**Title:** Toward Flat Space Holography via Interpolating Spacetimes

**Speakers:** Christian Ferko

**Collection/Series:** Quantum Fields and Strings

**Subject:** Quantum Fields and Strings

**Date:** May 20, 2025 - 2:00 PM

**URL:** https://pirsa.org/25050042

**Abstract:**

In this talk, I will describe recent work on holographic correspondences in spacetimes which interpolate from anti-de Sitter space in the deep bulk to asymptotic regions which share some properties with flat space. Examples include the linear dilaton throat in the F1-NS5 solution and the NCOS decoupling limit of the D1-D5 system. In both examples, null geodesics take infinite coordinate time to reach the boundary, the causal structure resembles that of Minkowski space, and we can sensibly study radiation near future null infinity. These spacetimes are good solutions of string theory and thus might be considered candidates for a top-down sort of celestial holography.

# Towards Flat Space Holography via Interpolating Spacetimes

(Based on work to appear, 2302.03041 with S. Sethi and C-K. Chang)

Christian Ferko

Northeastern University and IAIFI, the
Institute for Artificial Intelligence and Fundamental Interactions

Perimeter Quantum Fields and Strings Seminar
May 20, 2025

# Holography beyond $\mathrm{AdS}$.

While $\mathrm{AdS}/\mathrm{CFT}$ has been extensively studied, holography in spacetimes with non-$\mathrm{AdS}$ asymptotics is comparatively poorly understood.

In particular, the holographic dual to quantum gravity in spacetimes with a null boundary like Minkowski space – should such a dual exist – is believed to look quite different from field theories we are accustomed to.

It may be interesting to list some open questions about flat space holography which bear some connection to the subject of this talk.

# Simplest top-down example that isn't self-dual?

**56:** What is the simplest top-down model of celestial holography that isn't self-dual?

**HINT:** Is the asymptotically flat part of the bulk four or higher dimensional? If the bulk theory is a string theory, does the celestial dual admit a perturbative expansion as well as a 1/N expansion? Can it be obtained as a limit of AdS/CFT, perhaps in Mellin space?

***– David Skinner***

# Where are the black holes?

**24:** What is the celestial CFT state dual to black holes?

***– Andrea Puhm***

**63:** Is there a celestial dual of the Schwarzschild black hole, and how do we describe it?

***– Bin Zhu***

**50:** Can we add Black Holes to CCFT dynamically?

**HINT:** Compute Black Hole microstates.

***– Lucadam Ciamball***

**59:** What is the exact role of timelike infinity (i^+ and i^-) in flat space holography?

***– Marc Henneaux***

# Full dual vs. sector or limit?

**71:** Do we really expect flat space holography to give us a duality describing the full UV structure of the bulk physics or just the infrared part of it?

***– Francisco Javier Rojas Fernandez***

**72:** If so, like in AdS/CFT, could we find what is the expansion parameter that takes us away from field theory into more stringy effects?

***– Francisco Javier Rojas Fernandez***

**19:** Does one expect a holographic dual to flat space that is fully decoupled from gravity? In how many dimensions does it live and what are its locality properties?

**HINT:** String- theoretical realization of/insights into flat holography?

***– Monica Guica***

# One-slide summary of talk.

We present two holographic correspondences with the following properties:

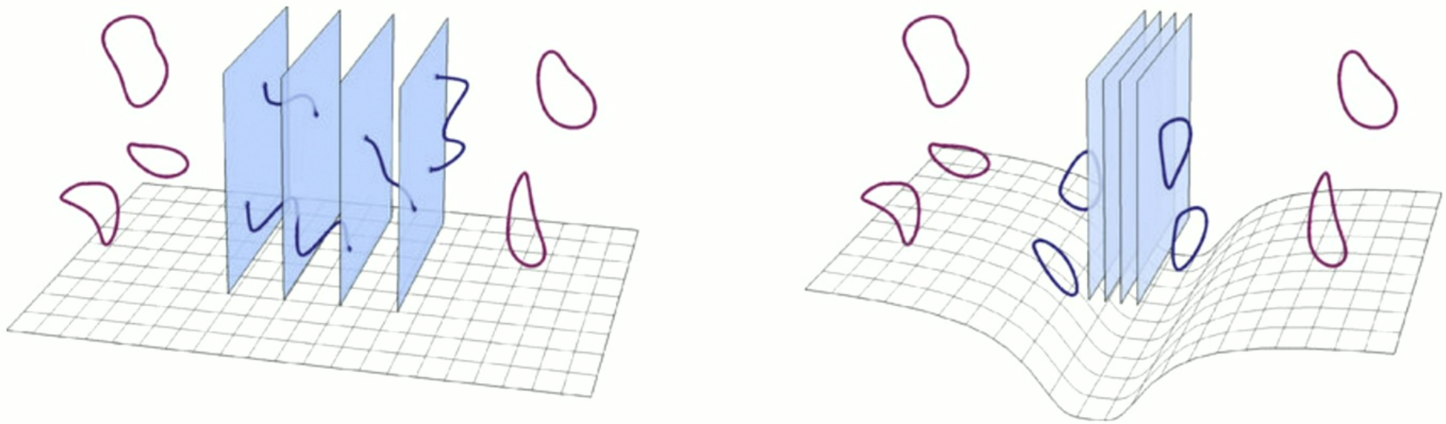
1. They are **top-down** constructions in string theory.
2. The bulk shares **some properties of flat space** like a notion of $\mathscr{I}^+$.
3. A **well-defined dual theory** is expected to exist. It is not a conformal field theory, but an irrelevant deformation of a CFT.
4. We can study **black hole solutions** and compare their energies and/or high-energy density with those of states in the dual.

This is not true flat-space holography.

But the above merits of these constructions might justify studying them as examples that teach us some lessons which could apply more generally.

# What are they? Partial decoupling limits of branes.

$\mathrm{AdS}$ holography was first discovered in the context of brane systems.



(Image credit: [Jahn, Eisert '21])

For example, a particular decoupling limit of the D1-D5 system yields an $\mathrm{AdS}_3 \times S^3 \times T^4$ background which is dual to a $\mathrm{CFT}_2$.

We adopt the same philosophy and engineer "interpolating" spacetimes using *partial decoupling limits* of brane systems, e.g. F1-NS5 and D1-D5.

# Roadmap.

**Goal**: describe two top-down constructions in which quantum gravity is expected to admit a holographic dual, which feature a notion of $\mathscr{I}^+$, and where one can study some aspects of black hole solutions.

The plan for the rest of the talk is as follows:

- ☑ Part 1: Introduction and context.
- ☐ Part 2: Asymptotically linear dilaton.
- ☐ Part 3: Asymptotically critical $B$-field.
- ☐ Part 4: Summary and future directions.

# Part 2: Asymptotically linear dilaton.

# One can do more with gravity plus a scalar.

Our first example is related to *linear dilaton* spacetimes, which have a scalar field $\Phi$ that varies linearly with an appropriate coordinate.

Such spacetimes are, in a sense, "just between" $\mathrm{AdS}$ and Minkowski. The entropy of large black holes in each case scales as:

$$S_{\mathrm{BH}}\Big|_{E\to\infty} \sim E^p \,, \qquad p = \begin{cases} \frac{d-2}{d-1} & \mathrm{AdS}_d \\ \frac{d-2}{d-2} = 1 & \mathrm{LD}_d \\ \frac{d-2}{d-3} & \mathrm{Mink}_d \end{cases} \,.$$

The conformal structure of $\mathrm{LD}_d$ is a null diamond like $\mathrm{Mink}_d$.

It has long been believed that the $\mathrm{LD}_7$ spacetime is holographically dual to $6d$ little string theory (LST). More recently, a $2d$ compactification of LST has been proposed to be described by a "single-trace $T\overline{T}$ deformation."

# Constructing asymptotically linear dilaton.

The starting point for our first interpolating spacetime is the F1-NS5 solution of type IIB supergravity, whose action is

$$S_{\rm IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int d^{10}x \, \sqrt{-g} \, e^{-2\Phi} \left( R + 4 \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \cdots \right) .$$

We initially study solutions of the form $\mathbb{R}_t \times S^1 \times \mathbb{R}^4 \times T^4$ where we

1. take $m_1$ fundamental strings which wrap the $S^1$, and
2. wrap $m_5$ NS5-branes on a $T^4$ and the $S^1$ of the strings.

The string-frame metric for these solutions takes the form

$$ds^2 = \frac{-dt^2 + dx_5^2}{f_1} + f_5 \left( dx_1^2 + \cdots + dx_4^2 \right) + \left( dx_6^2 + \cdots dx_9^2 \right) ,$$

where $f_1 = 1 + \frac{r_1^2}{r^2}$ and $f_5 = 1 + \frac{r_5^2}{r^2}$ are functions of the radial coordinate $r$ built from the transverse directions, $r^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$.

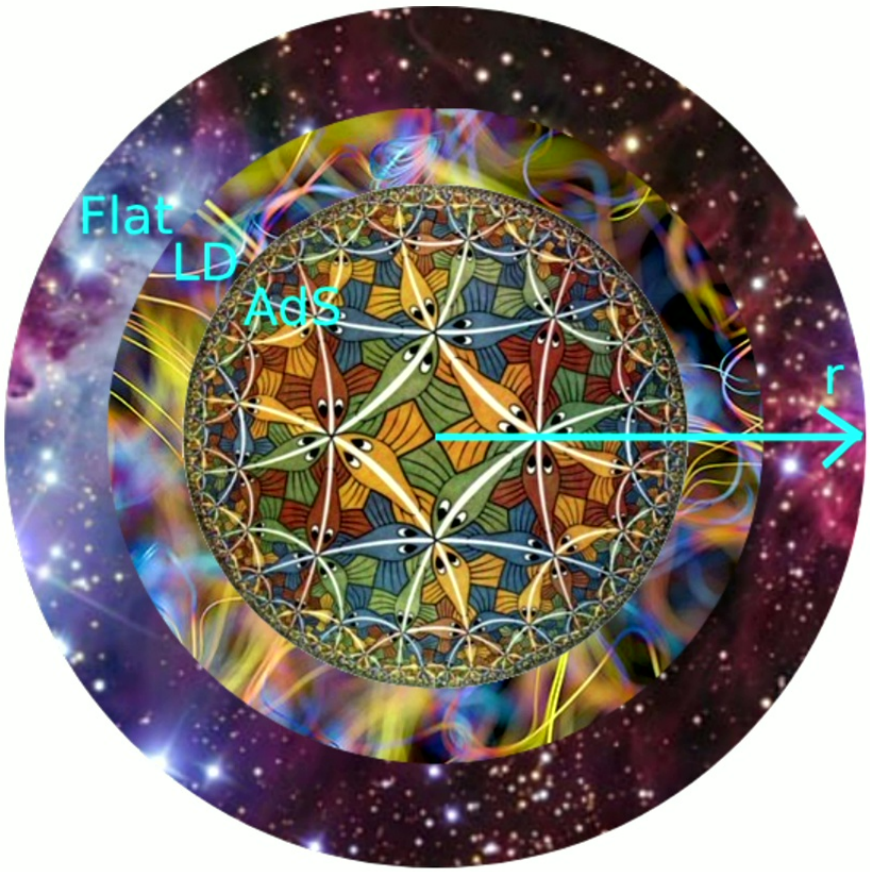
# Three regions.

The functions $f_1$ and $f_5$ depend on two dimensionful lengths $r_1$, $r_5$, which scale as $r_1^2 \sim m_1$ and $r_5^2 \sim m_5$. There are three qualitative regions:

1. For $r \gg r_1, r_5$, we are far from the strings and fivebranes. The spacetime looks like flat $\mathbb{R}^{1,4} \times S^1 \times T^4$.
2. For intermediate values of $r$, the solution looks like $\mathcal{M}_3 \times S^3 \times T^4$, where $\mathcal{M}_3$ is a linear dilaton spacetime.
3. For $r \ll r_1, r_5$, the space reduces to $\mathrm{AdS}_3 \times S^3 \times T^4$.

We can also "heat up" the solutions, putting a BTZ black hole of mass $M$ and spin $J$ in the deep bulk $\mathrm{AdS}_3$ region.

# Cartoon of undecoupled spacetime.



# Peeling the onion.

One can sequentially decouple, first stripping off the flat region to get an asymptotically linear dilaton (ALD) spacetime, and then peeling off that region to get $\mathrm{AdS}_3 \times S^3 \times T^4$. Our interest is in *partly* decoupled ALD.

To take this limit, define $r = g_s \hat{r}$ and take $g_s \to 0$, giving

$$\widehat{ds}^2 = \frac{-dt^2 + dx_5^2}{f_1} + \frac{r_5^2}{\hat{r}^2} d\hat{r}^2 + r_5^2 d\Omega_3^2 + ds_{T^4}^2 \, , \qquad e^{-2\Phi} = \frac{\hat{r}^2}{r_5^2} \left( 1 + \frac{\hat{r}_1^2}{\hat{r}^2} \right) \, .$$

This amounts to "dropping the 1" in $f_5 = 1 + \frac{r_5^2}{r^2}$, replacing $f_5$ with $\frac{r_5^2}{r^2}$, but retaining the full function $f_1$ which now takes the form $f_1 = 1 + \frac{\hat{r}_1^2}{\hat{r}^2}$.

The $S^3$ and $T^4$ have constant volume, so we dimensionally reduce to get a $3d$ solution with coordinates $(t, x_5, \hat{r})$. This is our focus in what follows.

# Firing a laser.

Converting from string to Einstein frame and dropping hats, the metric is

$$ds_E^2 = \frac{r^2}{\left(r^2 + r_1^2\right)^{3/4} r_5^{1/2}} \left(-dt^2 + dx_5^2\right) + \frac{\left(r^2 + r_1^2\right)^{1/4} r_5^{3/2}}{r^2} dr^2 \, .$$

Suppose we consider null geodesics fired radially outwards, which obey

$$ds_E^2 = -\frac{r^2}{\left(r^2 + r_1^2\right)^{3/4} r_5^{1/2}} dt^2 + \frac{\left(r^2 + r_1^2\right)^{1/4} r_5^{3/2}}{r^2} dr^2 = 0 \, ,$$

or

$$\frac{dt}{dr} = \frac{\sqrt{r^2 + r_1^2} r_5}{r^2} \, .$$

We wish to integrate this from $r_0$ to some $r_{\text{max}} \to \infty$ to see how long null geodesics take to go outwards to the boundary.

# Modified causal structure.

**Case 1**. If we decouple the ALD region, leaving us with $\mathrm{AdS}_3$, we replace $\sqrt{r^2 + r_1^2}$ by $r_1$, and

$$\Delta t = \int_{r_0}^{r_{\mathrm{max}}} dr \, \frac{r_1 r_5}{r^2} = r_1 r_5 \left( \frac{1}{r_0} - \frac{1}{r_{\mathrm{max}}} \right) ,$$

which is finite as $r_{\mathrm{max}} \to \infty$, as usual.

**Case 2**. Keeping the ALD region, we compute

$$\Delta t = \int_{r_0}^{r_{\mathrm{max}}} dr \, \frac{\sqrt{r^2 + r_1^2} r_5}{r^2} \sim \log \left( \frac{r_{\mathrm{max}}}{r_1} \right) + \cdots ,$$

where $\cdots$ are finite terms, sub-leading at large $r_{\mathrm{max}}$. Now $\Delta t$ diverges logarithmically, so the laser takes infinite time to reach the boundary.

# Adding black holes.

Adding a BTZ black hole of mass $M$ and spin $J = 0$ in the bulk gives

$$ds^2 = -\frac{1 - \frac{m}{r^2}}{\lambda + \frac{r_1^2}{r^2}}\, dt^2 + \frac{dx_5^2}{\lambda + \frac{r_1^2}{r^2}} + \frac{r_5^2}{r^2 - m}\, dr^2 + r_5^2\, d\Omega_3^2 + ds_{T^4}^2 \,,$$

$$e^{2\Phi} = \frac{r_5^2 \sqrt{1 + \frac{\lambda m}{r_1^2}}}{r_1^2 + \lambda r^2} \,, \qquad m = \frac{8 M G_3 r_1^2 r_5^2}{R^2} \,.$$

The three-form is fixed to be $H_3 = \frac{2m_5}{\sqrt{\alpha'}} \epsilon_{S^3} + \frac{32 m_1 \pi^4 \alpha'^3 g_s^2}{V_4 r_5^3} e^{2\Phi} \epsilon_3^{\mathcal{M}_3}$ which importantly satisfies appropriate flux quantization conditions.

A new parameter $\lambda$ has appeared which was absent before and which affects the linear dilaton slope. Taking $\lambda \to 0$ gives constant dilaton.

# Matching mass and energy.

We compute the mass of these spacetimes using the **covariant phase space formalism** of gravitational charges [Iyer, Wald '94] [Wald, Zoupas '99],

$$Q_\xi = \frac{R^2}{4 r_5^2 \lambda} \left( \sqrt{1 + \frac{8 \lambda M G_3 r_5^2}{R^2}} - 1 \right) .$$

The result matches the expectation for a single-trace $T\overline{T}$-deformed CFT,

$$E_n(\lambda) = \frac{R}{2\lambda} \left( \sqrt{1 + \frac{4 \lambda E_n(0)}{R}} - 1 \right) ,$$

up to identification of parameters.

Thus the masses of black hole solutions match the energies of states in the expected dual. The high-energy density of states is Hagedorn in both.

This concludes our first example of a top-down holographic correspondence which (i) has the causal structure of Minkowski, and (ii) permits a comparison between bulk black holes and boundary states.

# Part 3: Asymptotically critical $B$-field.

# A different interpolating spacetime.

Our ALD example is a spacetime which is expected to be holographic and which has a causal structure like flat space. But it has the disadvantage that we lack a *complete* definition of the dual deformed CFT.*

In the next example we *do* have a good definition of the dual, as a non-commutative open string theory (NCOS), where one can compute.

The starting point for this case is the extremal D1-D5 solution, with metric

$$ds^2 = \frac{1}{\sqrt{f_1 f_5}}\left(-dt^2 + dx_5^2\right) + \sqrt{f_1 f_5}\left(dr^2 + r^2\, d\Omega_3^2\right) + \sqrt{\frac{f_1}{f_5}} ds_{T^4}^2 \,,$$

where there are $\hat{n}_1$ D1-strings wrapping an $S^1$ and $\hat{n}_5$ D5-branes wrapping the $T^4$ and $S^1$, and again $f_1 = 1 + \frac{r_1^2}{r^2}$, $f_5 = 1 + \frac{r_5^2}{r^2}$. This is the S-dual of the previous starting point, but we will take a different decoupling limit.

*Except at $m_5 = 1$ where we can define the deformation of the symmetric product.

# Steps to generate the solutions.

Beginning from the D1-D5 system, we then do the following:

1. Perform a TsT transformation – a T-duality, "shift" diffeomorphism, and another T-duality – to activate a $B_2$-field parameterized by $\alpha$.
2. Impose flux quantization on the resulting solution, which is needed to be a good solution of string theory (as opposed to just supergravity).
3. Take an NCOS decoupling limit, defining

$$r = \tilde{r}\ell_s \,, \quad g = \tilde{g}\frac{b}{\alpha'} \,, \quad \cosh(\alpha) = \frac{b}{\alpha'} \,, \quad \tilde{x}^i = \frac{\ell_s}{b}x^i \quad i = 0, 5 \,,$$

and sending $\ell_s \to 0$ holding fixed $(\tilde{g}, b, \tilde{x}^i)$, where $\ell_s^2 = \alpha'$.

The resulting solutions have all fluxes $H_3$, $F_1$, $F_3$, $F_5$ turned on, in addition to the metric $g_{\mu\nu}$ and dilaton $\Phi$. We will focus mostly on the metric.

# The decoupled NCOS metric.

When the dust settles, we find a metric of the form

$$\frac{ds^2}{\alpha'} = \frac{\sqrt{f_1 f_5}}{f_1 f_5 - 1}\left(-d\tilde{t}^2 + d\tilde{x}_5^2\right) + \sqrt{f_1 f_5}\left(d\tilde{r}^2 + \tilde{r}^2\, d\Omega_3^2\right) + \sqrt{\frac{f_1}{f_5}}\sqrt{\widetilde{V}_4}\, \widetilde{ds}^2_{T^4} \,.$$

As before, the $3d$ part of this metric interpolates from $\mathrm{AdS}_3$ near $\tilde{r} = 0$ to different asymptotics at large $\tilde{r}$, and the coordinate time for a null geodesic to travel from a starting $\tilde{r}_0$ up to some $\tilde{r}_{\text{max}} \to \infty$ is

$$\Delta \tilde{t} = \int_{\tilde{r}_0}^{\tilde{r}_{\text{max}}} d\tilde{r}\, \frac{d\tilde{t}}{d\tilde{r}} \sim \log\left(\frac{\tilde{r}_{\text{max}}}{\tilde{r}_0}\right) + \cdots \,,$$

which diverges at large $\tilde{r}_{\text{max}}$. Again, radiation can escape from the bulk!

Even if we truncate to the $3d$ space parameterized by $(\tilde{t}, \tilde{x}_5, \tilde{r})$, this space is not asymptotically flat – the Ricci tensor has components which go to constants at large $\tilde{r}$. However, the Ricci scalar falls off like $\frac{1}{\tilde{r}^2}$.

# Bondi-like coordinates.

To study radiation in this spacetime, it is convenient to use variables similar to Bondi coordinates in asymptotically flat space.

The full expression is somewhat ugly,

$$u = t + \frac{(n_1 + n_5 \tilde{V}_4)}{m_1 \tilde{r} \sqrt{\tilde{V}_4}} \Bigg( \sqrt{n_1 n_5 + m_1 \tilde{r}^2} + \tilde{r} \sqrt{m_1} \log \left( \sqrt{n_1 n_5 + m_1 \tilde{r}^2} - \sqrt{m_1} \tilde{r} \right) \Bigg) ,$$

where $n_1$, $n_5, m_1$ are integers related to the starting $\hat{n}_1$, $\hat{n}_5$, and $B$-field.

But at leading order at large $\tilde{r}$ this takes the form

$$u = t - \sqrt{\frac{A_2}{A_1}} \log (\tilde{r}) ,$$

in terms of some constants $A_1$ and $A_2$.

# Scalar radiation near $\mathscr{I}^+$.

We make one more change of coordinates, trading $\tilde{r}$ for $\rho = \tilde{r}^{3/4}$, and find that the leading metric at large $\rho$ is

$$ds^2_{(0)} = \rho^2 \left(-A_1\, du^2 + A_1\, dx_5^2 + A_2\, d\Omega_3^2\right) - \frac{8}{3}\sqrt{A_1 A_2}\rho\, du\, d\rho\,,$$

with no $d\rho^2$ term since $\rho$ is now a null coordinate. The torus part of the metric is sub-leading at this order in a large-$\rho$ expansion.

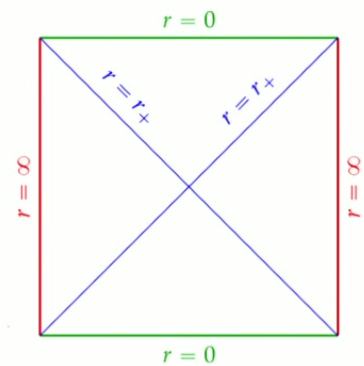
The leading part of the wave equation for a massless scalar $\chi$ is

$$\begin{aligned}\Box_{(0)}\chi^{(0)} = \Bigg[\sqrt{A_1 A_2}\left(32\frac{\partial}{\partial u} + 24\rho\frac{\partial^2}{\partial u\,\partial\rho}\right) - A_1\rho\left(33\frac{\partial}{\partial\rho} + 9\rho\frac{\partial^2}{\partial\rho^2}\right) \\ - 16A_2\frac{\partial^2}{\partial\tilde{x}_5^2} - 16A_1\,\Delta_{S^3}\Bigg]\chi^{(0)} = 0\,.\end{aligned}$$

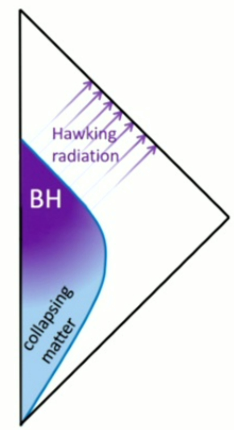
One can then study waves near $\mathscr{I}^+$, developing the leading & subleading solutions for $\chi$ at large $\rho$, as in flat space using Bondi coordinates.

# Adding a mass above extremality.

I have focused on the *extremal*, or zero-mass solutions. Like the ALD case, one can also consider *non-extremal* solutions, where one has turned on a non-zero temperature (or added a black hole in the bulk).




vs




In AdS, an evaporating black hole eventually comes to equilibrium with radiation. But in these interpolating spacetimes, Hawking radiation takes infinite time to reach the boundary and a black hole can evaporate.

One interesting future direction is to understand this more deeply.

# Part 4: Summary and future directions.

# Summary.

We have studied two classes of non-asymptotically $\mathrm{AdS}$ spacetimes which are nonetheless expected to have a holographic dual. In both cases, the causal structure is more similar to that of flat space.

1. Asymptotically linear dilaton spacetimes, a decoupling limit of F1-NS5, are believed to be dual to a single-trace $T\overline{T}$-deformed CFT.
2. Asymptotically critical B-field spacetimes, a limit of D1-D5, are expected to be dual to a non-commutative open string theory.

Both examples interpolate from an $\mathrm{AdS}$ bulk (with BTZ black holes) to an asymptotic region where null geodesics takes infinite coordinate time to reach the boundary and whose conformal structure is a null diamond.

One can therefore study radiation near $\mathscr{I}^+$ in these backgrounds.

## Future directions.

We have only initiated the study of interpolating spacetimes as toy models for flat space holography. There is still a lot to be done:

1. work out the precise dictionary between bulk observables near $\mathscr{I}^+$ and quantities in the dual theories;
2. understand the relationship, if any, between observables at $\mathscr{I}^+$ and near spatial infinity, perhaps via some sort of matching conditions;
3. identify other interpolating spacetimes with a $\mathscr{I}^+$, perhaps those in the 22-parameter family of [Georgescu, Guica, Kovensky '24].

Further work on this general class of holographic correspondences could perhaps teach us new lessons about genuine flat-space holography.

**Thank you for your attention!**