c^2}{\hbar\omega}$)

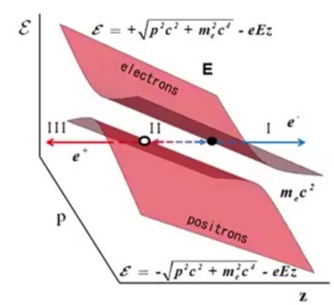
$\chi_\gamma = \dfrac{e\hbar^2\sqrt{-\left(F^{\mu\nu} k_\mu\right)^2}}{m_e^3 c^4} \approx a_0 \dfrac{\hbar^2 \omega_0 \omega_\gamma}{m_e^2 c^4}$ ($N\omega_0 + \omega_\gamma \rightarrow e^+ e^-$: multi-photon Breit-Wheeler process)

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (Tensor of electromagnetic field)



The invariant elecrtric and magnetic fields, $\mathfrak{a}$ and $\mathfrak{b}$, are expressed
in terms the Poincare invariants $\mathfrak{F} = F^{\mu\nu} F_{\mu\nu} = (\boldsymbol{B}^2 - \boldsymbol{E}^2)/2$ and $\mathfrak{G} = F^{\mu\nu} \tilde{F}_{\mu\nu} = \boldsymbol{B} \cdot \boldsymbol{E}$
(dual tensor equals $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$) as

$\mathfrak{a} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} + \mathfrak{F}}$ and $\mathfrak{b} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} - \mathfrak{F}}$ (vacuum polarization and el.-pos. pair creation)



Energy scaling for multi-PW LWFA (χ_e)

The first step towards finding the energy scaling for the LWFA electrons has been made by **Tajima and Dawson (1979)** who v

that the accelerated electron energy equals $\mathcal{E}_e = 2m_e c^2 \gamma_{ph}^2$, where factor γ_{ph} depends on the plasma density as $\gamma_{ph} = \omega / \omega_{pe} = \sqrt{n_{cr} / n}$,

where n_{cr} is the plasma critical density and $\omega_{pe} = \sqrt{4\pi n e^2 / m_e}$.

At the threshold of relativistic self-focusing, the expressions for optimal set of laser-plasma parameters in terms of the laser energy $\mathcal{E}_{las}$ and wavelength λ_0 are as follows

(**Valenta, et al 2025, Valenta and Bulanov 2025**).

$$\chi_e = \frac{e\hbar\sqrt{-\left(F^{\mu\nu}p_\mu\right)^2}}{m_e^3 c^4} \approx \frac{a_0}{a_S}\frac{p_e}{m_e c} > 1$$

$$\chi_\gamma = \frac{e\hbar^2\sqrt{-\left(F^{\mu\nu}k_\mu\right)^2}}{m_e^3 c^4} \approx a_0 \frac{\hbar^2 \omega_0 \omega_\gamma}{m_e^2 c^4} > 1$$

normalized laser amplitude $a_0 = 2.54$ (1)

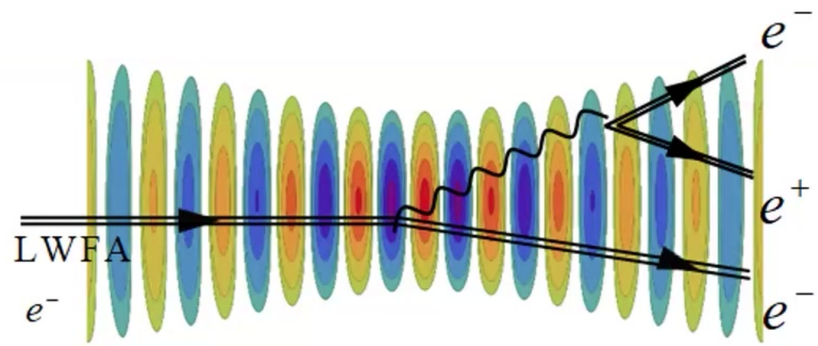
laser pulse waist $w_0 \approx 0.65\lambda_0(\mathcal{E}_{las} / \bar{\mathcal{E}})^{1/3}$ (2)

plasma density $n_e \approx 0.49 n_{cr}(\mathcal{E}_{las} / \bar{\mathcal{E}})^{-2/3}$ (3)

laser pulse duration $\tau_0 \approx 0.23(2\pi / \omega_0)(\mathcal{E}_{las} / \bar{\mathcal{E}})^{1/3}$ (4)

electron energy $\mathcal{E}_m = 4.53 m_e c^2 (\mathcal{E}_{las} / \bar{\mathcal{E}})^{2/3}$ (5)

acceleration length $l_{acc} = 0.77\lambda_0(\mathcal{E}_{las} / \bar{\mathcal{E}})$ (6)



Here $\bar{\mathcal{E}} = 29\ \mu J$ for $1\,\mu$m wavelength.

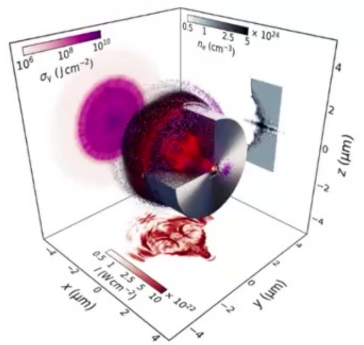
For $\mathcal{E}_{las}$=1 KJ, 1 μm wavelength laser can accelerate electrons up to 140 GeV.

The plasma density is $1.6 \times 10^{16}\,\mathrm{cm}^{-3}$. The acceleration length is 11.5 m.

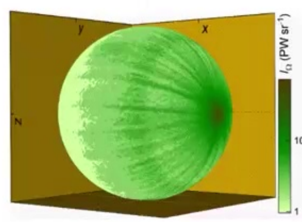
The laser pulse waist and length are equal to 140 μm and 110 μm respectively.

Gamma Flash (χ_γ)

T. Nakamura, et al, Phys. Rev. Lett. 108, 195001 (2012); K. V. Lezhnin, et al., Phys. Plasmas 25, 123105 (2018); P. Hadjisolomou, et al., Phys. Rev. E 104, 015203 (2021); P. Hadjisolomou, et al., Journal of Plasma Physics, 88, 905880104 (2022).

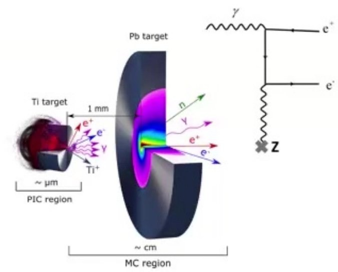


- **3D PIC: Pulse-Target Interaction** of an **ultra-intense** laser pulse with a foil target. 30% efficiency

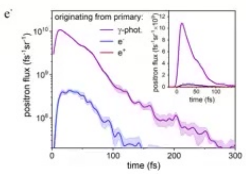


The γ -photon radiant intensity

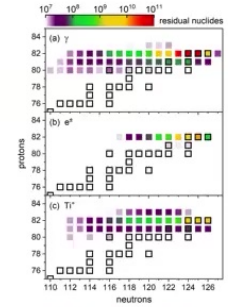
Electron-positron pairs and radioactive nuclei production by irradiation of high-Z target with γ-photon flash



Setup



Positron flux

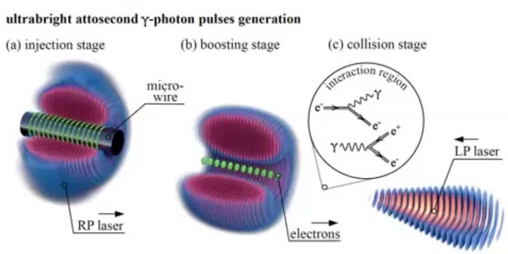


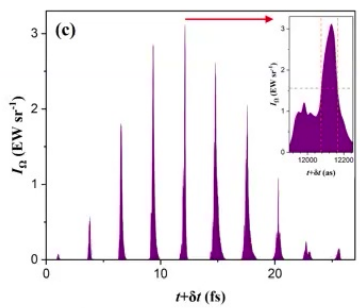
High Z nuclides

D. Kolenatý, et al., Electron-positron pairs and radioactive nuclei production by irradiation of high-Z target with γ-photon flash generated by an ultra-intense laser in the λ3 regime, Phys. Rev. Res. 4, 023124 (2022)

Attosecond GeV collimated γ-photon beams in collision of 25 PW+ 25 PW laser pulses

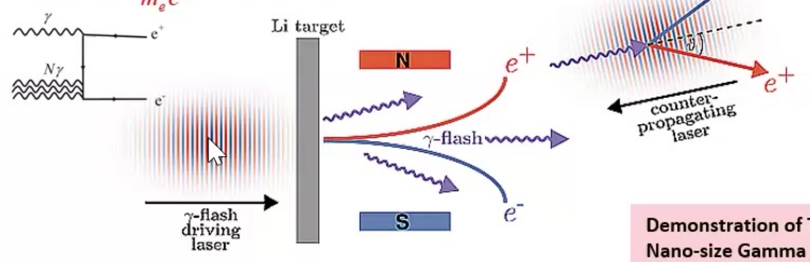
P. Hadjisolomou, et al. Physical Review E 111 (2), 025201 (2025)





All-optical nonlinear Breit-Wheeler pair production with γ- flash photons (Gamma-Gamma Collider)

$$\chi_\gamma \approx a_0 \frac{\hbar^2 \omega_0 \omega_\gamma}{m_e^2 c^4} \quad (N\omega_0 + \omega_\gamma \rightarrow e^+ e^-)$$

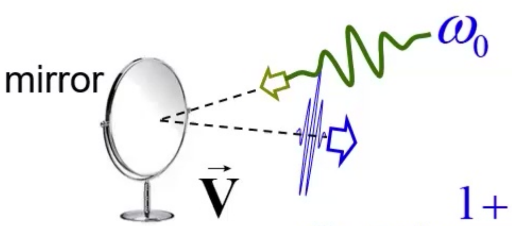


A.MacLeod, et al., All-optical nonlinear Breit-Wheeler pair production with flash photons, Physical Review A 107, 012215 (2023)

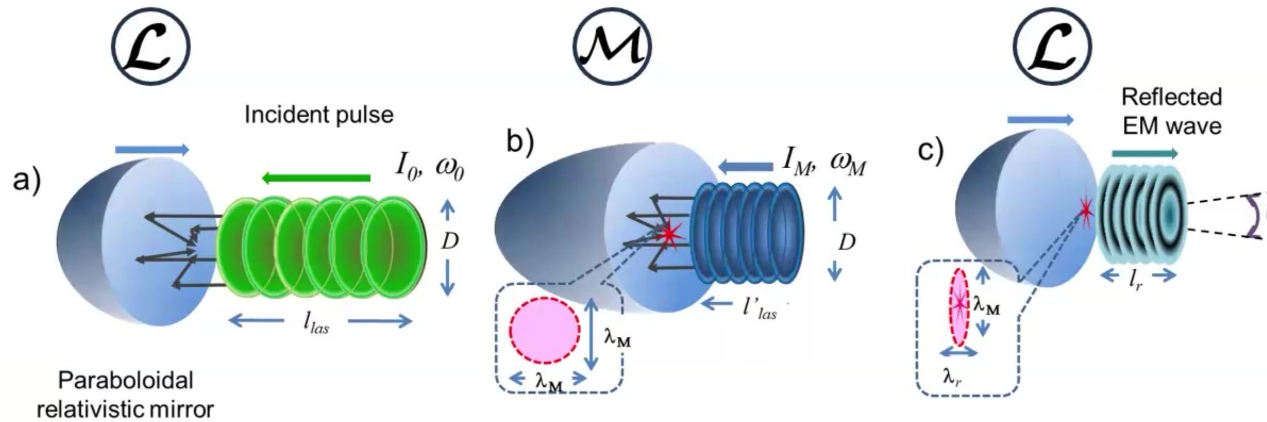
Demonstration of The Brightest Nano-size Gamma Source
AS Pirozhkov, A Sagisaka, K Ogura, EA Vishnyakov, AN Shatokhin, ...
arXiv preprint arXiv:2410.06537

Towards Schwinger Limit with Relativistic Mirrors (Relativisti

An electromagnetic wave reflected off a moving mirror acquires modified frequency and electric field as predicted by
A. Einstein, Ann. Phys. (Leipzig) 17, 891 (1905)



$$\omega_r = \omega_0 \frac{1+V}{1-V}$$

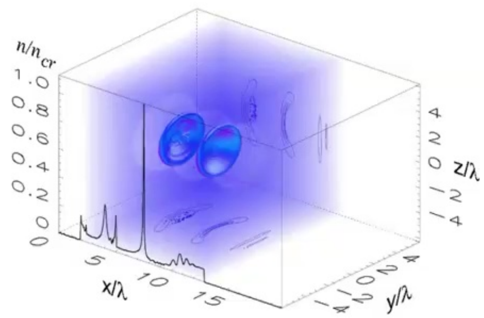


The concept of a flying relativistic mirror. a) Laboratory frame of reference $\mathcal{L}$. Before reflection from the mirror, the laser pulse propagates from right to left. b) Frame of reference where the mirror is at rest, $\mathcal{M}$. The incident laser pulse has a length $l'_{las} \approx l_{las}/2\gamma_M$ and wavelength $\lambda_M \approx \lambda_0/2\gamma_M$. After reflection from a parabolic mirror, the radiation is focused to the region of size $\approx \lambda_M$. c) Laboratory frame of reference $\mathcal{L}$. The reflected pulse is compressed by a factor $4\gamma^2_M$ and its wavelength becomes equal $\lambda_r \approx \lambda_0/4\gamma^2_M$. The focal region has the form of an ellipsoid with the longitudinal λ_r and transverse λ_M dimension, respectively. Radiation is collimated within the angle $\theta \sim 1/\gamma_M$.

S. V. Bulanov, T. Zh. Esirkepov, T. Tajima, Light intensification towards the Schwinger limit, *Phys. Rev. Lett*. 91, 085001 (2003)
G. Mourou, T. Tajima, S. V. Bulanov, Optics in the Relativistic Regime, *Rev. Mod. Phys.* 78, 309 (2006)
S. V. Bulanov, T. Zh. Esirkepov, M. Kando, A. S. Pirozhkov, and N. N. Rosanov, Relativistic Mirrors in Plasmas–Novel Results and Perspectives, *Physics Uspekhi* 56, 429 (2013)
S. V. Bulanov, T. Zh. Esirkepov, M. Kando, and J. Koga, Relativistic mirrors in laser plasmas (analytical methods), *Plasma Sources Sci. Technol*. 25, 053001 (2016)
H. Vincenti, T. Clark, L. Fedeli, P. Martin, A. Sainte-Marie, and N. Zaim, Plasma mirrors as a path to the Schwinger limit: theoretical and numerical developments, *Eur. Phys. J. Spec. Top.* 232 2303 (2023)

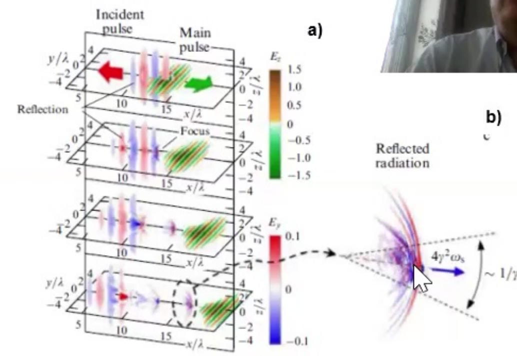
Relativistic Flying Mirror: Concept & Proof-of-Principle

3D Wake Wave:



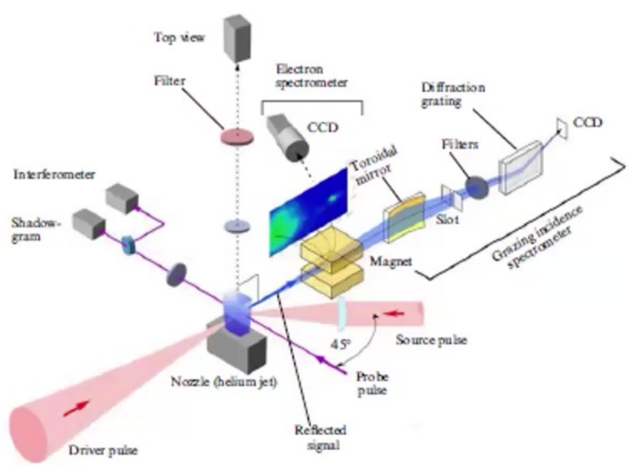
3D PIC:

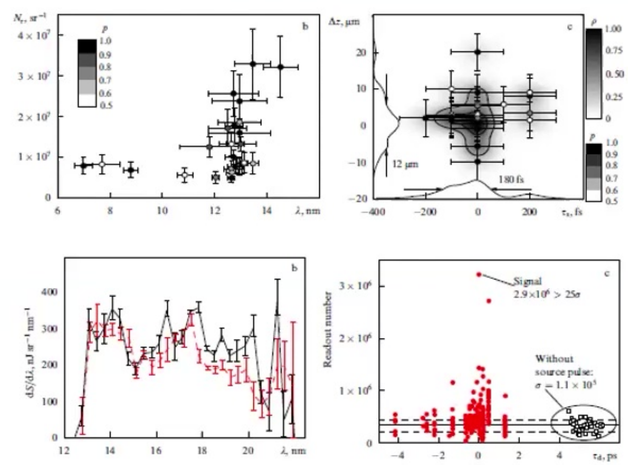
$$\omega_r = \omega_0 \frac{1+\beta_M}{1-\beta_M}, \quad I_r = I_0 \gamma_{ph}^6 \mathbf{R} \left(\frac{D}{\lambda_0}\right)^2$$



SVB et al., PRL 91, 085001 (2003); M.Kando et al., PRL 99, 135001 (2007); A. Pirozhkov, PoP 14, 123106 (2007); M.Kando et al., PRL 103, 235003 (2009); M.Lobet et al., Phys. Lett. A 377, 1114 (2013); SVB et al., Phys. Usp. (2013)

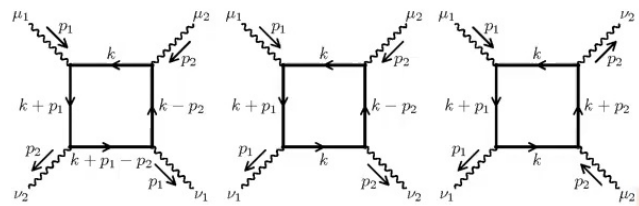
Experiments:





Photon–Photon Scattering

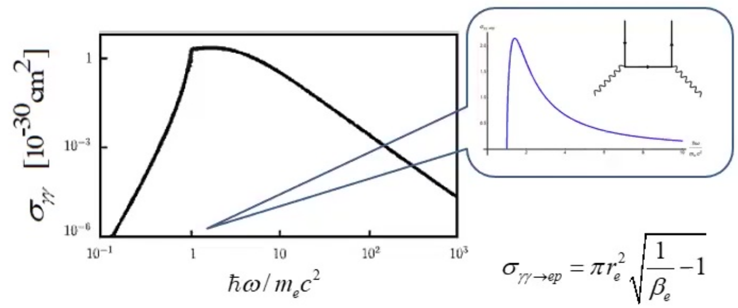
In the QED, photon-photon scattering occurs via creation-annihilation of virtual electron-positron pairs by two initial photons followed by annihilation of the pairs into final photons



R. Karplus and M. Neuman, Phys. Rev. 83, 776 (1951)
A. Akhieser, L. Landau and I. Pomeranchouk, Nature 138, 206 (1936)

Dependence of the photon-photon scattering cross section on the photon frequency (assume that $\omega_1 = \omega_2 = \omega_\gamma$)

$$\sigma_{\gamma\gamma} = \begin{cases} \left(\dfrac{973}{10125\pi}\right)\alpha^2 r_e^2 \left(\dfrac{\hbar\omega_\gamma}{m_e c^2}\right)^6 & \text{for } \hbar\omega_\gamma << m_e c^2 \\ \left(\dfrac{3}{12\pi}\right)^2 \alpha^2 r_e^2 \left(\dfrac{m_e c^2}{\hbar\omega_\gamma}\right)^2 & \text{for } \hbar\omega_\gamma >> m_e c^2 \end{cases}$$



$$\sigma_{\gamma\gamma\to ep} = \pi r_e^2 \sqrt{\frac{1}{\beta_e} - 1}$$

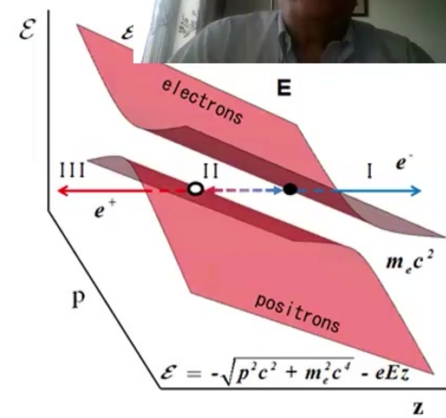
$$\beta_e = \sqrt{1 - \frac{2 m_e^2 c^4}{\hbar^2 \omega_1 \omega_2 (1 - \cos\varphi)}}$$

G. Breit and J. A. Wheeler, Physical Review 46, 1087 (1934)

Experiment:

ATLAS Collaboration, Nature Physics 13, 852 (2017)

Heisenberg-Euler Lagrangian



In the long wavelength and low frequency approximation ($|\partial_\mu A_\nu| / |A_\mu| << \lambdabar_C^{-1}$)
the Lagrangian describing the electromagnetic field in vacuum is

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'$$

where $\mathcal{L}_0 = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ gives the Maxwell equations.

The Heisenberg-Euler term

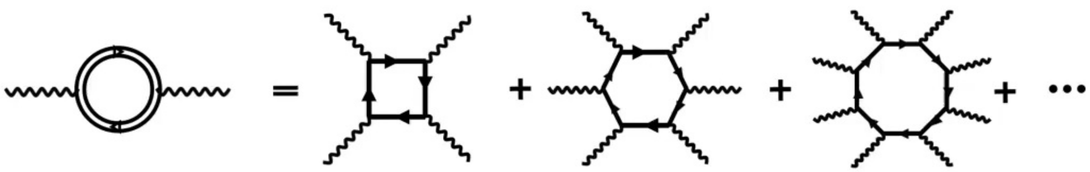
$$\mathcal{R}e\{\mathcal{L}'\} = -\frac{m_e^4}{8\pi^2}\int_0^\infty \frac{\exp(-\eta)}{\eta^3}\left\{1 - \frac{\eta^2}{3}\left(\mathfrak{e}^2 - \mathfrak{b}^2\right) - \left[\eta\mathfrak{e}\cot(\eta\mathfrak{e})\right]\left[\eta\mathfrak{b}\coth(\eta\mathfrak{b})\right]\right\} d\eta$$

$$\mathcal{I}m\{\mathcal{L}\} = \frac{m_e^4}{4\pi^3}\mathfrak{e}^2 \exp\left(-\frac{\pi}{\mathfrak{e}}\right)$$

The invariant fields $\mathfrak{a}$ and $\mathfrak{b}$ are

$$\mathfrak{e} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} + \mathfrak{F}} \text{ and } \mathfrak{b} = \sqrt{\sqrt{\mathfrak{F}^2 + \mathfrak{G}^2} - \mathfrak{F}}$$

Here we use the units $c = \hbar = 1$, and the EM field is normalized on the QED critical field E_S.



W. Heisenberg and H. Euler, Zeit. fuer Phys. 98, 714 (1936)

Radiation Corrections

N. B. Narozhny, Sov. Phys. JETP 28, 371 (1969); V. I. Ritus,

At the focus of 10 PW laser the field intensity can reach 10^{24}W/cm^2, i.e. $a_0 = 10^3$

Vacuum polarization changes the refraction index: radiation correction to the "photon mass" results in the dispersion equation for the EM wave frequency and wave vector

$$\omega^2 - k^2c^2 - \mu^2_{\parallel,\perp}\frac{c^4}{\hbar^2} = 0$$

where

$$\mu^2_{\parallel,\perp} = -\alpha m_e^2 \begin{cases} \left[\dfrac{11\pm3}{90\pi}\chi_\gamma^2 + i\sqrt{\dfrac{3}{2}}\dfrac{3\pm1}{16}\chi_\gamma \exp\left(-\dfrac{8}{3\chi_\gamma}\right)\right] & \text{for } \chi_\gamma \ll 1 \\ \left[\dfrac{5\pm1}{28\pi^2}\sqrt{3}\Gamma^4\left(\dfrac{2}{3}\right)(1-i\sqrt{3})(3\chi_\gamma)^{2/3}\right] & \text{for } \chi_\gamma \gg 1 \end{cases}$$

with $\alpha = e^2/\hbar c \approx 1/137$. When $\alpha\chi_\gamma^{2/3} \to 1$ the "photon mass" tends to m_e.

In the limit $\chi_\gamma \ll 1$ the difference between the vacuum refraction index and unity is

$$\Delta n_{\parallel,\perp} = \alpha\frac{11\pm3}{45\pi}\left(\frac{E}{E_S}\right)^2$$

i. e. the normalized phase velocity of the EM wave equals $\beta_{\parallel,\perp} = 1 - \varepsilon_{\parallel,\perp}(E/E_S)^2$ with $\varepsilon_{\parallel,\perp} = \alpha(11\pm3)/45\pi \approx 10^{-4}$.

Probing the QED Vacuum

E. B. Aleksandrov, A

Sov. Phys. JETP 62,

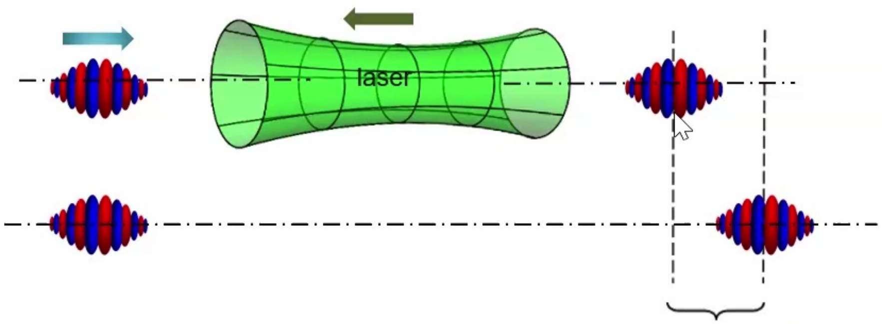
B. King, T. Heinzl, High Power Las. Sci. Eng. 4, 1 (2016)

F. Karbstein, Particles 3, 39 (2020)

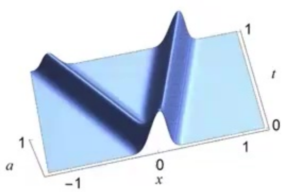
Vacuum polarization

$v = c(1-2\delta n), \qquad \delta n = 2\varepsilon_2 W^2 \approx \alpha\left(E_{l\alpha}/E_S\right)^2$

$\alpha = e^2/\hbar c = 1/137, \quad E_S = m_e^2 c^3/e\hbar$

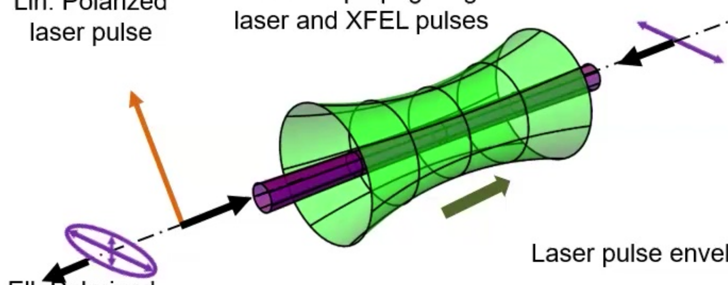


$(\omega+kc)(\omega-kc+2kc\varepsilon_2 W^2)=0$



$\Delta x = \alpha\left(E_{l\alpha}/E_S\right)^2 l_f$

Vacuum birefringence



T. Heinzl, et al., Opt. Express 267, 318 (2006)

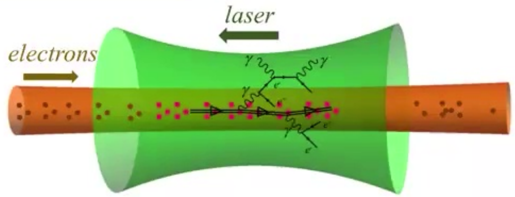
H.-P. Schlenvoigt, et al., Phys. Scr. 91, 023010 (2016)

B. Shen, et al., Plasma Phys. Contr. Fus. 60, 044002 (2018)

T. Tajima and R. Li, SPIE (2018)

Synergic Cherenkov Radiation-Compton Scatter

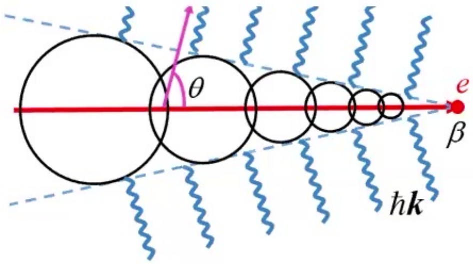
I.M. Dremin, JETP Lett. 76, 151 (2002); A. J. Macleod, A. Noble, and D. A. Jaroszynski, Phy
S. V. Bulanov, P. V. Sasorov, S. S. Bulanov, G. Korn, Synergic Cherenkov-Compton radiation,



$$\gamma_0 > \gamma_{Ch} = \frac{1}{\sqrt{2\Delta n_\pm}} = \sqrt{\frac{45\pi E_S^2}{\alpha(11\pm 3)E_0^2}} \approx 30\sqrt{\frac{I_S}{I_0}}$$

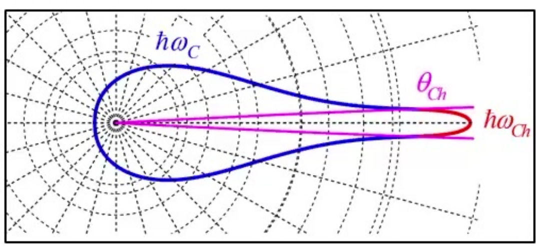
The laser intensity $I_0 = cE_0^2/4\pi$ in the focus region of 10 PW laser is equal to 10^{24} W/cm^2;
$I_S = cE_S^2/4\pi \approx 10^{29}\,W/cm^2$, i. e. the Cherenkov radiation threshold is exceeded for the electron energy above 10 GeV.

The Cherenkov cone with the angle $\theta_{Ch} = 2\sqrt{\varepsilon_\pm I_0/I_S}$ in the focus of 10 PW laser is $\approx 2\times 10^{-5}$.



$$\boldsymbol{p}_0 + \hbar\boldsymbol{k}_0 = \boldsymbol{p} + \hbar\boldsymbol{k},$$
$$m_e c^2\gamma_0 + \hbar\omega_{\gamma,0} = m_e c^2\gamma + \hbar\omega_\gamma,$$
$$\boldsymbol{k} = \frac{\boldsymbol{k}}{|\boldsymbol{k}|c} n_\pm \omega$$

At the high photon energy end, when $\chi_\gamma = \frac{E_0}{E_S}\frac{\hbar(\omega + k_x c)}{m_e c^2} >> 1$, the vacuum polarization effects and the Cherenkov radiation weaken.
As a result, the photons with the energy above $\hbar\omega_\gamma = m_e c^2 E_S / E_0$ are not present in the radiation. For 10 PW laser parameters this energy is 100 MeV.

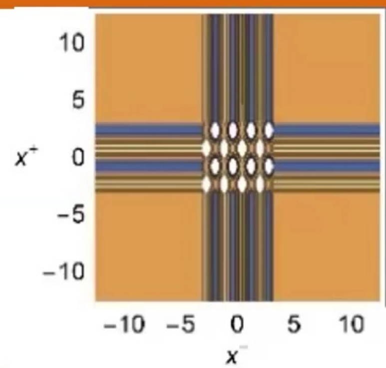


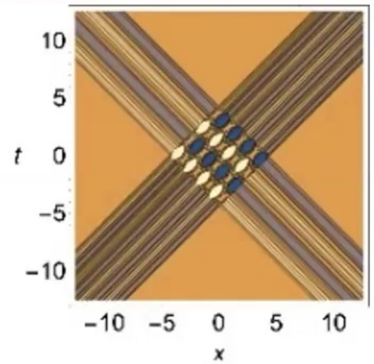
High Order Harmonics in QED Vacuum

In Dirac's light-cone coordinates
the Lagrangian variation

$$\partial_+(\partial\mathcal{L}/\partial u)+\partial_-(\partial\mathcal{L}/\partial w)=0$$

yields system of **quasilinear**
nonlinear equations for EM wave





Di Piazza A, Hatsagortsya
Harmonic generation fro
Phys. Rev. D 72 085005

Lundstrom E, Brodin G, Lundin J, Marklund M, Bingham R, Collier J, Mendonca J T, Norreys P 2006 Using high-power lasers for detection of elastic photon-photon scattering Phys. Rev. Lett. 96 083602

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Sasorov P V, Esirkepov T, Pegoraro F, Bulanov S V, Generation of High Order Harmonics in Heisenberg-Euler Electrodynamics NJP 23 (10), 105003 (2021)

$$\partial_+ w-\partial_- u=0$$

$$\left[1-4\varepsilon_2 uw-9\varepsilon_3 u^2w^2+\dots\right]\partial_+ u-\left[\varepsilon_2 w^2-3\varepsilon_3 uw^3+\dots\right]\partial_- u-\left[\varepsilon_2 u^2-3\varepsilon_3 u^3 w+\dots\right]\partial_+ w=0$$

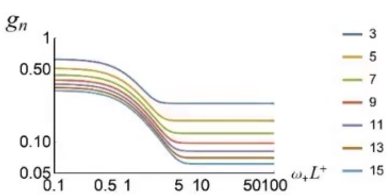
with $\varepsilon_2=(2/45\pi)\alpha$ and $\varepsilon_3=(32/315\pi)\alpha$ and $W_0=E_0/E_S$.

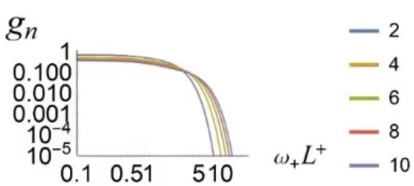
HOH & Mixing

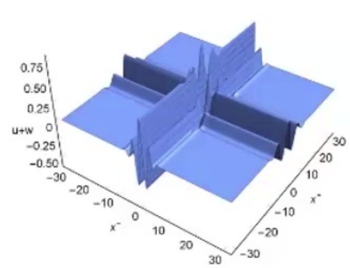
Perturbation theory: $u_0(x^-)+\varepsilon_2 u_1+\varepsilon_3 u_2\dots$ and $w_0(x^+)+\varepsilon_2 w_1+\varepsilon_3 w_2+\dots$ yields

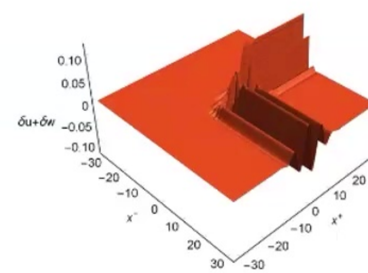
$$u(x^-,x^+)=u_0(x^-)+\varepsilon_2\left(u_0^2(x^-)w_0^2(x^+)+\partial_- u_0\int^{x^+} w_0^2(s)ds\right)-\frac{3}{2}\varepsilon_3\left(u_0^3(x^-)w_0^3(x^+)+\partial_-(u_0^2)\int^{x^+} w_0^3(s)ds\right)+\dots$$

$$w(x^-,x^+)=w_0(x^+)+\varepsilon_2\left(u_0^2(x^-)w_0^2(x^+)+\partial_+ w_0\int^{x^-} u_0^2(s)ds\right)-\frac{3}{2}\varepsilon_3\left(u_0^3(x^-)w_0^3(x^+)+\partial_+(w_0^2)\int^{x^-} u_0^3(s)ds\right)+\dots$$









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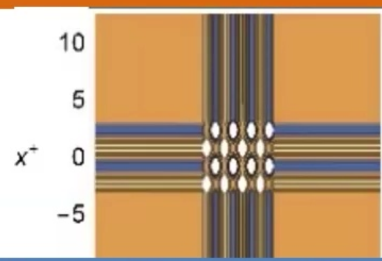
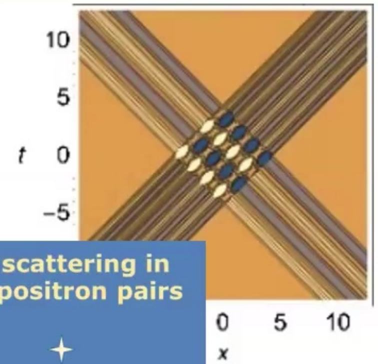
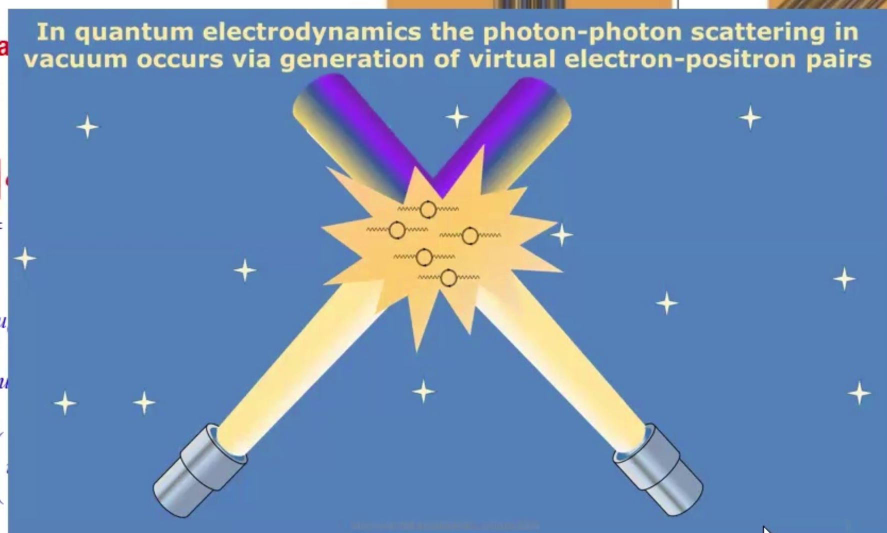
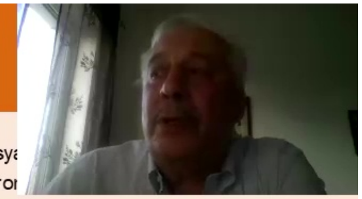
$w(x^-,x^+)=w_0(x^+)+\varepsilon_2\Big($









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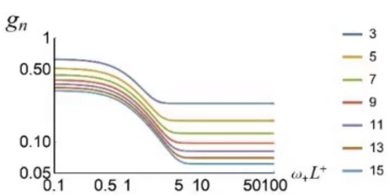
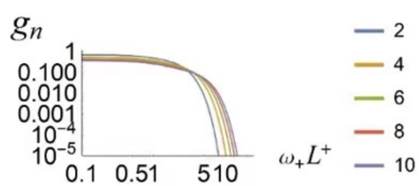
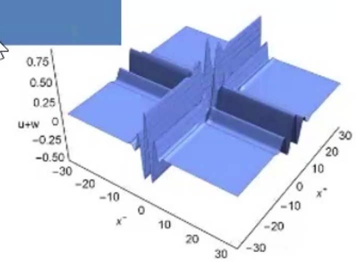
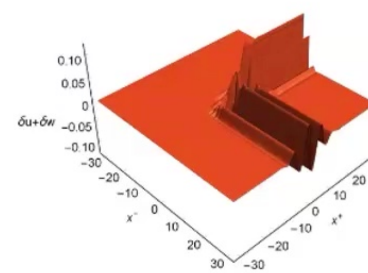








Nonlinear EM waves in the QED vacuum

H. Kadlecova, G. Korn, S. V. Bulanov, Electromagnetic s
Phys. Rev. D 99, 036002 (2019)
F. Pegoraro and S. V. Bulanov, Hodograph solutions of tl
electrodynamics in the quantum vacuum, Phys. Rev. D 100, 036004 (2019)
F. Pegoraro and S. V. Bulanov, Nonlinear, nondispersive wave equations: Lagrangian and Hamiltonian functions in the Hodograph transformation, Phys. Lett.A 384,126064 (2020)

We use Dirac's light-cone coordinates

$$x^+ = \frac{x+t}{\sqrt{2}}, \quad x^- = \frac{x-t}{\sqrt{2}}, \quad \partial_\pm = \frac{\partial}{\partial x^\pm}$$

Lagrangian variation $\partial_+(\partial\mathcal{L}/\partial u) + \partial_-(\partial\mathcal{L}/\partial w) = 0$ yields system of nonlinear equations for EM wave

$$\partial_+ w - \partial_- u = 0$$

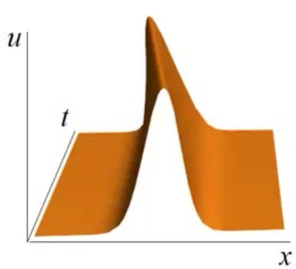
$$\left[1 - 4\varepsilon_2 (W_0 + w) u - 9\varepsilon_3 (W_0 + w)^2 u^2\right]\partial_+ u - \left[\varepsilon_2 (W_0 + w)^2 + 3\varepsilon_3 (W_0 + w)^3 u\right]\partial_- u - \left[\varepsilon_2 u^2 + 3\varepsilon_3 (W_0 + w) u^3\right]\partial_+ w = 0$$

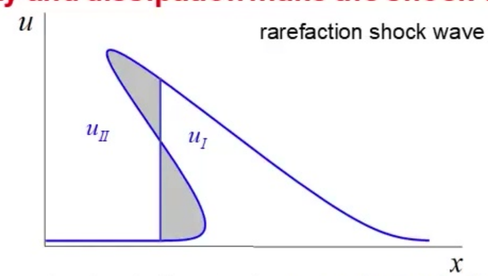
with $\varepsilon_2 = (2/45\pi)\alpha$ and $\varepsilon_3 = (32/315\pi)\alpha$ and $W_0 = E_0/E_S$.

Analizing this system with the hodograph transform, self-similar solutions, etc. gives

$$\partial_t u + \left(v_W - 2\left(4\varepsilon_2^2 + 3\varepsilon_3\right)W^3 u\right)\partial_x u = 0$$

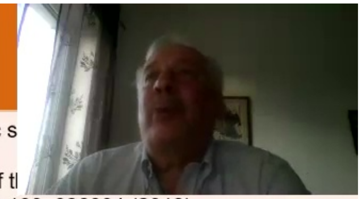
Riemann wave solution describes the nonlinear EM wave steepening and breaking leading to the shock wave formation: **the nonlinearity and dissipation make the shock wave**.





Shock wave front velocity equals $v_{SW} = c(1 - 2\delta n) - (4\varepsilon_2^2 + 3\varepsilon_3)W^3(u_I + u_{II})$

Nonlinear EM waves in the QED vacuum



We u…

…, Electromagnetic s…
…graph solutions of t…
…um, Phys. Rev. D 100, 036004 (2019)
…near, nondispersive wave equations: Lagrangian and
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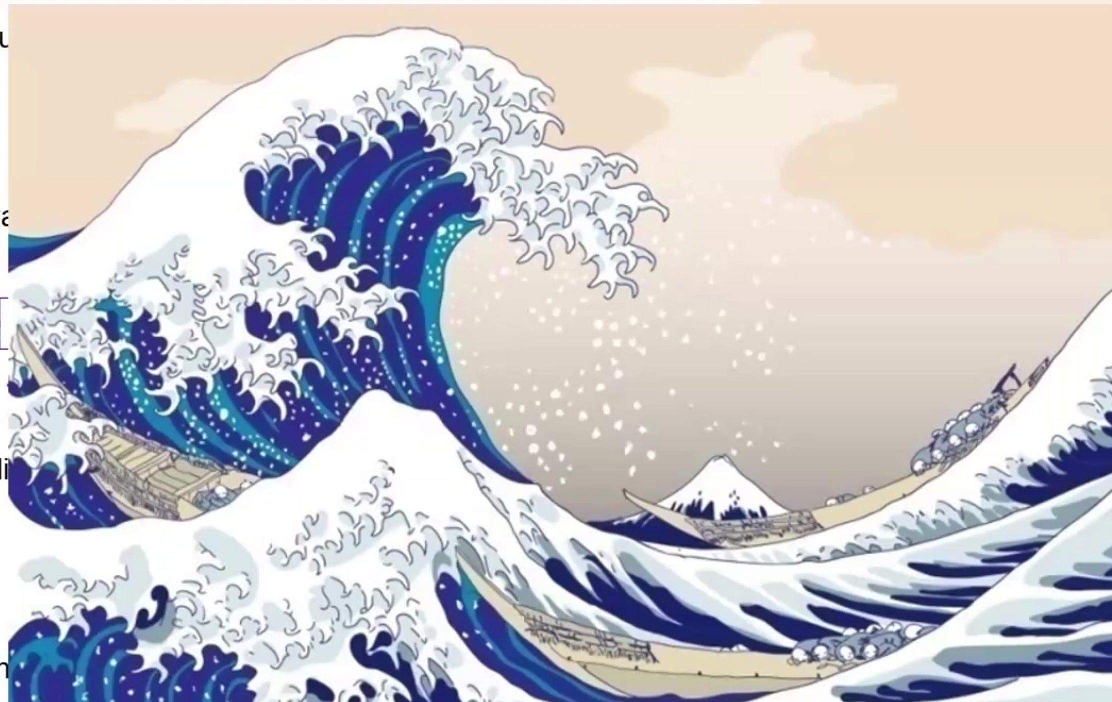


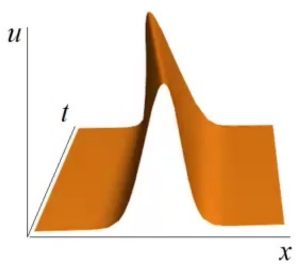
Lagra… …s for EM wave

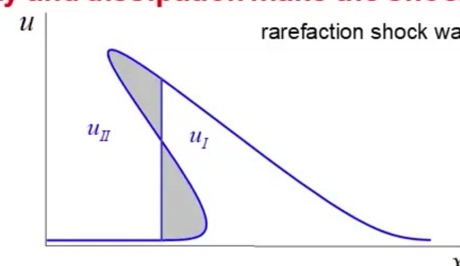
$$\ldots V_0 + w)u^3 \big]\partial_+ w = 0$$

with …

Anali…

Riem…
leading to the shock wave formation: **the nonlinearity and dissipation make the shock wave**.





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Electromagnetic Solitons in QED Vacuum

S. V. Bulanov, P. V. Sasorov, H. Kadlecova, S. S. Bulanov, G. Korn, Electromagnetic solitons in quantum vacuum,

The nonlinearity and dispersion make the soliton

The photon invariant mass:

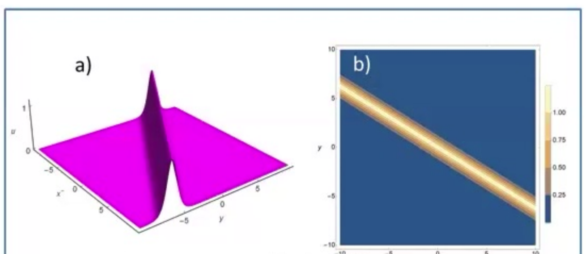
$$\mu^2 = -\alpha m_e^2 \left\{ \frac{7}{45\pi}\left[\chi_\gamma^2 + \frac{1}{3}\chi_\gamma^4\right] + i\sqrt{\frac{3}{2}}\frac{1}{4}\chi_\gamma \exp\left(-\frac{8}{3\chi_\gamma}\right)\right\}$$

The term $\frac{1}{3}\chi_\gamma^4$ describes the dispersion effects

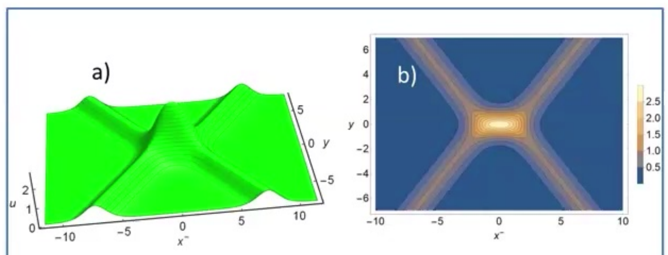
Implementation of nonlinearity and dispertion effects to the wave equation results in the Kadomtsev-Petviashvili equation (known as 2D or 3D Korteveg de Vries equation)

$$\partial_-\left[\partial_+ u + \left(\frac{4e^2}{45\pi}W_0^2 + \frac{32\sqrt{2}e^2}{105\pi}W_0^3 u\right)\partial_- u + \frac{8e^2}{135\pi m_e^2}W_0^4\partial_{---}u\right] = -\frac{1}{2}\partial_{yy}u$$

Typical KP solitons :

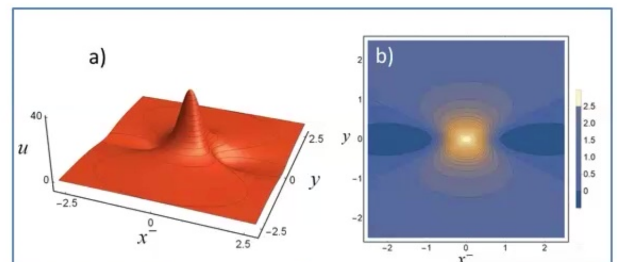


single soliton



double soliton

ROGUE WAVE



lump soliton

D. J. Korteweg and G. de Vries, Philos. Mag. 39, 422 (1885); B. B. Kadomstev and V. I. Petviashvili, Sov. Phys. Dokl. 15, 539 (1970); G. Biondini and D. E. Pelinovsky, Kadomtsev-Petviashvili equation, Scholarpedia 3, 6539 (2018)

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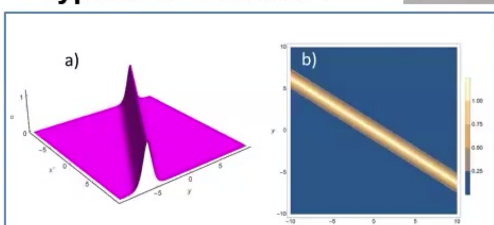

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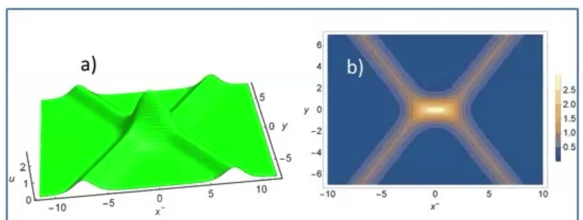

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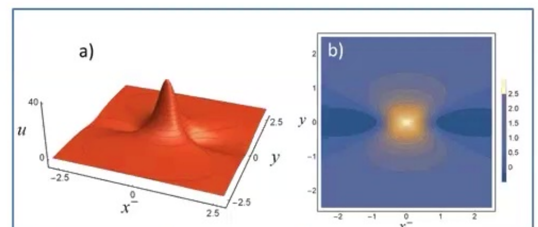
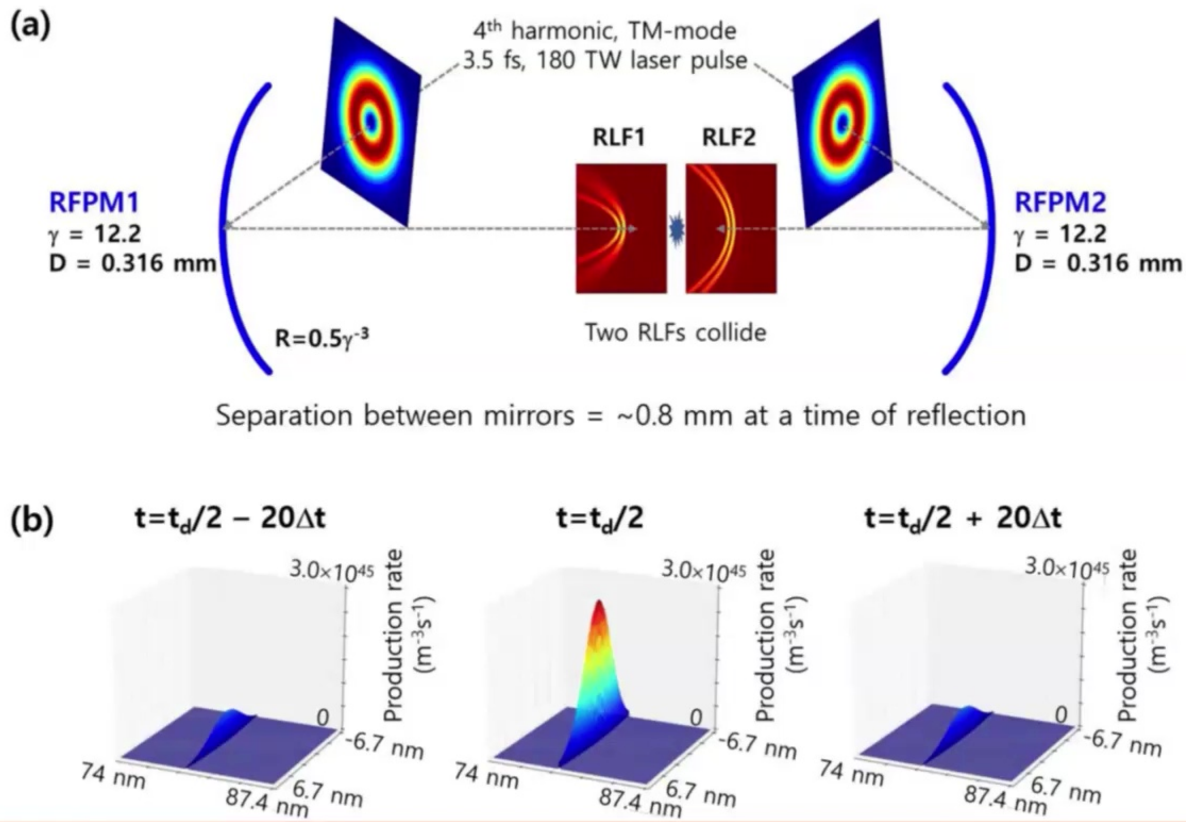
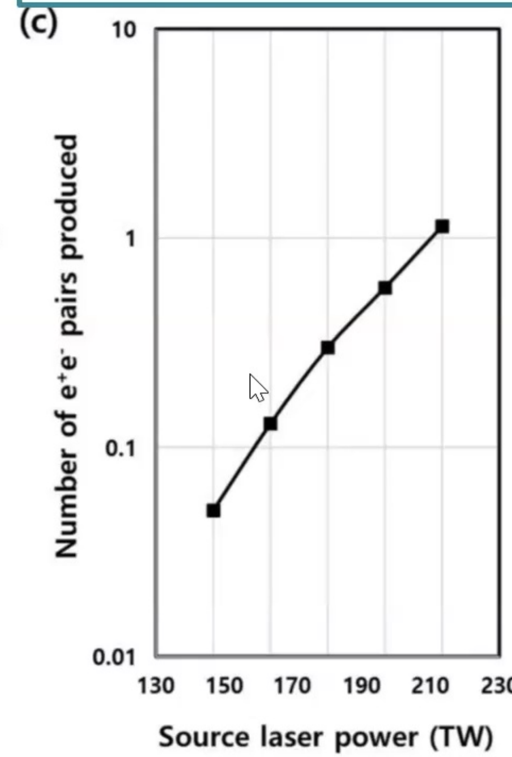


lump soliton

Approaching the Schwinger Limit

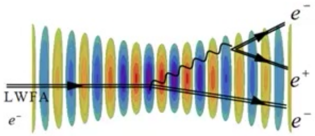
$$W \approx \frac{\Delta x^3 \Delta t}{\lambda_C^4} \frac{E^2}{E_S^2} \exp\left(-\pi \frac{E_S}{E} \right)$$





S. S. Bulanov, T. Zh. Esirkepov, A. G. R. Thomas, J. K. Koga, and S. V. Bulanov, Schwinger Limit Attainability with Extreme Power Lasers, *Phys. Rev. Lett.* 105, 220407 (2010);
T. M. Jeong, Bulanov, S. V., Sasorov, P., Valenta, P., Korn, G., Esirkepov, T., Koga, J., Pirozhkov, A., Kando, M., Bulanov, S. S., Relativistic flying laser focus by a laser-produced parabolic plasma mirror, *Phys. Rev. A* 104, 053533 (2021);
T. M. Jeong, S. V. Bulanov, R. Shaisultanov, and P. Hadjisolomou, Toward superstrong fields with a relativistic curved plasma mirror, *Phys. Rev. A* 111, 032218 (2025).

Planning of Experiments on Collision of L3-L4 Laser Pulses (1-100 GeV LWFAe+10^{23} W/cm

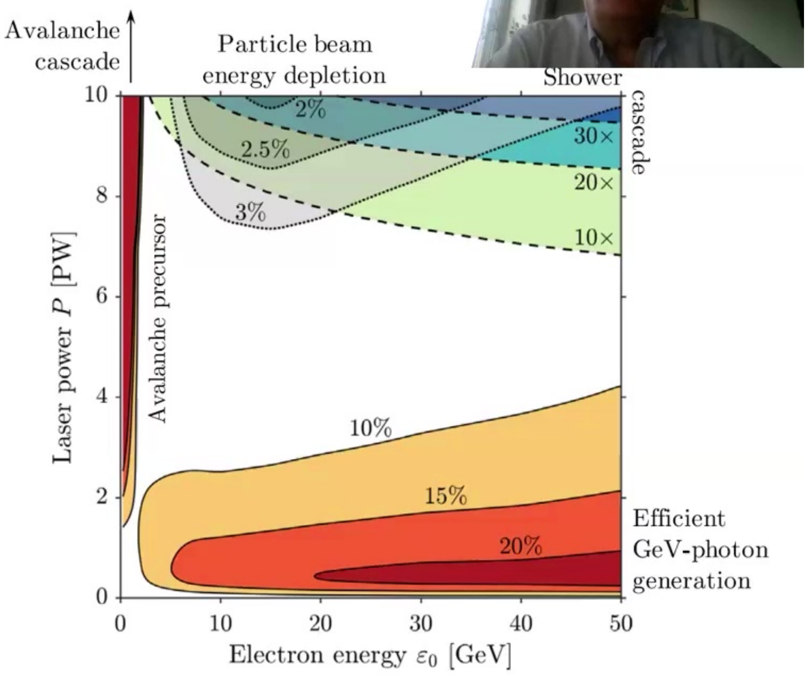


L3 HAPLS

Laser pulse Energy:	30 J
Laser pulse Duration:	<30 fs
Laser pulse Peak Power:	1 PW
Repetition Rate:	10 Hz

L4 ATON

Laser pulse Energy:	1500 J
Laser pulse Duration:	150 fs
Laser pulse Peak Power:	10 PW
Repetition Rate:	<0.02 Hz



(i) high-energy photon generation [contours of the number of high-energy photons (solid)], (ii) shower type cascade [contours of the number of *e−e+* pairs (dashed)], and (iii) electron-beam-energy depletion [contours of the final electron-beam energy as percentage of the initial electron-beam energy (dotted)]. (1) the efficient generation of multi-GeV photons using electron beams of high energy and (2) an avalanche precursor.

J. Magnusson, A. Gonoskov, M. Marklund, T. Zh. Esirkepov, J. K. Koga, K. Kondo, M. Kando, S. V. Bulanov, G. Korn, C. G. R. Geddes, C. B. Schroeder, E. Esarey, and S. S. Bulanov, Multiple colliding laser pulses as a basis for studying high-field high-energy physics", Phys. Rev. A 100, 063404 (2019);
A.Gonoskov, T. G. Blackburn, M. Marklund, and S. S. Bulanov, Charged particle motion and radiation in strong electromagnetic fields, Rev. Mod. Phys. 94 045001 (2022);
E.Rockafellow, B.Miao, J.E.Shrock, A.Sloss, M.S.Le, S.W.Hancock, S.Zahedpour, R. C.Hollinger, S.Wang, J.King, P.Zhang, J.Šišma, G.M.Grittani, R.Versaci, D.F.Gordon, G.J.Williams, B.A.Reagan, J.J.Rocca, H.M.Milchberg, Development of a high charge 10 GeV laser electron accelerator, Physics of Plasmas 32 (5) (2025)

CONCLUSION

- The QED vacuum in the long - wavelength limit behaves as continuous medium with the optical properties determined by interacting electromagnetic fields

- In the high photon energy range the QED vacuum is a medium possessing dispersion and dissipation

- The vacuum can be considered as a "plasma" of virtual electron - positron pairs and high energy photon mixture. The electron - positron pairs may become real when the electric field approaches the Schwinger limit : $E \rightarrow E_S$ ($\chi_\gamma >> 1$) forming "Lepton - Gamma Plasma".

- We extend the field of applications of the methods of Nonlinear Wave Theory to the QED vacuum

- Relativistic Flying Mirrors (Relativistic Catoptrics) pave a way towards critical QED field limits

SEVERAL PROBLEMS TO DISCUSS, CHOOSE AND COLLA

- *Theoretical and experimental developing of novel methods of high - energy charged particle acceleration (LWFA electrons & RPA ions)*

- *Demonstating and using the high - energy photon generation within the framework of Gamma Flash concept (theory & experiment)*

- *Work on Relativistic Catoptrics for approaching the Schwinger field limit :* $E \rightarrow E_S\ (\chi_\gamma >> 1)$

- *Probing the texture of QED vacuum (vacuum polarization, nonlinear EM waves in QED vacuum, finding optimal field configurations)*

- *Better understanding of the counterplay between the Breit - Wheeler and Schwinger mechanisms of boiling vacuum*

- *Lowering the field intensity required to observe the Schwinger electron - positron pair creation (multi - pulse collision,* 4π *dipole field...)*

Thank you for listening to me!