Title: Probing Quantum Features of Gravity in Tabletop Regime and Reassessing the I.I.D. Assumption in Quantum Probability Assignments

Speakers: Linqing Chen

Collection/Series: Quantum Gravity

Subject: Quantum Gravity

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Abstract:

Recent advancements in tabletop experiments may offer the first empirical proof that gravity is not classical. In the first part of my talk, I will present two effects that overcome the current limitations of Newton potential phenomenology, involving generic quantum sources of gravity. These effects are derived using a field-basis formulation of linearised gravity, which is particularly suited for describing superposition of macroscopically distinct gravitational field configurations in the low energy regime. This formalism also offers a natural setting for exploring the gauge symmetries. In particular, I will discuss the construction of linearised quantum diffeomorphism transformations by extending the notion of quantum reference frames to quantum fields.

Probing quantum features of gravity in tabletop regime & reassessing the I.I.D. assumption in probability assignments

Based on projects with F. Giacomini (ETH), C. Rovelli,
C. Brukner and R. Simmons (IQOQI Vienna)

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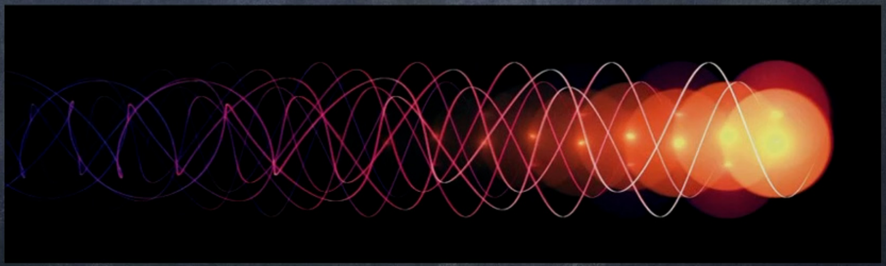
Perimeter QG Seminar May 30, 2025

- There are many candidate theories for quantum gravity (string theory, loop quantum gravity, causal set theory, asymptotic safety ….), but so far there is no rigorous argument **nor experimental evidence** yet to prove that gravity is indeed quantum.
- The traditional expectation: quantum gravitational effect could only show up at very high energy or short length scale - still far out of reach from the current technology.



[Image credit: Perimeter Institute]

- Recent years, the advancements on the low energy table-top experiments give us a lot of hope!

Recent advancements on the table-top experiments

- Fine measurement of classical gravitational field sourced by small mass: **10^-5 kg** (current status 10^-8 kg)

[Measurement of gravitational coupling between millimetre-sized masses, T. Westphal, H Hepach *et al.* ***Nature*** 2021]

- Quantum control of macroscopic quantum states in the lab: Using optical levitation combined with quantum ground-state cooling techniques, experimentalists can control the quantum trajectory of a 10^-18 kg nanoparticle (towards 10^-12 kg in a few years)

[Quantum control of a nanoparticle optically levitated in cryogenic free space , F. Tebbenjohanns, M. Mattana, *et al.* ***Nature*** 2021]
[Real-time optimal quantum control of mechanical motion at room temperature L. Magrini, P. Rosenzweig, *et al.* ***Nature*** 2021]

- Delocalised quantum source: 10^-16 kg optically levitated nanoparticle delocalised over several nanometers, on timescales of milliseconds.

[Fast Quantum Interference of a Nanoparticle via Optical Potential Control , L. Neumeier, M. Ciampini *et al.* ***PNAS*** 2024]

- Preparing massive quantum states in large superposition

[Quantum superposition of molecules beyond 25 kDa, Y.Fein, P. Geyer, et al. ***Nature Physics*** 2019],
[Quantum superposition at half meter scale, Y.Fein, P. Geyer, et al. ***Nature Physics*** 2019]

Hope from the recent experimental advancements

Fine measurement of classical gravitational field sourced by small mass: measuring 10^-8 kg this year.	⟷	Quantum control of macroscopic quantum states: towards 10^-12 kg in a few years.

@ Markus Aspelmyers lab in IQOQI-Vienna

- When the two regimes close, experimental advancement will reach a point that we could measure physical effect of gravitational field generated by quantum matter source - an observational window for the genuine low energy quantum gravity effect!

When did we experimental prove that EM field is quantum?

- The photoelectric effects 1905 — then it was generally believed that there is ``quanta of light"
- In 1916, Millikan measured the value of Planck's constant, from the photoelctric effect.
- 1960s Jaynes et al. developed a a semi-classical theory which could account for the effect observed so far with classical electromagnetism, with quantum effect contributed by matter.

[E.T. Jaynes and F.W. Cummings, Comparison of quantum and semiclassical radiation theories with application to the beam maser, Proceedings of the IEEE **51** (1963) 89.
M.D. Crisp and E. Jaynes, Radiative effects in semiclassical theory, Physical Review **179** (1969) 1253.]

- 1973 Clauser sent single photons through two interferometers, to test the violation of Cauchy-Schwarz inequality. This is considered the first test that really differentiated the classical and quantum field-theoretic predictions for the photoelectric effect.

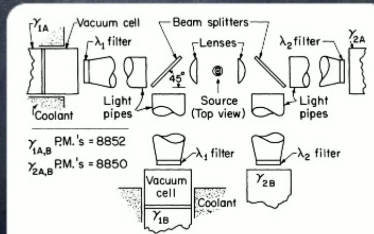


FIG. 2. Schematic diagram of our apparatus.

[J.F. Clauser, Experimental distinction between the quantum and classical field-theoretic predictions for the photoelectric effect, Phys. Rev. D **9** (1974) 853.]

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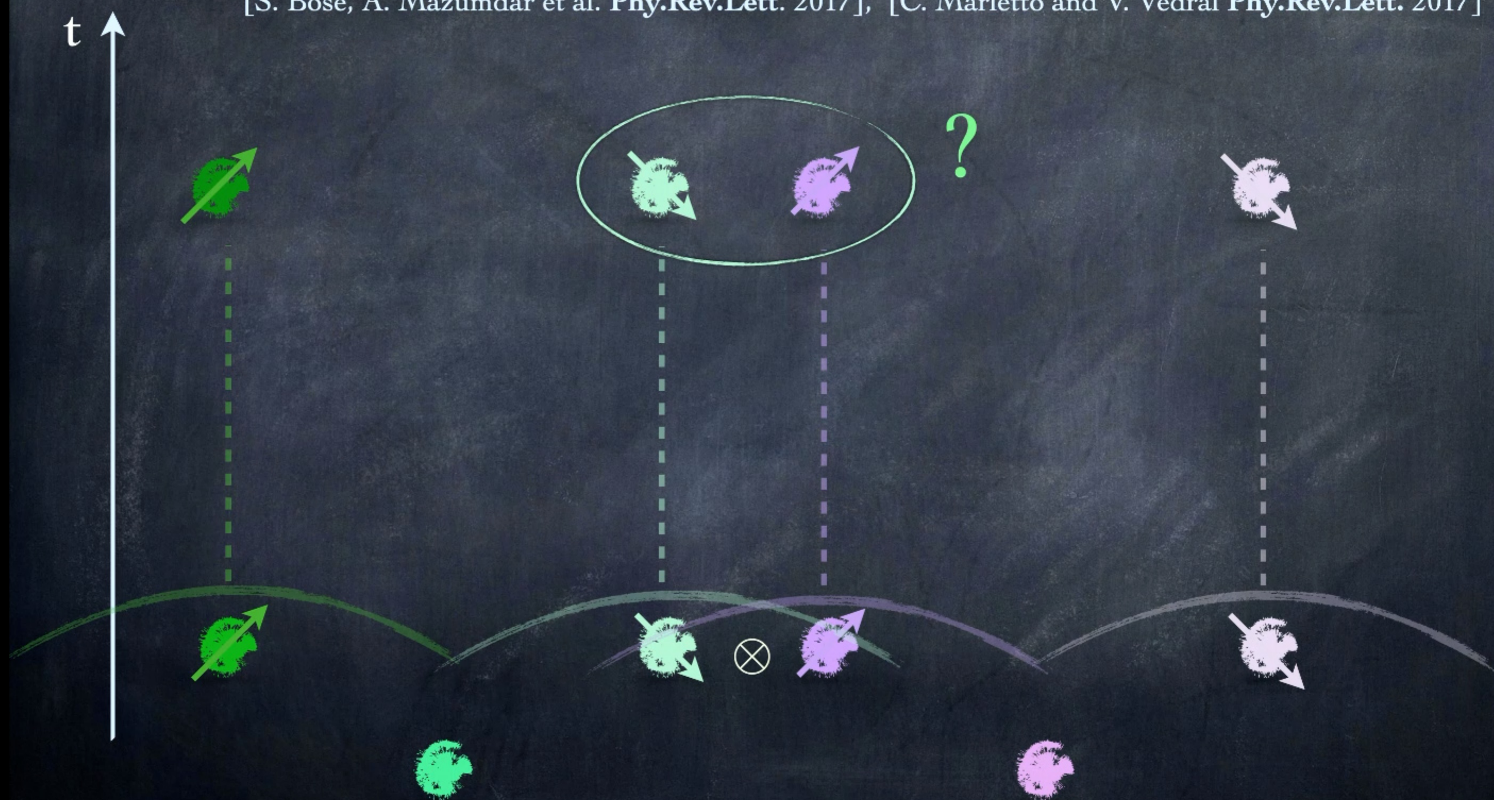
The Gravity-Induced-Entanglement experiment (BMV)

[S. Bose, A. Mazumdar et al. **Phy.Rev.Lett**. 2017], [C. Marletto and V. Vedral **Phy.Rev.Lett.** 2017]



10^-14 kg, 100 micrometers

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The Gravity-Induced-Entanglement experiment

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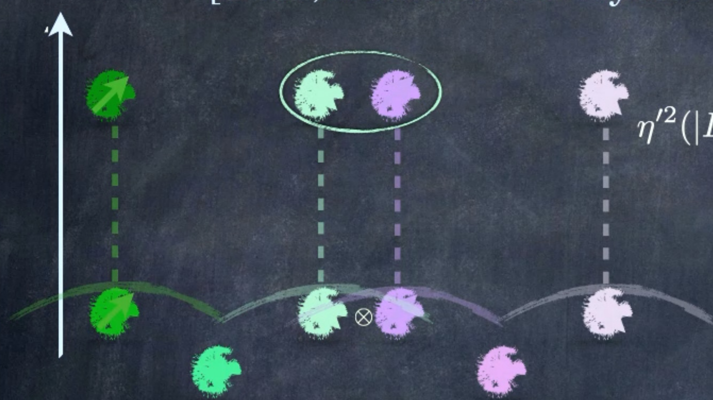

$$\eta'^2(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + e^{i\Delta\phi}|R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2)$$

$$\eta(|L\rangle_1 + |R\rangle_1) \otimes \eta(|L\rangle_2 + |R\rangle_2)$$

If there is detected entanglement

A key theorem in quantum information: entanglement between two systems cannot be created by Local Operations and Classical Communication (LOCC)

The gravitational field has to be nonclassical in nature.

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Subtle debates regarding the GIE: a spectrum of views

GIE only involves Newton potential, it has nothing to do with QG, it does not involve true quantum gravitational degrees of freedom.

GIE alone does not prove gravity is non-classical, we needs extra assumption.

GIE implies detection of macroscopic superposition of spacetimes.

GIE implies detection of virtual gravitons.

Quantum Newtonian field carries quantum information and mediates the entanglement.

Newtonian field entanglement implies gravitons-induced entanglement.

Many researchers contributed to the discussion in the last 7 years: Anastopoulos, Aspelmeyer, Barker, Belenchia, Bengyat, Bhatar, Blencowe, Bose, Brukner, Carney, Castro-Ruiz, Chen, Christodoulou, Cooper, Di Biagio, Galley, Geraci, Hackermüller, Howl, Hu, Huggett, Iyer, Kent, Kim, Krisnanda, Lami, Linneman, Liu, Mahesh, Marletto, Marshman, Martín-Martínez, Mazumdar, Milburn, Morley, Müller, Mummery, Naik, Pal, Paterek, Paternostro, Pedernales, Perche, Pitalúa-García, Plenio, Qvarfort, Rovelli, Schneider, Schut, Selby, Serafini, Sillanpää, Tam, Taylor, Toros, Ulbricht, Vedral, Wald, Yant etc.

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Debates regarding the GIE experiment

The subtleties of the GIE interpretation can be summarised as the following:

- An interaction given by Newtonian potential can effectively explain GIE, in the case that we don't have a mediator, LOCC cannot apply;

$$\hat{V}_N(\hat{x}_L - \hat{x}_R) = -G\frac{m_L m_R}{|\hat{x}_L - \hat{x}_R|}$$

- The only element of gravitational theory required in the GIE proposal is the Newton potential, which is compatible with a low-energy and non-relativistic limit of **classical** general relativity, but it does not probe the field nature of gravity.
- Then we need more input or joint experiments before claiming any quantumness of gravitational field.

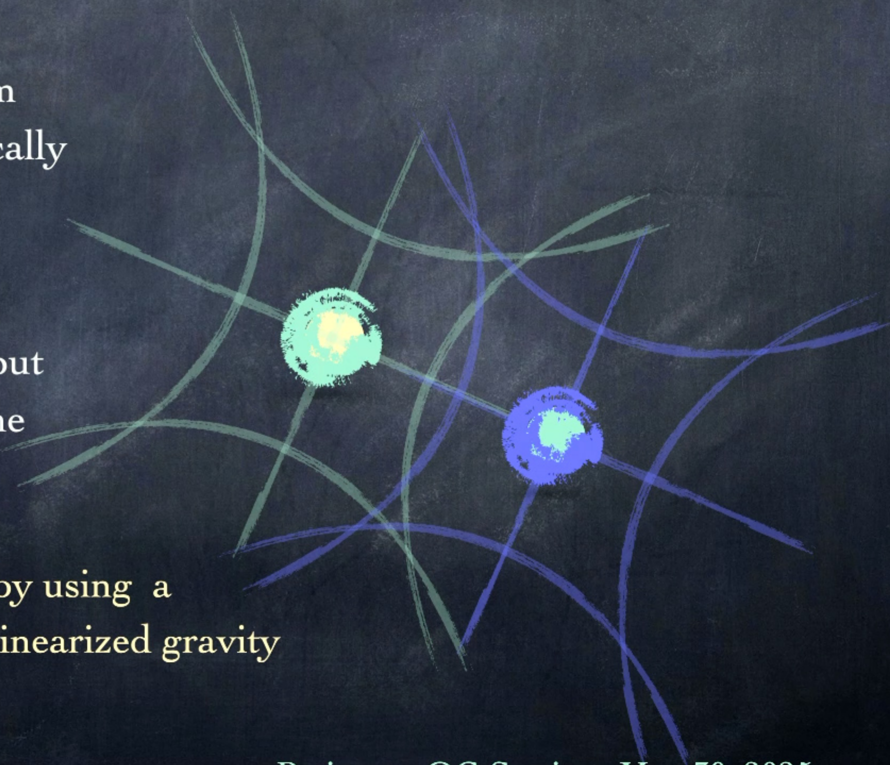
Can we provide stronger evidence for the quantum signature of gravity, that is beyond the predictions given by Newton potential?

Part 1: Questions

- What is a useful theoretical framework that is suitable for giving predictions and devising new proposals for the table-top experiments?

- How to describe the quantum superposition of macroscopically distinct gravitational fields?

- What quantum gravitational source could tell us more about the gravitational field than the semiclassical source?



I will answer those questions by using a quantum field formulation of linearized gravity in the field basis.

A theoretical framework for the table-top experiment

- What is the physical regime we are studying? Low energy quantum matter source, weak quantum gravitational field.

$\mapsto$ The linearised gravity is sufficient for this,

1. The quantum state of gravitational field can be well-defined from the first principle.
2. All UV-complete quantum gravity models have to agree with the physical predictions in this regime.

- No gauge fixing for the spacial diffeomorphisms, in order to understand the role of gauge symmetry in the quantum state.

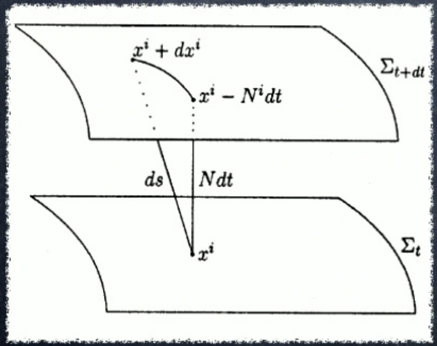
- Instead of the usual Fock basis in the standard QFT textbook, we choose to use the field basis: $\hat{h}_{ij}(x)|h_{ij}\rangle = h_{ij}(x)|h_{ij}\rangle$, because it is especially convenient to describe macroscopically distinct field configurations.

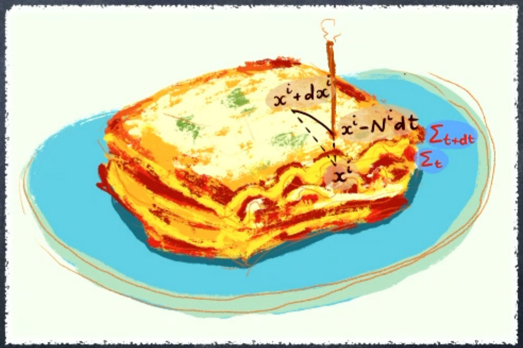
The Hamiltonian for linearised gravity

- We perform canonical quantisation of metric perturbation. The spacetime metric is cast in the 3+1 decomposition (known as ADM formulation)

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

$\kappa = 16\pi G/c^4$ $\qquad N \to 1 + \kappa n \qquad \gamma_{ij} = \delta_{ij} + \kappa h_{ij} \qquad N^i \to 0 + \kappa n^i$





Art credit: Nuriya Nurgalieva @ ETH

- We only fix the temporal part to bring out the constraints for Hamiltonian analysis:

n, n^i are fixed to be zero: $h_{0\mu} = 0$

The Hamiltonian for linearised gravity

- Linearising the full gravitational Hamiltonian, we obtain

Canonical variables: $\{h_{ij}(\mathbf{x}), \pi^{kl}(\mathbf{x}')\} = \delta^k_{(i}\delta^l_{j)}\delta^3(\mathbf{x}-\mathbf{x}')$

$$H_{G+M} = \kappa \int d^3x \; (\pi_{kl}\pi^{kl} - \pi^2/2) + \qquad (\kappa = 16\pi G/c^4)$$
$$+\frac{1}{4\kappa}\int d^3x \left(\partial_k h_{ij}\partial^k h^{ij} - \partial_i h \partial^i h - 2\partial^k h_{ik}(\partial_j h^{ij} - \partial^i h) - 4n_i \mathcal{G}^i - 4n\mathcal{C}\right).$$

- For **static** matter source: relevant information is given by $\hat{T}_{00}$ of matter field
- Four constraints from diffeomorphisms - the gauge symmetry of gravity

→ $\hat{\mathcal{C}}^i := \partial_j \hat{\pi}^{ij} = 0.$ Transversality: ensures spacial diffeormophisms

→ $\hat{\mathcal{C}} := -\partial_i \partial^i \hat{h}^T - \kappa \hat{T}_{00} = 0$ Gravitational "Gauss law"

- We solve the eigenstate of the Hamiltonian within the constraints surface:

$$\hat{\mathcal{G}}^i|\Psi_\rho\rangle = 0 \qquad \hat{\mathcal{C}}_\rho|\Psi_\rho\rangle = 0, \qquad \hat{H}|\Psi_\rho\rangle = \mathcal{E}_\rho|\Psi_\rho\rangle$$

The quantum state of gravitational field

LQC, F. Giacomini, C. Rovelli, Quantum 7, 958 (2023)

- The solution is expressed in the Schrödinger representation in the **field basis.**
- In the representation which diagonalises operator h_{ij} $\quad \Psi[h_{ij}] := \langle \Psi | h_{ij} \rangle$

$$\Psi_\rho[h_{ij}] = \eta\, \delta[h^T - \mathsf{h}^T_\rho] \exp\left\{ -\frac{1}{4\kappa\hbar} \int \frac{d^3k}{(2\pi)^3}\, |\vec{k}|\, h^T_{ij}(\vec{k}) h^{ij}_T(-\vec{k}) \right\}.$$

$h^T_{ij}(k) = P^k_i P^j_l h_{kl}(k)$

- In the π_{ij} representation $\Psi[\pi_{ij}] := \langle \Psi | \pi_{ij} \rangle$

$$\Psi_\rho[\pi_{ij}] = \eta \exp\left\{ -\frac{i}{2\hbar} \int \frac{d^3k}{(2\pi)^3} \pi_T(\vec{k}) \mathsf{h}^T_\rho(\vec{k}) - \frac{\kappa}{\hbar} \int \frac{d^3k}{(2\pi)^3} \frac{1}{|k|} \left(\pi^T_{ij}(\vec{k}) \pi^{ij}_T(-\vec{k}) - \frac{1}{2} \pi_T(\vec{k}) \pi_T(-\vec{k}) \right) \right\}.$$

"Gravitational Gauss Law" — Gaussian of the transverse modes

- New ground state as a displacement by the solution of the Gauss constraint:

$$\mathsf{h}^T_\rho(\vec{x}) = \frac{\kappa}{4\pi} \int d^3y \frac{\rho(\vec{y})}{|\vec{x} - \vec{y}|}$$

Vacuum state: coherent state

The Hamiltonian for linearised gravity

- Linearising the full gravitational Hamiltonian, we obtain

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The quantum state of gravitational field

LQC, F. Giacomini, C. Rovelli, Quantum 7, 958 (2023)

- The inner product is positive definite.

- The energy eigenvalue $\hat{H}|\Psi_\rho\rangle = \mathcal{E}_\rho|\Psi_\rho\rangle$

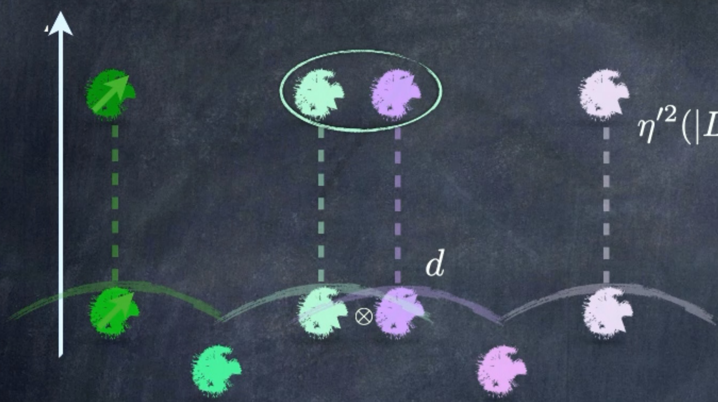
$$\mathcal{E}_\rho = \underbrace{\hbar \int \frac{dk^3}{(2\pi)^3}|\vec{k}|\delta(0)}_{\text{Vacuum energy,}} - \underbrace{\frac{1}{8\kappa}\int \frac{d^3k}{(2\pi)^3}\frac{\rho^2(\vec{k})}{|\vec{k}|^2}}_{\text{Gravitational self-energy}} = \mathcal{E}_{vac} - \frac{1}{8\kappa}\int d^3x\, d^3y\; \frac{\rho(\vec{x})\rho(\vec{y})}{|\vec{x}-\vec{y}|}$$

- New ground state as a displacement by the solution of the Gauss constraint:

$$\mathsf{h}^T_\rho(\vec{x}) = \frac{\kappa}{4\pi}\int d^3y \frac{\rho(\vec{y})}{|\vec{x}-\vec{y}|}$$

Vacuum state: coherent state

The limit to the BMV phase



$$\eta'^2(|L\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2 + e^{i\Delta\phi}|R\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2)$$

$$\Delta\phi_G = -\frac{Gm^2 t}{8\hbar d}$$

- Superposition of semiclassical localised matter source gives us the superposition of macroscopically distinct geometries in the linearised gravity regime:

$$\Psi_{x_n^i}[x_n, \pi_{ij}] = \sum_i C_i \prod_{n=1,2} \delta(x_n - x_n^i)\cdot\psi_{vac}\cdot \exp\left\{-\frac{i}{2\hbar}\int d^3x \pi_T(\vec{x}) \mathrm{h}^T_{x_n^i}(\vec{x})\right\}.$$

- Generating entanglement is a genuine quantum effect coming from the quantum Newtonian fields. Only considering the quantisation of radiative d.o.f freedom is not enough for the phenomena.

In this table-top experiment regime, can we provide stronger evidence that doesn't need graviton emission, and it is beyond the predictions given by the Newton potential?

Here we identify two physical effects:

- → The first one comes from the quantum delocalised sources interacting with quantum gravitational field.
- → The second effect stems from gravitational commutator.

LQC with F. Giacomini, arXiv:2402.10288

Semi-classical Localized Source

- Semi-classicality: $[\hat{x},\hat{p}]|\alpha_{sc}\rangle \approx 0.$ for any relevant operational procedure.
- Localizability: the resolution of the apparatus is larger than the width of the Gaussian of the source: $\hat{x}|\alpha_{sc}\rangle \approx x_{sc}|\alpha_{sc}\rangle$

- $\hat{T}_{00}$ and $\hat{x}$ are mutually diagnolizable: $\hat{T}_{00}|\alpha_{sc}\rangle = \rho(x_{sc})|\alpha_{sc}\rangle,$

- Superposing semiclassical localised sources: $|\psi\rangle_{sc} = \int d\mu(\alpha)\psi(x)|\alpha_x\rangle_{sc},$ the result gravitational state gives us the "superposition of semiclassical spacetimes" in the recent literature.

- The states of gravitational fields are orthogonal - macroscopically distinguishable. However, a tiny shift of the source makes the gravitational quantum state perfectly orthogonal.

The defect from such approximation:

$\langle\alpha_x|\alpha_{x+\epsilon}\rangle_S \neq 0$

But $\langle\Psi_x|\Psi_{x+\epsilon}\rangle_{S+G} = 0$

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Quantum Source for gravitational field

- For quantum source: $[\hat{x}, \hat{p}]|\alpha_s\rangle \neq 0.$ $\langle \Delta x \rangle_{\alpha_s}, \langle \Delta p \rangle_{\alpha_s}$ not negligible

- $\hat{T}_{00}$ and $\hat{x}$ are **not** mutually diagnolisable

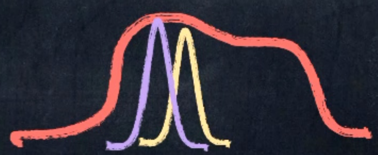
  $|\psi\rangle_s = \int d\mu(E) \psi_E |E\rangle_s$ eigenstates of $\hat{T}^S_{00}$ are in general non-local in x

- The scalar constraint: $\hat{\mathcal{C}}|\Psi\rangle_{G+S} := (\partial_i \partial^i \hat{h}^T + \kappa \hat{T}_{00})|\Psi\rangle_{G+S} = 0$

  ↓

  If the mass source is not an eigenstate of the Hamiltonian, the gravitational field must involve superposition of different metrics!

- The gravitational field of a delocalised quantum source is not given by a superposition of semiclassical localised one:



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Semiclassical mass source v.s. generic quantum source

Superposition of localised Gaussians

General quantum source

$\langle \Delta x \rangle_{\alpha_{cs}}, \langle \Delta p \rangle_{\alpha_{cs}} \ll$ experimental resolution

$[\hat{x}, \hat{p}]|\alpha_{sc}\rangle \approx 0.$

$\langle \Delta x \rangle_{\alpha_s}, \langle \Delta p \rangle_{\alpha_s}$ not negligible

$[\hat{x}, \hat{p}]|\alpha_s\rangle \neq 0.$

$$\hat{T}^{00}(x)|\phi_S\rangle = \sum_i c^2 \rho(\vec{x} - \vec{x}_i)|x_i\rangle \quad \neq \quad \hat{T}^{00}(x)|E\rangle_S = E(\vec{x}, t)|E\rangle_S$$

The entangling phase for generic quantum sources

- Now let us consider the scenario in which two static delocalised quantum sources A and B interact gravitationally:

- In the temporal gauge $h_{0\mu} = 0$, the interaction Hamiltonian is zero (at leading order of G)

$$\hat{H}_I^{(tot)} = -\frac{1}{2}\int d^3x\, \hat{h}_{\mu\nu}(\vec{x})(\hat{T}_A^{\mu\nu}(\vec{x}) + \hat{T}_B^{\mu\nu}(\vec{x})) = 0 \ \ !$$

The Gauss constraint fully encoded the interaction:

$$\hat{\mathcal{C}} := -\partial_i\partial^i\hat{h}^T - \kappa\hat{T}_{00}^A - \kappa\hat{T}_{00}^B = 0$$

- After solving the quantum state of gravitational field, for each branch of superposition, the eigenvalue of the gravitational Hamiltonian reads:

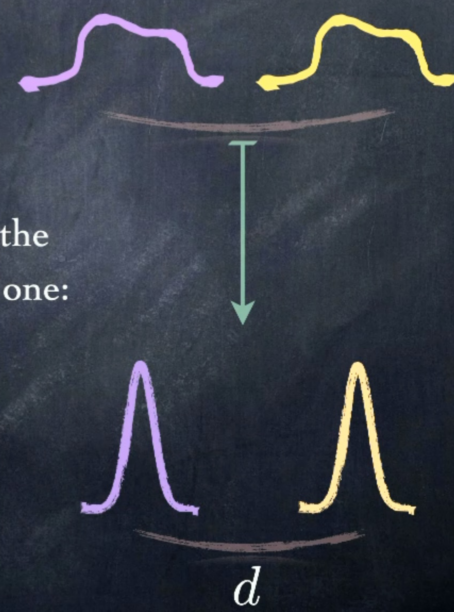
$$\mathcal{E}_{ABG} = \mathcal{E}_{vac} + \mathcal{E}_A + \mathcal{E}_B - \frac{\kappa}{4\pi}\int d^3x\, d^3y\, \frac{E_A(\vec{x})E_B(\vec{y})}{|\vec{x}-\vec{y}|}.$$

The entangling phase for generic quantum sources

- The entangling phase in the final state $\int d\mu(E_A)d\mu(E_B)e^{i\Delta\phi_G(E_A,E_B)}....|E\rangle_A|E\rangle_B|g_E\rangle_G$

in general, the phase depends on the details of how each energy eigenvalue distribute in space. Tuning the potential of the quantum source would help us to identify the signal.

$$\Delta\phi_G(E_A,E_B) = -\frac{\kappa t}{4\pi\hbar}\int d^3x d^3y \frac{E_A(\vec{y})E_B(\vec{x})}{|\vec{y}-\vec{x}|}.$$



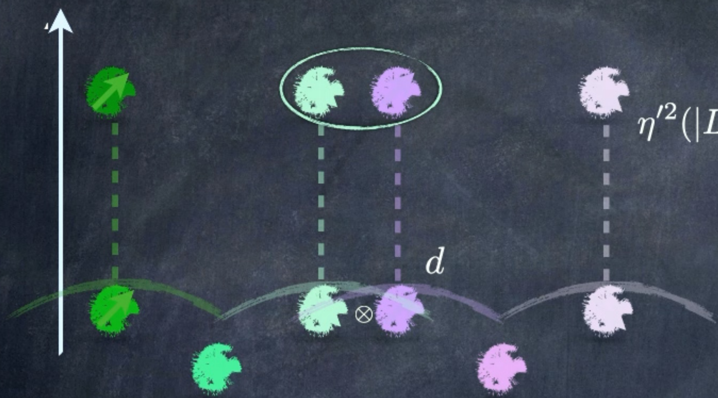
$$E_\alpha(\vec{x}) = c^2 m_\alpha \delta(\vec{x}-\vec{x}_\alpha),$$

When we restrict the quantum source to be the semiclassical localised one:

$$\Delta\phi_G = -\frac{G m_A m_B t}{\hbar d}$$

The phase predicted by the Newton potential.

The limit to the BMV phase



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- Superposition of semiclassical localised matter source gives us the superposition of macroscopically distinct geometries in the linearised gravity regime:

$$\Psi_{x_n^i}[x_n, \pi_{ij}] = \sum_i C_i \prod_{n=1,2} \delta(x_n - x_n^i)\cdot\psi_{vac}\cdot\exp\left\{-\frac{i}{2\hbar}\int d^3x\,\pi_T(\vec{x})\mathrm{h}^T_{x_n^i}(\vec{x})\right\}.$$

- Generating entanglement is a genuine quantum effect coming from the quantum Newtonian fields. Only considering the quantisation of radiative d.o.f freedom is not enough for the phenomena.

Remarks

- For general quantum source, the analytical form of the phase cannot be mimicked by most common models in which a quantum description of the gravitational field is not adopted:

  1) Interaction through Newton potential. 

  2) The Schrödinger-Newton equation

  3) When gravity is kept classical but couples to quantum matter, the time evolution is a stochastic open-system dynamics. (E.g. works by J.Oppenheim)

- The field aspect of gravity is probed with the delocalised quantum source. Observing such functional form of the entangling phase is a stronger evidence that gravity needs to have a quantum description.

Testing quantum commutator of the gravitational field

- A quantum probe initially prepared in a superposition of different momentum eigenstates. There is a gravitational field generated by a quantum source. The interaction is described by:

$$\hat{H}_I = -\frac{1}{2}\int d^3x \hat{h}^{ij}(x)\hat{T}^P_{ij}(x)$$

- Due to the gravitational commutator $[\hat{h}_{ij}(\vec{x}), \hat{\pi}^{kl}(\vec{x}')] = i\hbar\delta^k_{(i}\delta^l_{j)}\delta^3(\vec{x}-\vec{x}')$ the gravitational Hamiltonian and interacting Hamiltonian does not commute:

$$\frac{1}{\sqrt{2}}(|\gamma_{p_A}\rangle + |\gamma_{p_B}\rangle)$$

$$[\hat{H}_G, \hat{H}_I]|\Psi\rangle_{S+G} = i\hbar\kappa\int d^3x\left(\hat{\pi}^T_{ij}(\vec{x}) - \frac{1}{2}P_{ij}\hat{\pi}^T(\vec{x})\right)\hat{T}^{ij}_P(\vec{x})|\Psi\rangle_{S+G}.$$

Testing quantum commutator of the gravitational field

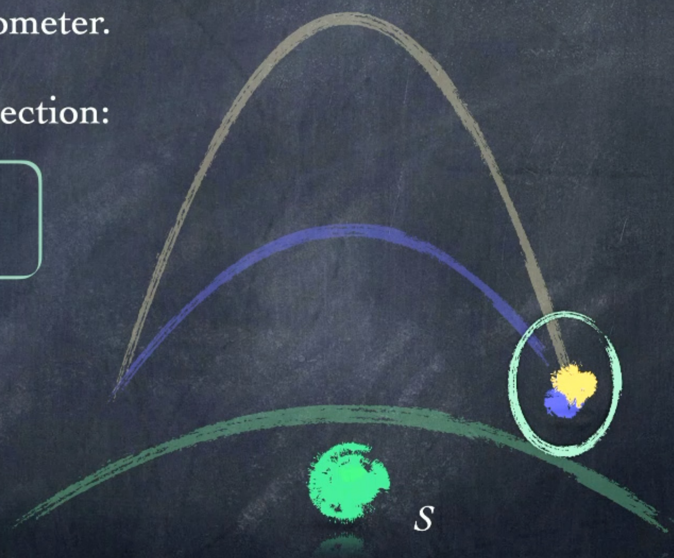
- The momenta were fine tuned in a way such that after the quantum probe flying through the gravitational field, the recombine at the same time t in order to test the relative phase through the interferometer.

- The relative phase has a quantum correction:

$$\Delta\varphi \supset \frac{\kappa t^3}{\hbar}(\alpha f_1[T_P^{ij}] + \alpha^2 f_2[T_P^{ij}])$$

α labels the order of the commutator.

- This phase scales nonlinear with time, same order of coupling as the entangling phase.



- Testing such quantum effect will be a more explicit indicator that the linearised gravity is a quantum field.

Summary I

- The limitation of Newton potential phenomenology: the non-local interacting potential is compatible with a low-energy and non-relativistic limit of classical general relativity, and it does not probe the field nature of gravity.
- For generic quantum source, the functional form of the phase cannot be reproduced with Newtonian potential nor with any limit of classical general relativity.
- When we consider a moving probe, the effect of quantum commutator of gravitational field shows up in the entangling phase as an higher order term. Probing these effects in the entangling phase would be a more explicit test of the gravitational field as a quantum mediator.
- If future experiments observed entanglement production, but incompatible with the analytical form of the two effects we predicted, it implies something new or unexpected for quantum gravity!

Recent efforts towards new protocols and effects:

New protocols and ideas for testing the quantum nature of gravity? How to provide stronger & more explicit theoretical evidences in this regime of table top experiments?

- Non-Gaussianity signature in Bose-Einstein condensate [R.Howl, V. Vedral, ***PRX Quantum 2021***]
- Quantum gravitational effects in quantum gas (ensemble of ultra-cold atoms) [S. Haine, ***New J. Phys.*** 2021]
- Stimulated and spontaneous single-graviton processes in massive quantum acoustic resonators (Weber bar) [Tobar, Manikandan, Beitel, Pikovski, ***Nature Comm.*** 2024]
- Thermodynamical observable: the heat capacity of Bose gas [T. Strasser, M Christodoulou, R.Howl, C.Brukner, arXiv ***2024***]

….and many more. Novel protocols will come from a closer dialogue between experimentalists and theorists!

- Further reads on the state of the art:
  Review: Massive quantum systems as interfaces of quantum mechanics and gravity,
  S. Bose, I. Fuentes, A. Geraci, et al. Rev. Mod. Phys. 97, 015003 2025

Part 2: Foundational questions of quantum-sourced gravitational fields

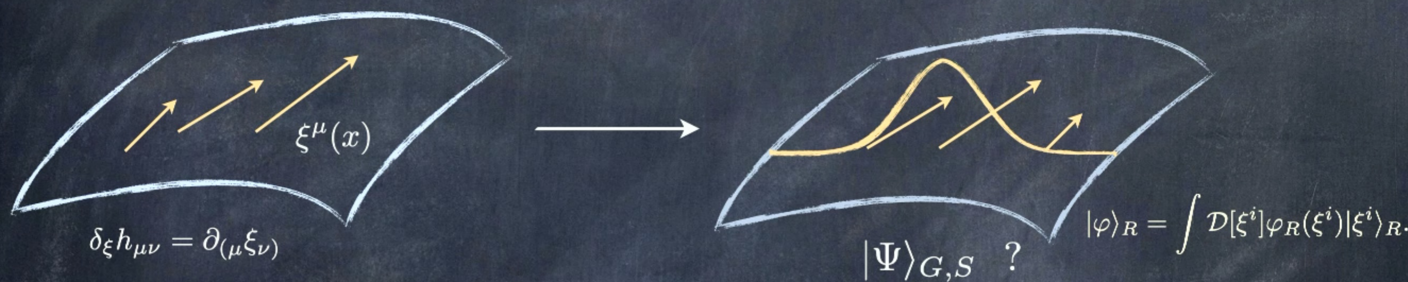
Lin-Qing Chen, IQOQI & Uni Vienna

Perimeter QG Seminar May 30, 2025

Quantum linearised diffeomorphisms

LQC, F. Giacomini, in progress.

- How does the notion of diffeomorphisms change in QG, compare to classical general relativity?
- A first step of our investigation: diffeomorphisms generated by quantum system



Classical linearised diffeomorphisms generated by a vector field $\xi^\mu(x)$

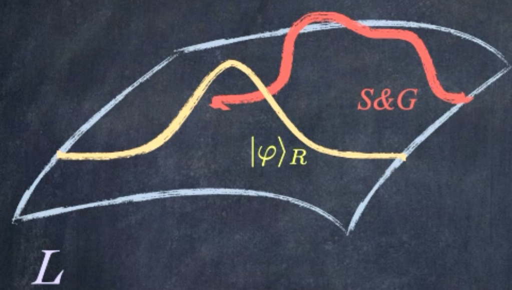
Can we make sense of diffeomorphisms generated by a quantum vector field?

- This can be formulated by a generalisation of the quantum reference frame transformation. [F. Giacomini, E. Castro-Ruiz, Č Brukner *Nat. Commun. 10, 494 (2019)*]

Quantum linearised diffeomorphisms

LQC, F. Giacomini, in progress.

- Quantum linearised diffeomorphisms transformation (spacial) generated by a quantum vector field: $|\varphi\rangle_R = \int \mathcal{D}[\xi^i]\varphi_R(\xi^i)|\xi^i\rangle_R.$



$$\hat{S}_{L\to R} := \mathcal{P}_{RL} \cdot e^{-i\int d\vec{x}\hat{T}^{S}_{0i}(\vec{x})\hat{\xi}^{i}_{R}(\vec{x})} \cdot e^{-i\int d\vec{x}\hat{\pi}^{G}_{ij}(\vec{x})\partial^{(i}\hat{\xi}^{j)}_{R}(\vec{x})}$$

- Remove the background: describing quantum state of gravitational field and its observables relative to quantum (coordinate) fields, which also back-react gravitationally.

By applying and generalising the perspective neutral approach of quantum reference frames:

[A.Vanrietvelde, P. Hoehn, F. Giacomini, and E. Castro-Ruiz. Quantum, 4:225, 2020, etc]

Relational approaches are necessary for background independence

Evidences:

- Relational observables, gravitationally-dressed observables

  [B.Dittrich, C.Rovelli, S.Giddings, W. Donnelly, P. Höhn, L. Freidel etc.]

- Type reductions of von Neumann algebra of QFT in subregions

  [V. Chandrasekaran, R. Longo, G. Penington, E. Witten, K. Jensen, J. Sorce, A. J. Speranza, J. De Vuyst, S. Eccles, P. A. Hoehn, J. Kirklin, C. J. Fewster, D. W. Janssen, L. D. Loveridge, K. Rejzner, J. Waldron etc.]

My projects:

- Events and localisation needs to be defined relative to certain structure; reconciling the difference between QI and GR, as well as the long lasting debate regarding the indefinite causal order protocols. [V. Vilasini, L-.Q. Chen, L. Ye, R. Renner, arXiv:2505.21797]

- Probability assignment in quantum measurement?

  In preparation with C. Brukner and R. Simmons

The i.i.d assumption in probability assignments

In preparation with C. Brukner and R. Simmons

- In quantum measurements, the probabilities infered from them are based on the assumption that the ensemble are prepared and measured in the same way – following an identical and independant distribution (i.i.d).

- It underlies the statistical foundation of quantum state tomography: "learning" what the quantum state is by performing many measurements on identically prepared systems:

$$\sum_i p_i \rho_i^{(1)} \otimes \rho_i^{(2)} \ldots \otimes \rho_i^{(N)}$$

  Born rule connects the state we assigned to, with the observable probability.

- Practically i.i.d fails when there is entanglement or correlation within ensemble, or there is memory effect between operations etc. Open field of research.
- As a basic assumption, it should not be any fundamental reason it cannot be fulfilled in certain idealisation.

- To make sense of any quantum states in the quantum gravity regime, we must, at least in principle, be able to define tomography (even if we cannot perform them).

The relative frequency operator

- The quantitative relationship between the i.i.d. and probability assignment is explicitly manifested in the relative frequency operator.

  [Finkelstein, D., 1963, "Logic of Quantum physics", Transactions of the NY Academy of Sciences, 25: 621- 637
  J. Hartle "Quantum Mechanics of Individual Systems", Am. Jour. Phys., 36, 704-712, (1967).]

- Although an individual measurement outcome is inherently unpredictable, the relative frequencies of an outcome become definite in the limit of an infinite ensemble of i.i.d systems:

$$|S\rangle^{\otimes N} := |S\rangle_1 \otimes |S\rangle_2 \otimes .... \otimes |S\rangle_N$$

- Supposingly we study observable A with eigenstates $\{|i\rangle\}$

$$\hat{F}_N^k(\hat{A}) := \sum_{i_1,i_2....i_N}^{d} \sum_{\alpha=1}^{N} \frac{\delta_{k,i_\alpha}}{N} \otimes_\alpha |i_\alpha\rangle\langle i_\alpha|$$

  The probability of observing outcome a_k becomes the eigenvalue of the joint state:

$$\lim_{N\to\infty} \hat{F}_N^k(\hat{A})|S\rangle^{\otimes N} = \lim_{N\to\infty} |\langle k|S\rangle_\alpha|^2 |S\rangle^{\otimes N}.$$

- In particular, for non-commuting observables $[\hat{A}, \hat{B}] \neq 0$, $\lim_{N\to\infty} [\hat{F}_N(\hat{A}), \hat{F}_N(\hat{B})]|S\rangle^{\otimes N} = 0$

The i.i.d assumption in quantum gravity

The first level — the entanglement from gauge constraints

- For N copies of identical static quantum sources $S^{(N)}$, we have the total wave function:

$$|\Psi\rangle_{S^{(N)},G} = \eta \int \mathcal{D}[\pi_{ij}] \prod_{\alpha}^{N} \int d\mu(E_\alpha)\phi_\alpha(E_\alpha) \exp\left(-\frac{i}{2\hbar}\int d^3x \pi_T(\vec{x}) \mathrm{h}^T_{E_\alpha}(\vec{x})\right) \Psi_{vac}[\pi_{ij}]|E\rangle_\alpha|\pi_{ij}\rangle_G$$

- The i.i.d assumption does not hold for this ensemble. One can check that:

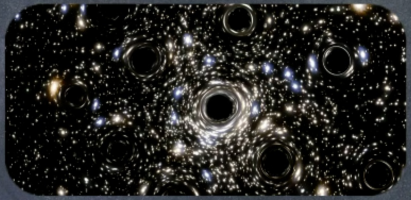
$$[\hat{T}_{00}, \hat{x}_S] \neq 0 \qquad \lim_{N\to\infty} [\hat{F}_N(\hat{T}_{00}), \hat{F}_N(\hat{x}_S)]|\Psi\rangle_{S^{(N)},G} \neq 0,$$

- Practical resolution one may propose: put sources far enough from each other, or:

That's how i.i.d ensemble of charged particles are prepared, despite of Gauss constraints.

The i.i.d assumption in quantum gravity

- However, this seems a more fundamental problem rather than practical problem that is unique for quantum gravity:



The second level — the lack of split property and back-reaction:

- For the ensemble preparation, the lack of split property states that one cannot assign quantum states independently in a subregion and its complement.
- The universal coupling of gravity ensures that any operation and measurement all have memory effect to the next in the sequence.
- Related issues have also been discussed as QG violates assumptions of Cencov's theorem — the Fisher information metric is the unique metric invariant under sufficient statistics. The authors resolution is to make the Born rule/information metric dynamical!

[Information Metrics and Possible Limitations of Local Information Objectivity in Quantum Gravity, P. Berglund, A. Geraci T. Hubsch, D. Mattingly, D. Minic arXiv:2501.19269v1]

The outline of one proposal of resolution

- We start with a local intertial frame R_0
- After preparing the first massive quantum state ρ_1 and performing measurement (relative to R_0), we preform reference frame transformation $S_{0\to 1}$ into local inertial frame R_1

……

- After preparing the α th massive quantum state ρ_α and performing measurement (relative to $R_{\alpha-1}$), we preform reference frame transformation into local inertial frame R_α
- The ensemble becomes $\rho^{(N)} := \rho_{1|R_0}\tilde{\otimes}\rho_{2|R_1}\tilde{\otimes}....\tilde{\otimes}\rho_{N|R_{N-1}}$

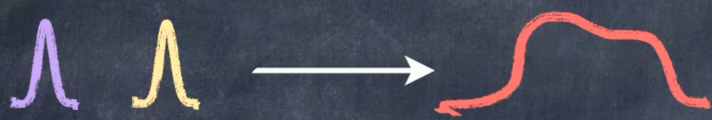
  The relative frequency operator: $\hat{F}_N^k := \sum_{i_1,i_2....i_N}^{d}\sum_{\alpha=1}^{N}\frac{\delta_{k,i_\alpha}}{N}\tilde{\otimes}_{\alpha|R_{\alpha-1}}\hat{\Pi}^i_{\alpha|R_{\alpha-1}}$

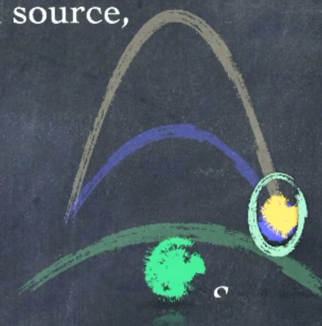
  in which $\hat{\Pi}^i_{\alpha|R_{\alpha-1}}$ is the measurement projector into the subspace of observable $\hat{A}_{\alpha|R_{\alpha-1}}$

- The correlation due to diffeomorphisms constraint and back-reaction shifts into the correlations among QRF; the existence of local inertial frame assures the reset of operations to i.i.d.

Summary III

- Solving quantum state of gravitational field with quantum source, identifying two quantum effects:

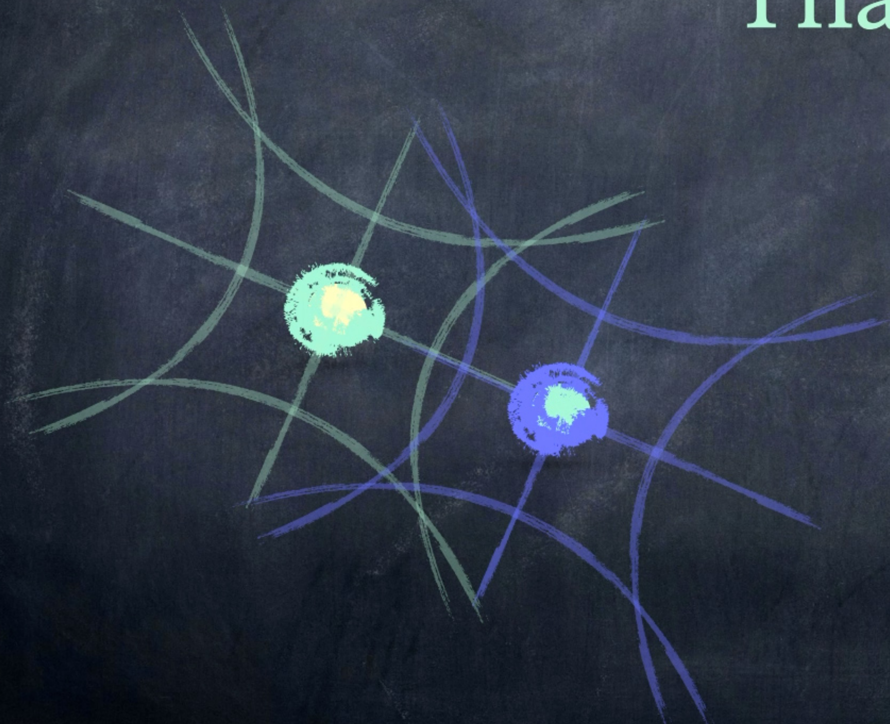




- Linearised quantum diffeomorphisms

- I.i.d assumption in quantum gravity: problem and one resolution.

Thank you!



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